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MODELLING OF THE DYNAMICS OF A HYDRAULIC POSITIONAL CYLINDER

R. DINDORF (KRAKÓW)

The present paper describes modelling of dynamics, results of simulation and experimental investigation of a hydraulic positional cylinder. The method of bond graph is used for modelling of the dynamics of the considered cylinder design. First, an initial digital simulation is performed in order to calculate basic dynamic characteristics of the cylinder. Next, an additional damping system is applied which affects dynamic characteristics of the considered cylinder. Finally, results of the model simulation are verified experimentally.

1. Introduction

Hydraulic positional cylinders make it possible to obtain some precisely definite positions of the piston. These cylinders find application in moving parts of industrial manipulators and robots or they can perform auxiliary functions in the drives of technological machines, e.g. in switching over the shift wheels, gears and couplings in gear-boxes. Examples of different hydraulic positional cylinders are presented, among others, in papers [3, 5]. Positional cylinders have to satisfy such requirements as: high reliability and speed, precision of performance, irrespective of the properties of the working fluid and the rate of piston motion. The technique of the control of a positional cylinder presented in this paper is similar to the application of hydraulic stepping motors and is classified as the so-called track program control [5]. This control consists in the fact that one of the ports of the cylinder is connected with a return line to the tank. The piston moves in the desired direction to reach the point precisely opposite this port. In the steady state process, the pressure on both sides of the piston is equal, and fluid flows through the holes to the tank. In order to secure proper dynamic properties of the positional cylinder, an additional damping is introduced. It may be achieved by: introduction of an additional leak between the chambers of the cylinder, viscous friction, Coulomb friction or other damping system. Each of the above mentioned kinds of damping has its advantages and disadvantages. Additional leak can cause the deterioration of accuracy in positioning of the piston. Viscous friction depends on the temperature of the working fluid and on the accuracy of construction of the cylinder. Coulomb friction leads to an increase in the positioning error, which results in a decrease in the positioning accuracy. Bond graphs [2, 6] are used to model the dynamics of the positional cylinder. In selecting a method of modelling it is taken into consideration that the dynamic structure of bond graphs is closely related to the functional structure of the hydraulic system. Since accumulation of kinematic and potential energies on bond graphs are described by means of a function of time, these graphs are applicable to digital simulation.

2. Design solution and modelling of dynamics of the positional cylinder

In the adopted design of a hydraulic positional cylinder, presented in a simplified way in Fig. 1, four positions of the piston are considered. The positional cylinder is fed from the source of constant pressure. In the inlets to the right and left chambers of the cylinder, throttle valves (of the capillary and orifice type) of constant diameter are fixed. A sleeve with properly made grooves and holes is located in the cylinder. Turning the sleeve by angles 0° and 180°, one obtains positions 1 and 3. Turning it by angles 90° and 270°, positions 2 and 4 are obtained. In each position of the piston, the cylinder is connected with the return line to the tank through four holes. The piston assumes the left-hand position 1 or 2 when control shutoff valve I is open and control shutoff valve II is closed, and it assumes the right-hand position 3 or 4 when the control shutoff valve I is closed and the control shutoff valve II is open. For example, opening of the control shutoff valve I and closing of the shutoff valve II reduces the pressure p_1 and moves the piston from the right to the left-hand position. As a result of volume increase in chamber 1 and volume decrease in chamber 2, pressure p_1 increases and pressure p_2 decreases. The pressure in the left and right chambers of the cylinder increases and decreases in turns until the steady position of the piston is achieved. The piston position is established after achieving the state of equilibrium of the forces acting on the cylinder piston.

In modelling of the dynamics of a positional cylinder, the following simplifying assumptions have been adopted: discharge pressure p_0 is constant ($p_0 = \text{const}$), pressure p_z in the outlet line to the tank equals the atmospheric pressure (it can also be written that $p_z = 0$), temperature change ΔT of the working fluid has a negligible influence on the cylinder characteris-

