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Identification of Johnson-Cook Equation Constants using Finite Element Method

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The objective of this work is to develop an identification technique of the Johnson-Cook equation constants for copper Cu-ETP samples. In this paper we describe a method of constant identification using the Taylor impact test and finite element analysis. Nowadays the most popular method of constant identification method is the split Hopkinson pressure bar technique. This method is quite easy but needs very expensive laboratory equipment. To implement this method we have to make a lot of different tests in order to have enough information about dynamic properties. We decided to prepare an identification process algorithm using the Taylor impact test as a basic type of experimental and numerical simulation.

Key words: Johnson-Cook material model, finite element analysis, identification and optimization procedures.

1. Introduction

The Taylor impact test is a one of the oldest of the experimental methods from which we can measure some materials properties. This kind of experiment made its beginning during the Second World War [1]. This method of experiment was used to measure dynamic yield stress. It was important at that time because weapon constructors needed more information about dynamic material properties in order to design better protection barriers.

For many years this kind of experiment was used only for that propose. When the new era of computer techniques started it opened a new door for the Taylor impact test. At that point in time this experiment was not only used to measure dynamic yield stress. Computer techniques and special machines for measurement, high-speed cameras for example, have found a new use for the Taylor impact test. Those new techniques of measurement were used to calculate the propagation of plastic and elastic wave velocities. Information about those parameters is very important if we want to describe the dynamic behavior of materials. The importance of these parameters grows when computer techniques are used to make simulations of material deformation under dynamic loads. Explicit finite element method was used during the identification process. Researchers using computer simulations can make many different experiments but there is a one big problem. To make these numerical experiments we need information about the constitutive equation which can be used to describe the dynamic behavior of materials in the plastic range.

In this paper we present how the results of laboratory experiments were connected with numerical simulation. In this connection between results of laboratory experiments and numerical simulation is a method which can be used for calculating the constants of the constitutive equation.

2. Taylor impact test

The Taylor impact test is one of a few methods that can be used to calculate dynamic parameters. This experimental method is connected with dynamic deformation of cylindrical samples on a rigid wall (Fig. 1).

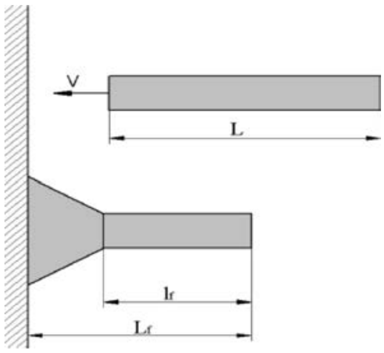

Fig. 1. Illustration of the idea of the Taylor impact test [1].

The result of laboratory experiments consists of a plastically deformed sample. The deformation is at one end of the sample only. Information about the size of this deformation is necessary for calculation of the dynamic yield stress. The calculation can be made using the simple equation [1]:

$$Y = \frac{\frac{1}{2}\rho V^2 \left(1 - \frac{l_f}{L}\right)}{\left(1 - \frac{L_f}{L}\right) \ln\left(\frac{L}{l_f}\right)}. \tag{2.1}$$

To calculate the dynamic yield stress we only need information about the impact velocity V, the length of the undeformed part of sample l_f, and L, L_f – the initial and final length of the sample, respectively.

Information about dynamic yield stress is insufficient if we want to make a computer simulation, but this knowledge is sufficient when we classify the material. During the numerical simulation, we need more than just the dynamic yield stress. All information necessary for analysis with finite element method we have from the constitutive equation.

3. Johnson-Cook model

Johnson-Cook equation is one of the most popular of the material models that can be used for calculation of the relationship during deformation with strain rate order of 10^3 s^{-1}, or higher. Equation (3.1) illustrates a mathematical interpretation of the Johnson-Cook material model [1, 4]. That is:

$$\sigma = [A + B\varepsilon^n] \cdot \left[1 + C \ln\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right)\right] \cdot \left[1 - \left(\frac{T - T_0}{T_m - T_0}\right)^m\right]. \tag{3.1}$$

The model presented by Johnson and Cook in the nineteen-eighties is very simple and is connected by three different parts. The first part describes the relationship between strain and stress; the second part of the equation presents the relationship between strain rate and stress; and the last part of the Johnson-Cook equation connects the stress value with material temperature during plastic deformation (temperature material softening). The Johnson-Cook equation has five constants: A, B, C, n and m [1, 4]. These five constants are characteristic for the given material. Information about these five constants can be used for calculating material dynamic response during plastic deformation. The Johnson-Cook material model gives information about the strain-stress curve in the plastic range, so it is really useful during computer simulation. This material model has one more thing that made it very popular: we can use it for describing almost any type of material. Moreover, it is a very useful material model because it is easy to use and needs only five constants to describe material stress-strain characteristics.

