ENGINEERING TRANSACTIONS • Engng. Trans. • 45, 3–4, 533–537, 1997
Polish Academy of Sciences • Institute of Fundamental Technological Research

B R I E F   N O T E S

TENSILE STRENGTH OF COMPOSITES REINFORCED WITH DISPERSED FIBRES DETERMINATION OF REINFORCEMENT

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The title subject was analysed using the damage theory, assuming that the distribution of the fibre strength is rectangular. The derived relations can be applied for design purposes. An example of calculations is given.

Several papers have been recently published on the mechanics of composites reinforced with dispersed fibres, e.g., [1, 2, 3, 4]. However, the fibre volume fraction was established by consecutive trials, according to the desired strength.

The aim of the paper is to present the relations proposed by the author on the analytical approach to the determination of tensile strength and calculation of the necessary reinforcement.

The fibres have to control the cracking due to various loads which are unavoidable and difficult for quantitative estimation, for example these caused by distortion, shrinkage and temperature variations. At the beginning, the microcracks appear which are controlled and arrested by dispersed fibres. But with the increase of the loads, a continuous macrocrack is formed where total load is transferred by the transversal fibres. The tensile strength of the composite is understood as a limit state of a system of fibres in a crack.

The bearing capacity of a fibre is variable and depends upon its anchorage in the matrix.

In Fig. 1 a model of the crack with fibres is presented. It can be deduced from the figure that the force R transferred by a single fibre is always perpendicular to the crack, because the fibres are slim and are bent in the crack to preserve the direction of the crack extension.

Any position of the fibre with respect to the crack has the same probability. They are characterized by two parameters: α – angle and a – length of anchorage.

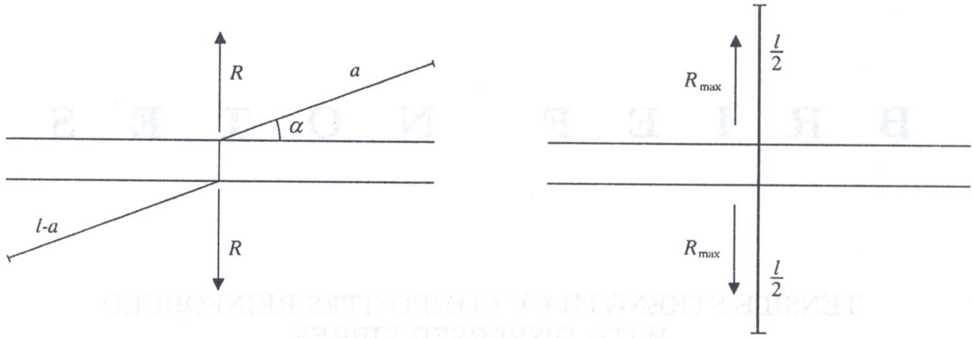

FIG. 1.

The force R reaches its minimum when the fibre is in the plane of the crack or when its end is tangent to the crack. The maximum corresponds to the situation when the fibre is perpendicular and a is equal to half of the fibre length., [5, 6]. The force R as a function of a and α can be represented by the following relation:

$$R(\alpha, a) = R_{\max}(a/0.5l)\sin\alpha, \tag{1}$$

where $R_{\max}$ – maximum force in the fibre, a – shorter section of the fibre as divided by the crack, l – length of the fibre, α – angle between fibre and crack.

Every value of $R(\alpha, a)$ except $R_{\max}$ can be obtained from an infinite number of combinations of values of two parameters α and a. Since any position of the fibre is of equal probability, then also every value of the force R within the limits $(0, R_{\max})$ is equally probable. Consequently, the density function $\varphi(R)$ of probability of the limit value of the force in the fibres crossing a crack is constant:

$$\varphi(R) = 1/R_{\max}\,. \tag{2}$$

This relation is obtained from Fig. 2, where

$$\varphi(R)R_{\max} = 1.$$

This expresses the fact that within the limits $(0, R_{\max})$, all values of the force in a fibre are of the same probability. This distribution is valid for any cross-section of a composite element.

The fracture process of the fibres in a crack is developing in a following way. First the fibres which have the weakest anchorage, are pulled out of the matrix. Then, the load is transferred onto other fibres. With the increase of the load, all fibres one after another are pulled out. The bearing capacity of the system is exhausted when the process is developing spontaneously without any increase of the external load. Description of such a process corresponds well to the damage theory, [7, 8].