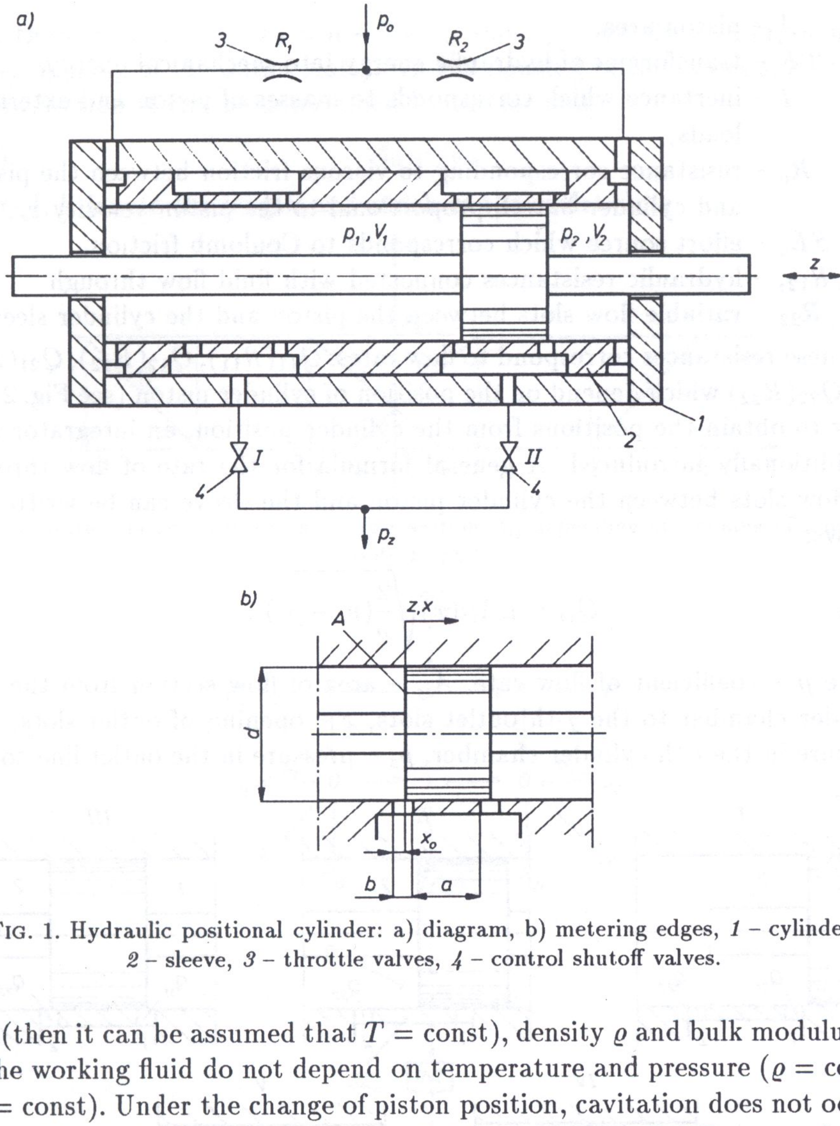

FIG. 1. Hydraulic positional cylinder: a) diagram, b) metering edges, *1* – cylinder, *2* – sleeve, *3* – throttle valves, *4* – control shutoff valves.

tics (then it can be assumed that $T = \text{const}$), density ϱ and bulk modulus E_c of the working fluid do not depend on temperature and pressure ($\varrho = \text{const}$, $E_c = \text{const}$). Under the change of piston position, cavitation does not occur, internal leaks in the cylinder are negligible due to small difference of pressure on both sides of the piston, pressure losses and fluid compressibility do not occur in the supply lines.

The method of bond graphs is used to model the dynamics of a positional cylinder. In creating a bond graph of the cylinder, the following notations are introduced:

SE_p – energy effort source which corresponds to pressure p_0,
R_1, R_2 – hydraulic resistances of throttle valves,
C_1, C_2 – hydraulic capacitances in cylinder chambers,

A – piston area,
TF – transformer of hydraulic energy into mechanical energy,
I – inertance which corresponds to masses of piston and external loads,
R_v – resistance corresponding to viscous friction between the piston and cylinder barrel, proportional to the piston velocity v_z,
SE_c – effort source which corresponds to Coulomb friction,
$R_{11}, R_{12}, R_{21}, R_{22}$ – hydraulic resistances connected with fluid flow through variable flow slots between the piston and the cylinder sleeve.

These resistances correspond to flow rates: $Q_{11}(R_{11}), Q_{12}(R_{12}), Q_{21}(R_{21})$ and $Q_{22}(R_{22})$ which depend on the position of cylinder piston (see Fig. 2). In order to obtain the positions from the cylinder position, an integrator INT is additionally introduced. A general formula for the rate of flow through the flow slots between the cylinder piston and the sleeve can be written as follows:

$$Q_{ij} = \mu A_{ij}(x)\sqrt{\frac{2}{\varrho}(p_i - p_z)}\,, \tag{2.1}$$

where μ – coefficient of flow rate, A_{ij} – area of flow section from the i-th cylinder chamber to the j-th outlet slots, x – opening of outlet slots, p_i – pressure in the i-th cylinder chamber, p_z – pressure in the outlet line to the tank.