4. Laboratory equipment

This work is experimental and numerical. In the experimental part of this work, the Taylor impact test was used as a source of measuring the dynamic properties of materials. The most important information used during the numerical procedure was: the impact velocity and sample final shape after deformation on a rigid wall. Laboratory experiments were done on a special experimental setup which was built for the Taylor impact test. Figure 2 presents the helium gas gun setup, as prepared for the Taylor impact test.

a) b)

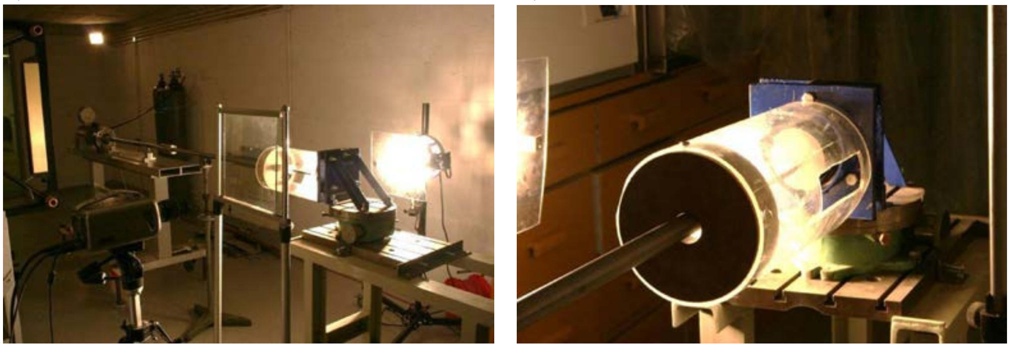

Fig. 2. Picture of the helium gas gun: a) The Taylor impact test setup; and b) a rigid target with protective surface.

After the Taylor impact test, the Copper Cu-ETP samples were deformed on one end. The final shape after deformation was measured using a coordinate measuring machine (a ZEISS measuring machine used) [2]. Figure 3 shows this machine (Fig. 3a) and a sample during the measurement process (Fig. 3b).

a) b)

Fig. 3. a) The Carl Zeiss measuring machine; b) the sample shape measuring process [2].

The coordinate measuring technique was used to measure the shape of the sample after the Taylor impact test. During this process we measured the final length L_f, the length of undeformed sample part l_f and the final diameter $\emptyset_f$ at several points. Information about these parameters was important in the second part of this work. The next part of this work is related to numerical simulation which was implemented here using the explicit solver of the "Ansys Autodyn" program. The numerical simulation was used to calculate the process of deformation during the Taylor impact test (Fig. 4).

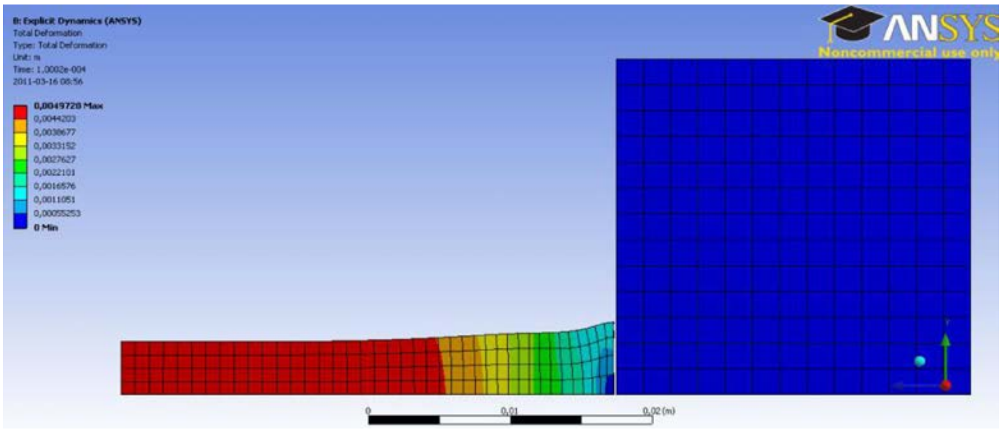

Fig. 4. Results of numerical simulation of the Taylor impact test at a hitting speed $v_o = 130$ m/s (total deformation value).