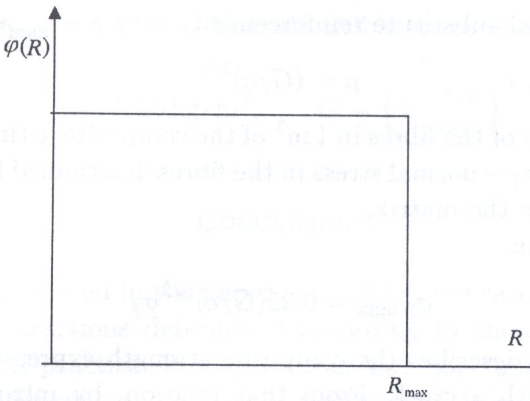

FIG. 2.

The notion of the nominal stress σ_n may be introduced for the system of fibres, and it is understood as the total force transferred by the fibres per unit surface of a crack,

$$\sigma_n = LR(1 - R/R_{\max}), \tag{3}$$

where L – mean number of the fibres in a unit of the cross-section of the composite.

Relation (3) expresses the situation when the nominal stress due to external load is σ_n, the bearing capacity of the most loaded fibres is R, and the cross-section of the damaged system of fibres is equal to the value in parantheses, where $R/R_{\max}$ is the relative damage.

The maximum value of the nominal stress transferred by the system of fibres in a crack may be found by differentiation of relation (3) with respect to R and by assuming that it is equal to 0,

$$1 - 2R/R_{\max} = 0. \tag{4}$$

It may be concluded that the maximum of the nominal stress $\sigma_{n\,\max}$ is obtained for $R = 0.5R_{\max}$. When the value $\sigma_{n\,\max}$ is achieved, all fibres with forces below $0.5R_{\max}$ are pulled out. Then the bearing capacity of the system is

$$\sigma_{n\,\max} = 0.25LR_{\max}. \tag{5}$$

Since L is the number of fibres in $1\,\mathrm{cm}^2$ and $R_{\max}$ can be expressed as a product of the area of cross-section of a single fibre and the maximum stress in the fibres determined from the condition of its anchorage in the matrix, we obtain:

$$LR_{\max} = \mu\sigma_f, \tag{6}$$

where μ – orthogonal substitute reinforcement,

$$\mu = (G/c)^{2/3}, \tag{7}$$

G (in kg/m^3) – mass of the fibres in 1 m^3 of the composite, c (in kg/m^3) – specific weight of the fibre, σ_f – normal stress in the fibres determined from the condition of fibre anchorage in the matrix.

Finally, we obtain:

$$\sigma_{n\,\max} = 0.25(G/c)^{2/3}\sigma_f\,. \tag{8}$$

The relation (8) describes the composite strength expressed as a limit state of the structure with a crack. From that relation, by introducing the safety coefficients, we obtain a formula to determine G – the necessary amount of fibres for a structure with a crack:

$$G = \left(4\frac{\sigma_{\max}}{\sigma_f}\frac{\gamma_1}{\gamma_2}\right)^{3/2} c, \tag{9}$$

where $\sigma_{\max}$ – maximum nominal stress due to prescribed load of the structure, γ_1 – coefficient of the overloading (higher than 1), γ_2 – safety coefficient against pull-out of the fibre from the matrix (lower than 1).

From the relation (9) we can determine the necessary volume of the fibres for the structure with cracks to support the given load and the related nominal stress, with suitable safety coefficients. The above formulation is general and can be applied, among others, for different quasi-continuous structures, i.e. structures with cracks. In case of the composite materials in which the cracks are inadmissible and the fibres are used only as a means against unforeseeable cracks, the volume of fibres is determined from the assumption that after crack opening, the fibres in the crack support nominal stresses equal to the matrix tensile strength.

Example of calculation

Let us determine the fibre reinforcement for a composite material in which the cracks are inadmissible. Then the requirement must be fulfilled:

$$\gamma_1\sigma_{\max} = f_t\,,$$

where f_t – tensile strength of the matrix (concrete) in direct tension.

The structure is subjected to bending at small depth of the cross-section, therefore there is an increase of the tensile strength at bending, [9]. The strength is estimated for the designing of the structure as follows:

$$f_{tf} = 1.5 f_t = 4.4 \text{ MPa};$$

it means that $\gamma_1 \sigma_{\max} = 4.4\,\mathrm{MPa}$. The admissible stress in the fibres is

$$\gamma_2 \sigma_f = 600\,\mathrm{MPa}, \qquad c = 7850\,\mathrm{kg/m^3}, \qquad G = \left(4 \cdot \frac{4.4}{600}\right)^{3/2} 7850 = 39.4\,\mathrm{kg/m^3}.$$

Conclusions

The relations proposed in the paper are valid in the case of short term loads. The fibre volume fractions determined according to these relations are close to those applied in practice. It may be therefore concluded that the physical meaning of the phenomena is correctly represented. However, the influence of creep of the bond that appears in the long term processes is not considered and it has some influence on the durability of structures. The paper may serve as a starting point for such an analysis.

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Received March, 19, 1997; new version September 8, 1997.