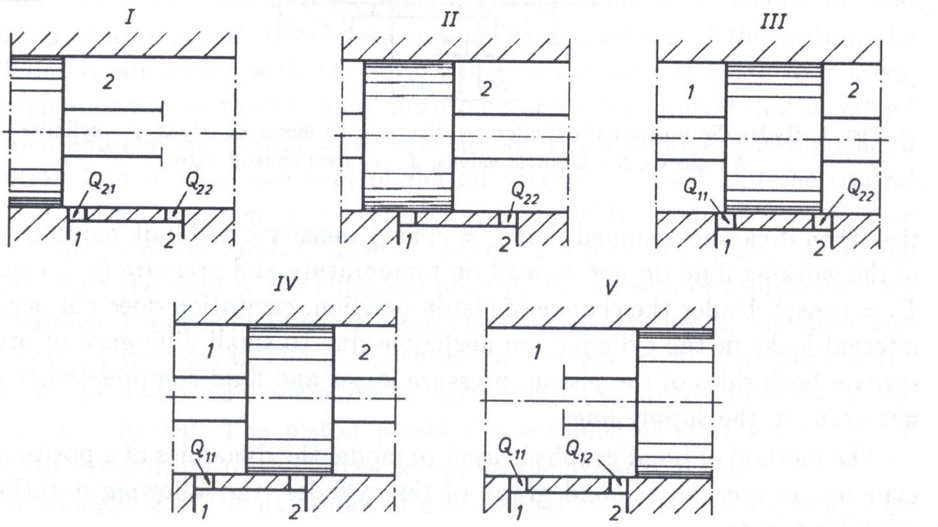

FIG. 2. Diagram of different positions of cylinder piston with respect to outlet slots.

The dependence of flow section A_{ij} on opening x of outlet slots 1 and 2 is represented on static characteristics in Fig. 3. The following notations are used there $x_1 = b - x_0$, $x_2 = a + b - x_0$ and $x_3 = a + 2b - x_0$ (x_0 denotes initial opening of the outlet slots). On full opening of the outlet slots 1 and 2, flow section A can be determined by the formula:

$$A_0 = lb\,, \tag{2.2}$$

where l, b denote the length and width of the outlet slots.

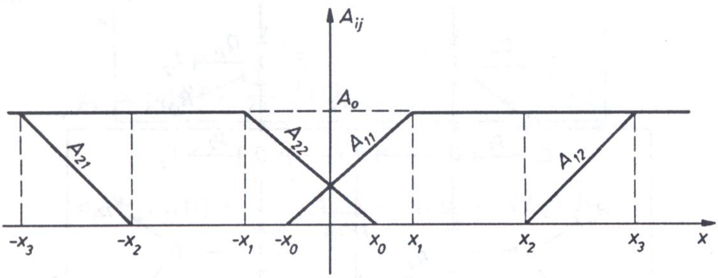

FIG. 3. Static characteristics of flow cross-section A_{ij} depending on opening of x outlet slots 1 and 2.

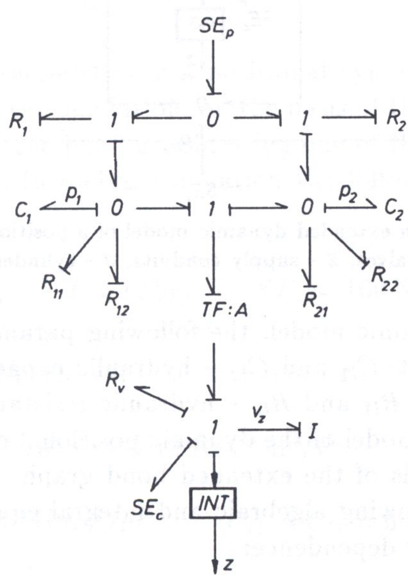

FIG. 4. Bond graph of a positional cylinder.

The bond graph of a hydraulic positional cylinder, for the previously determined parameters, is represented in Fig. 4. After analysing the created model of the dynamic positional cylinder it has been decided to extend it by conduits between the throttle valves and the cylinder. Such extension is justified by the fact that the conduits can have significant lengths. The phenomena which occur during damping of the piston vibrations accompanied by return flow to conduits, should be also taken into consideration.