In Fig. 4 the total deformation of a copper sample is shown after the Taylor impact test. During this numerical analysis the sample hit the rigid wall at a speed of 130 ms^{-1}.

5. Methodology of identification Johnson-Cook equation constants

The aim of this research was to prepare a methodology of investigation of the constants of the Johnson-Cook equation. In this paper we proposed using the Taylor impact test and computer identification as a method of calculating the parameters of the Johnson-Cook equation. Figure 5 illustrates the algorithm of identification and optimization process.

The identification algorithm presented in Fig. 5 was tested on copper cylindrical samples. The samples were 60 mm long and 12.05 mm in diameter. Samples were accelerated to a speed between 100–180 ms^{-1} by a helium gas gun. After the laboratory experiments the samples were deformed at one end. In the next step of the identification process we measured the final sample shape. Information about the final length, maximum diameter and length of the undeformed part of the sample were used at next point of the identification procedure. The Ansys Autodyn computer program was used to make an explicit finite element method calculation. The initial conditions for computer simulation were taken

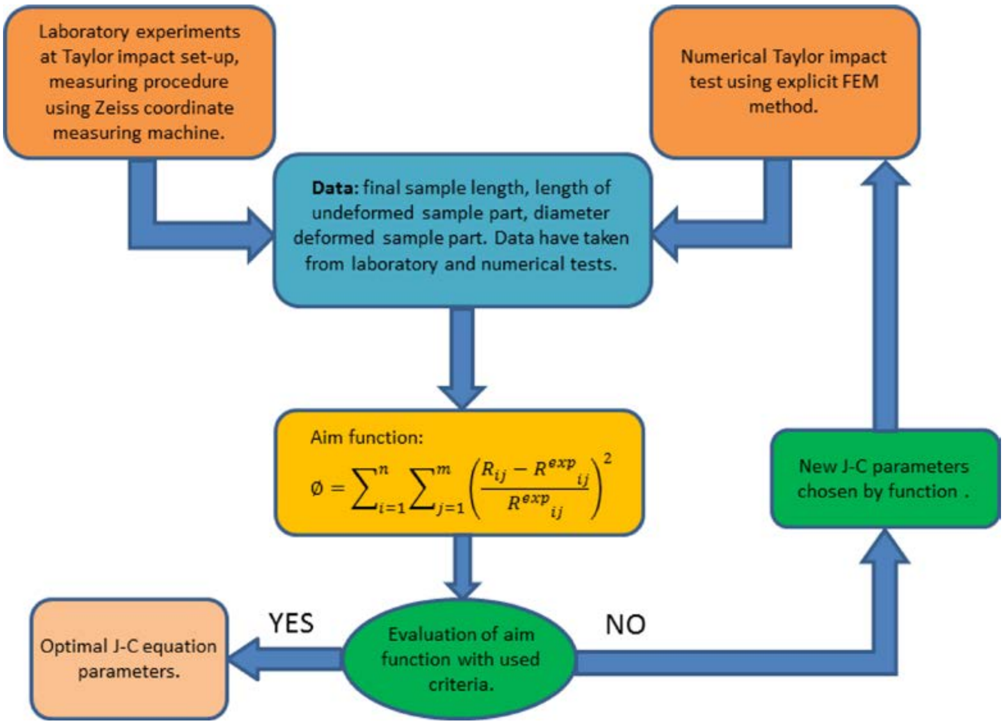

Fig. 5. Schematic of the identification algorithm used for numerical analyses.

from the laboratory experiments. The initial geometry of the sample and the impact velocity are necessary parameters for the numerical simulation, since without these parameters the computer simulation would not make sense. Information about the impact speed was taken from the high-speed camera recording. During the laboratory experiment all tests were recorded using a Phantom V12 high-speed camera and after, the numerical simulation of the Taylor impact test starts. At that point we can not yet make use of the identification and optimization procedure, as we have not declared the aim function which will be used during the identification process. To do this we connect the numerical Taylor test simulation with an optimization program. In optimization program a genetic algorithm was used for the identification procedure. A typical optimization procedure like gradient methods didn't give the correct results to our problem. To solve the problem of the identification Johnson-Cook parameters, we propose the following equation as an aim function:

$$f\left(R_{ij}\right) = \sum_{i}^{n}\sum_{j}^{m}\left(\frac{R_{ij} - R_{ij}^{\mathrm{exp}}}{R_{ij}^{\mathrm{exp}}}\right)^2. \tag{5.1}$$

During identification of parameters of the Johnson-Cook equation, the optimization program looked for a minimum of the aim function. The aim function is a sum of errors between numerical and experiment parameters. Values of R_{ij} represent data from numerical simulation and $R_{ij}^{\exp}$ represents data from the laboratory experiment. For our considerations we propose that the identification procedure be made on the final sample shape after the deformation with different impact speeds. The variable index i represents a geometric parameter, for example, the sample's final length value; and variable index j represents different impact speeds.