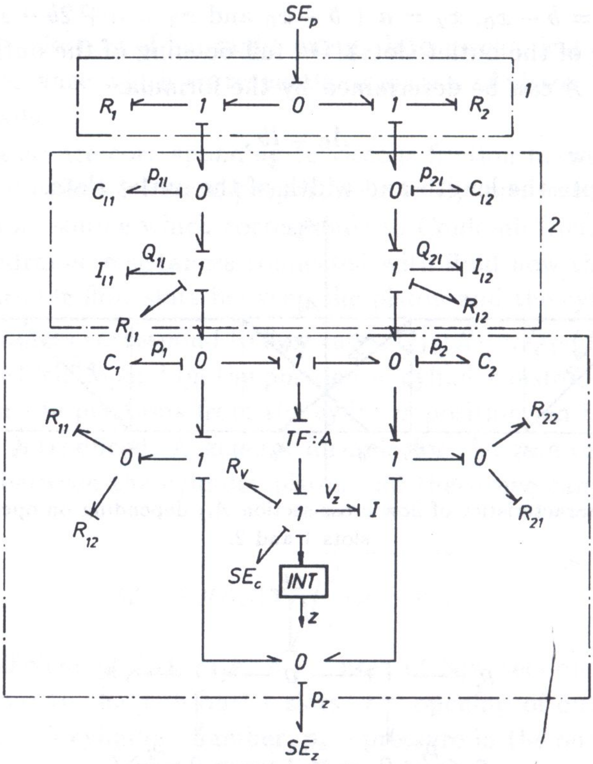

FIG. 5. Bond graph of an extended dynamic model of a positional cylinder: *1* – throttle valves, *2* – supply conduits, *3* – cylinder.

In the extended dynamic model, the following parameters of conduits will be taken into account: C_{l1} and C_{l2} – hydraulic capacitances, I_{l1} and I_{l2} – hydraulic inertances, R_{l1} and R_{l2} – hydraulic resistances. The bond graph of such an extended model of the dynamic positional cylinder is represented in Fig. 5. On the basis of the extended bond graph, represented by Fig. 5, we can write the following algebraic and integral equations resulting from the adopted causality dependence:

- for the throttle valves

$$Q_i = \frac{1}{R_i}\sqrt{|p_0 - p_{li}|}\,\mathrm{sign}(p_0 - p_{li}); \tag{2.3}$$

- for the conduits

$$Q_{li} = Q_{li}(0) + \frac{1}{I_{li}}\int_0^t (p_{li} - \Delta p_{li} - p_i)\,dt, \tag{2.4}$$

(2.5) $$p_{li} = p_{li}(0) + \frac{1}{C_{li}} \int_0^t (Q_i - Q_{li})\, dt,$$

(2.6) $$\Delta p_{li} = R_{li} Q_{li}\,;$$

- for the cylinder

(2.7) $$p_i = p_i(0) + \frac{1}{C_i} \int_0^t (Q_{li} - A v_z - Q_{i1} - Q_{i2})\, dt,$$

(2.8) $$v_z = v_z(0) + \frac{1}{I} \int_0^t (A \Delta p - R_v v_z - S E_c)\, dt,$$

(2.9) $$z = z(0) + \int_0^t v_z\, dt.$$

The dynamic characteristics of a positional cylinder can be determined by the method of digital simulation, on the basis of the dynamic model represented by means of the bond graph, using one of the available simulation programs, e.g. CSSL. In digital simulation the following parameter values were introduced:

$$A = 0.77\,10^{-3}\,\mathrm{m}^2, \qquad I = 12\,\mathrm{kg}, \qquad SE_c = 100\,\mathrm{N}, \qquad p_0 = 15\,\mathrm{MPa},$$

$$R_1 = R_2 = 0.41\,10^9\,\mathrm{Pa\,s/m^3}, \qquad \eta = 0.062\,\mathrm{Pa\,s}, \qquad E_c = 895\,\mathrm{MPa},$$

$$C_1 = C_2 = 0.85\,10^{-14} \div 0.42\,10^{-13}\,\mathrm{m^3/Pa}, \qquad \varrho = 850\,\mathrm{kg/m^3},$$

$$I_{l1} = I_{l2} = 1.1\,\mathrm{MPa\,s^2/m^5}, \qquad C_{l1} = C_{l2} = 0.8\,10^{-14}\,\mathrm{m^3/Pa},$$

$$R_{l1} = R_{l2} = 1.28\,10^9\,\mathrm{Pa\,s/m^3}.$$

After reverse control of the shutoff valves I and II, when the cylinder piston changes its position, the dynamic characteristics are determined to show the variations of pressure difference $p_1(t) - p_2(t)$ in cylinder chambers, displacements $z(t)$ and velocities $v_z(t)$ of the piston. Examples of the dynamic characteristics are presented in Fig. 6. Requirements for a positional cylinder involve high precision of positioning of the piston and strong and possibly fast damping of its vibrations.

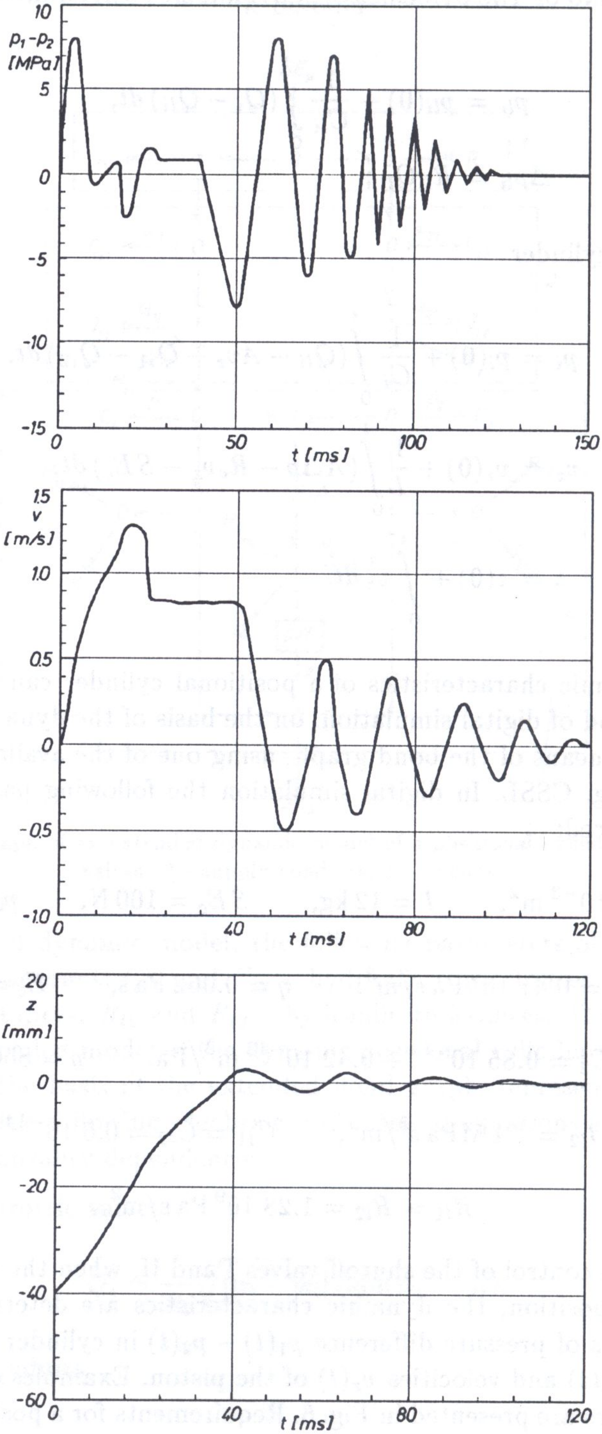

FIG. 6. Dynamic characteristics of a positional cylinder.

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3. MODELLING OF DYNAMICS AND INVESTIGATION OF A POSITIONAL CYLINDER WITH A DAMPER

In order to obtain the required dynamic properties of the hydraulic positional cylinder, an additional damping system (damper) has been introduced to the control system. The positional cylinder with a hydraulic damper is represented in Fig. 7. The adopted damper has a design solution applicable in a hydraulic cylinder [1]. This damper consists of a damping valve which performs the function of a bypass valve between two cylinder chambers, four return valves and an accumulator. In a steady state, the damping valve is