6. Results of identification procedure

The result of our work is found values for the Johnson-Cook equation parameters. Calculation was done using the finite element method and an optimization algorithm. Copper Cu-ETP Johnson-Cook parameters were identified during this procedure. Table 1 illustrates values of these parameters.

Table 1. Values of the Johnson-Cook equation parameters calculated during the identification process.

Constant	Value
A	100 MPa
B	263 MPa
n	0.23
C	0.029
m	0.98

Before our experiments began copper samples were annealed at temperature 500°C for one hour. This process was undertaken because in literature [1] we found information about Johnson-Cook parameters for this kind of material. Table 2 presents Johnson-Cook parameters reported from the literature.

Table 2. Johnson-Cook parameters reported from literature [1].

Constant	Value
A	90 MPa
B	292 MPa
n	0.31
C	0.025
m	1.09

In our tests we made ten laboratory experiments with the same type of sample. As can be seen the results of our work (Table 1) are different from those of Table 2; however, we can not say that the values in Table 1 are incorrect because the differences between the parameters in Table 1 and 2 are too small. To make some assessment of those parameter values we prepared a verification method.

In the present work we decided to compare the shape of the sample after the laboratory experiment with the shape of sample calculated in numerical simulation. Figure 6 presents two curves which show the shape of samples after the deformation process.

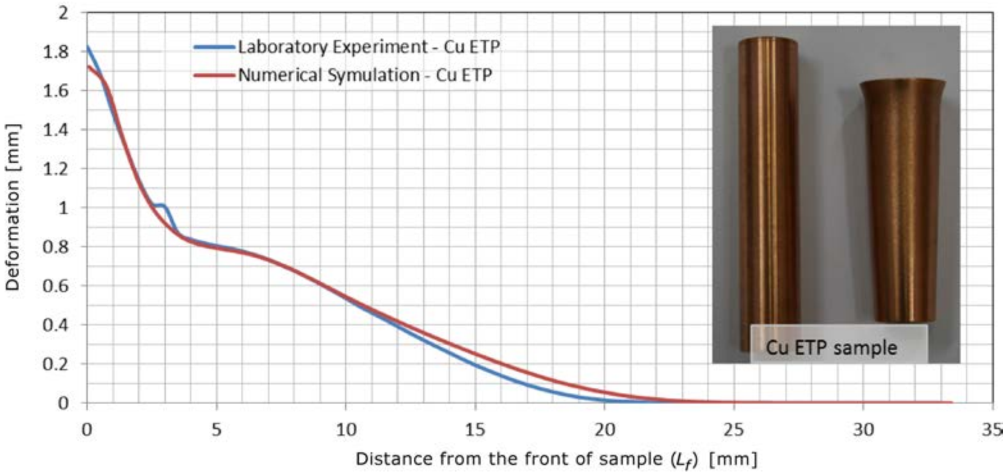

FIG. 6. Sample shape after plastic deformation during the Taylor impact test [5].

The result of comparing these two curves shows small differences. The biggest differences are in the front of the sample and in the place where the plastic wave has stopped. The error in the front of the sample is a result of an incorrect value for the friction coefficient used during the computer simulation. The error in the second place is a result of different Johnson-Cook equation parameters. For better comparison we calculate the value of the area under the curves. This information was used for a global error calculation. The fault was calculated as the quotient of the difference between values under the curves. The fault value is 3.8%.

In conclusion we can say that the proposed method of calculation of the Johnson-Cook equation parameters is practical and can be used as an alternative to the Hopkinson pressure bar test. The main advantage of this method over the Hopkinson pressure bar method is that it does not need a few different types of experiments to identify material model parameters. The proposed method needs only one type of experiment (the Taylor impact test) for the whole identification procedure. The only problem is that our method needs some kind of verification procedure; a solution to which is being investigated in our laboratory. At this point of our work we can say that the presented method has great potential and can be used as a main identification method like the Hopkinson pressure bar technique.

Acknowledgment

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