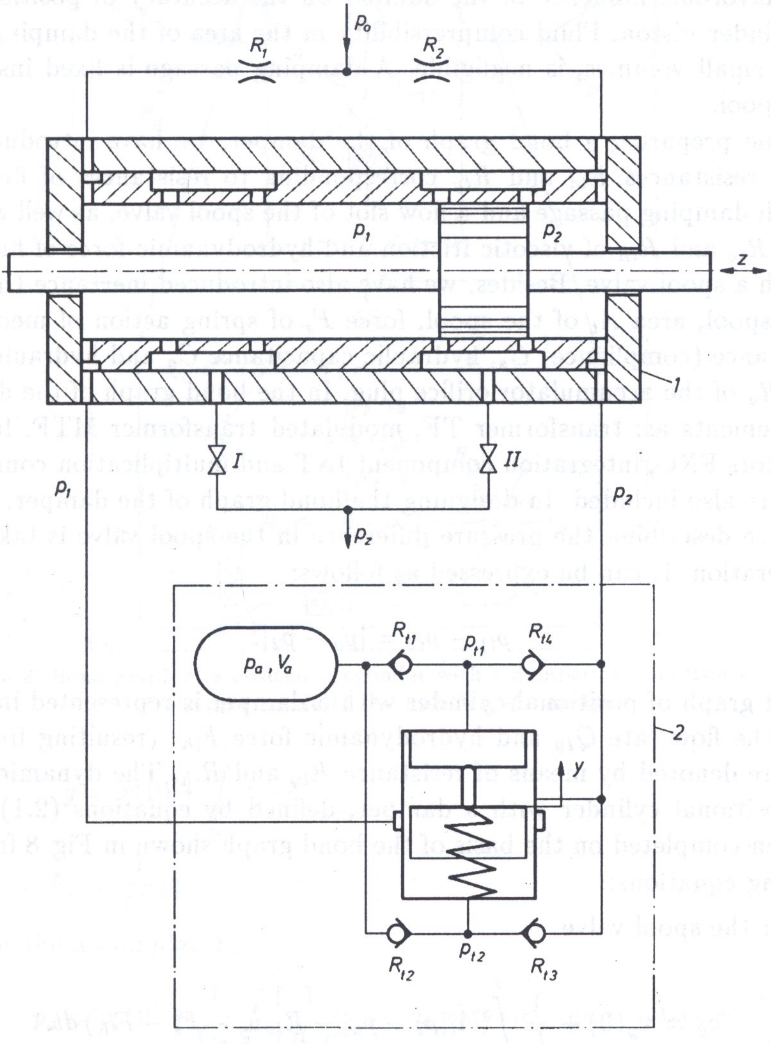

FIG. 7. Diagram of a positional cylinder with damper: *1* – cylinder, *2* – damper.

closed. Due to the application of return valves, the damping valve acts depending on higher pressure in a given cylinder chamber. The damping valve opens when the force resulting from the difference of pressures p_{t1} and p_{t2} is higher than the force of the spring. The accumulator performs the function of a pressure vessel in the damper. After damping, the valve opens and an additional fluid flow occurs between the two cylinder chambers. It is responsible for the damping effect of pressure pulsation caused by the change of position of the piston. The damper has a favourable effect on the dynamic characteristics of the cylinder, but it can also contribute to energy losses and positional errors. The design of the damping valve with a spool reduces the unfavorable influence of the damper on the accuracy of positioning of the cylinder piston. Fluid compressibility in the area of the damping valve, due to small volumes, is negligible. A damping passage is fixed inside the valve spool.

While preparing a bond graph of the damper, we have introduced hydraulic resistances R_{td} and R_{ty} corresponding to resistances of fluid flow through damping passage and a flow slot of the spool valve, as well as resistances R_{tv} and R_{th} of viscotic friction and hydrodynamic force of fluid flow through a spool valve. Besides, we have also introduced inertance (mass) I_t of the spool, area A_t of the spool, force F_s of spring action of mechanical capacitance (compliance) C_s, hydraulic capacitance C_a and hydraulic resistance R_a of the accumulator orifice plug. In the bond graph of the damper, such elements as: transformer TF, modulated transformer MTF, function generators FNC, integration component INT and multiplication component MUL are also included. In designing the bond graph of the damper, the dependence describing the pressure difference in the spool valve is taken into consideration. It can be expressed as follows:

$$p_{t1} - p_{t2} = |p_a - p_2|. \tag{3.1}$$

A bond graph of positional cylinder with a damper is represented in Fig. 8, where the flow rate Q_{ty} and hydrodynamic force F_{th} (resulting from this flow) are denoted by means of resistance R_{ty} and R_{th}. The dynamic model of a positional cylinder with a damper, defined by equations (2.1)–(2.9), has been completed on the basis of the bond graph shown in Fig. 8 from the following equations:

- for the spool valve

$$v_y = v_y(0) + \frac{1}{I_s} \int_0^t (A_t|p_2 - p_a| - R_{tv}v_y - F_s - F_{th})\, dt, \tag{3.2}$$

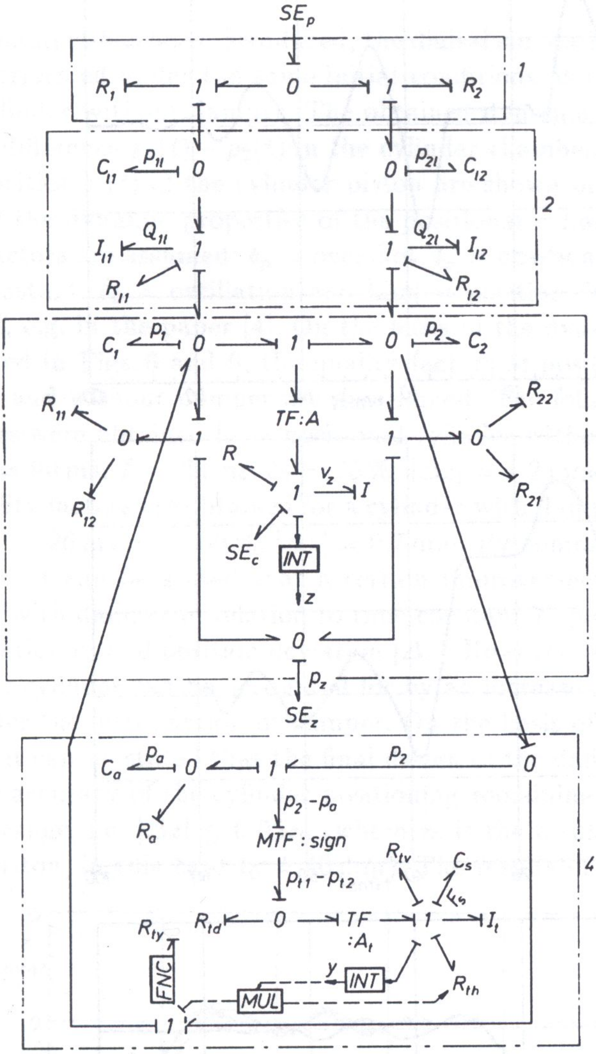

FIG. 8. Bond graph of a positional cylinder with a damper: *1* – throttle valves, *2* – supply conduits, *3* – cylinder, *4* – damper.

(3.3) $$y = y(0) + \int_0^t v_y \, dt,$$

(3.4) $$F_s = \frac{1}{C_s} y;$$

- for the accumulator

(3.5) $$p_a = p_a(0) + \frac{1}{C_a} \int_0^t \left[\frac{1}{R_{td}} (p_2 - p_a) \operatorname{sign}(p_2 - p_a) + A_t v_y - \frac{1}{R_a} p_a \right] dt.$$

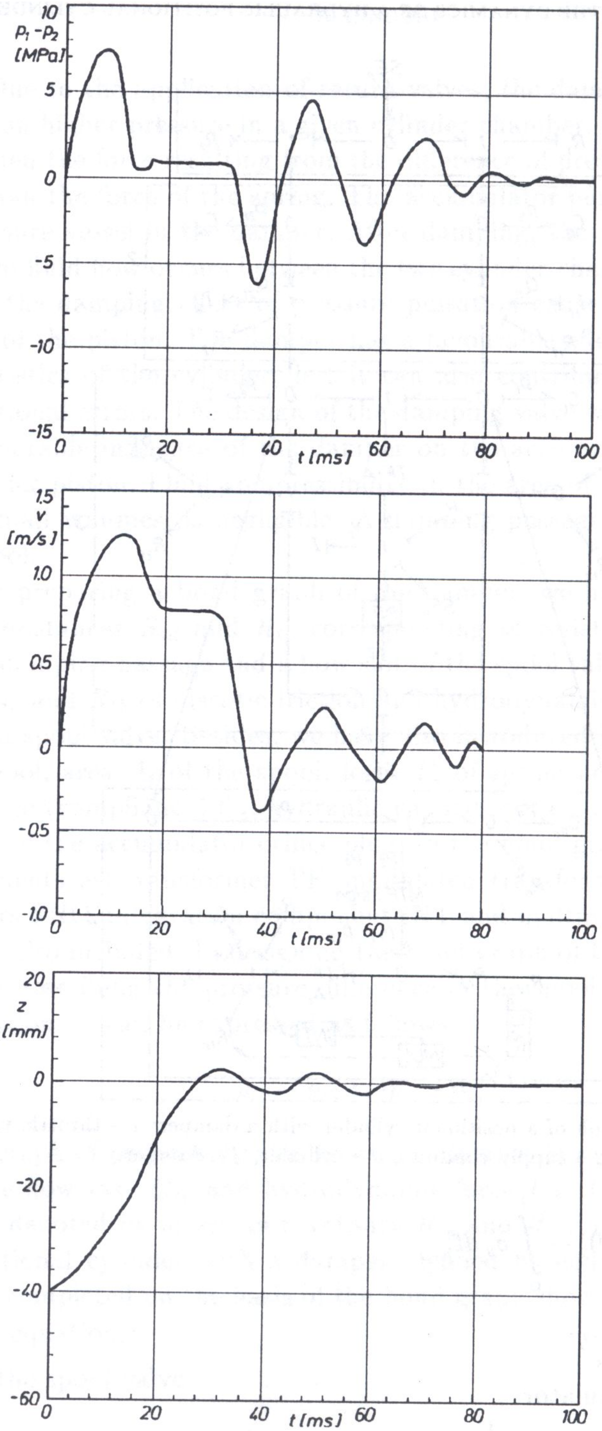

FIG. 9. Dynamic characteristics of positional cylinder with damper.

Once the damper has been introduced, the digital simulation of positional cylinder is performed under the same initial conditions as those used for a positional cylinder without damper. The obtained dynamic runs including: the pressure difference $p_1(t) - p_2(t)$ in the cylinder chambers, displacement $z(t)$ and velocities $v_z(t)$ of the cylinder piston are shown in Fig. 9. For the estimation of the dynamic properties of the positional cylinder, the following quality factors are assumed: δ_p = overshot, t_p = positional setting time, T = time constant, δ_0 = oscillation and $|\Delta z|$ = position deviation, which are described, e.g. in the paper [4]. On the basis of the dynamic characteristics presented in Figs. 6 and 9, the quality factors of positional cylinders with damper and without damper are determined. The following values of quality factors were obtained for a positional cylinder without the damper: $\delta_p = 5\,\%$, $t_p = 90\,\mathrm{ms}$, $T = 33\,\mathrm{ms}$, $\delta_0 = 75\,\%$, $|\Delta z| = 1.2\,\mathrm{mm}$. The following values of quality factors are obtained for a cylinder with damper: $\delta_p = 7.5\,\%$, $t_p = 65\,\mathrm{ms}$, $T = 26\,\mathrm{ms}$, $\delta_0 = 50\,\%$, $|\Delta z| = 0.7\,\mathrm{mm}$. By comparing the above quality factors it can be stated that a certain improvement was obtained for a cylinder with damper in relation to time constant T, positional setting time t_p, oscillation δ_0 and position deviation $|\Delta z|$. However, increase in overshot δ_p of this cylinder can be accounted for by an increase in the damping coefficient after the introduction of damper. On the basis of the performed investigation it can be stated that the final design of the damper is selected correctly, the accuracy of the cylinder positioning remaining within the admissible 5 % range, i.e. $|\Delta z| \leq 0.05 z_0$ (where z_0 is the displacement of the positioning piston, in this case $z_0 = 40\,\mathrm{mm}$). The results of digital simula-

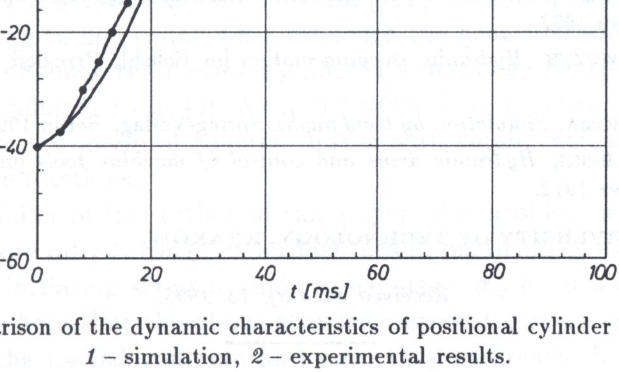

FIG. 10. Comparison of the dynamic characteristics of positional cylinder with damper: *1* – simulation, *2* – experimental results.

tion are verified experimentally. Examples of dynamic characteristics which represent the motion $z(t)$ of the cylinder piston obtained during simulation and experimental investigations are presented in Fig. 10. Convergence of the simulation and experimental results proves that all the parameters of the hydraulic positional cylinder with damper have been properly selected.

4. Conclusions

1. The design of a hydraulic positional cylinder presented in this paper enables us to obtain many different positions of the piston by exchanging the sleeve with properly made outlet slots.

2. Bond graphs, applicable to digital simulation, are used to model the dynamics of a hydraulic positional cylinder.

3. The dynamic model of a positional cylinder has been extended by supply conduits and an additional damping system (damper).

4. Initial simulation analysis of a positional cylinder showed that for improving its dynamic characteristics, additional damping should be used.

5. It results from the dynamic characteristics of a positional cylinder that change of the position of the piston of a cylinder with damper occurs faster and under well attenuated vibrations.

6. The dynamic properties of a hydraulic positional cylinder with a damper are confirmed experimentally.

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KRAKÓW UNIVERSITY OF TECHNOLOGY, KRAKÓW.

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