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THEORETICAL AND EXPERIMENTAL STUDIES ON ELASTO-PLASTIC DEFORMATION OF AN ELEMENT WITH SURFACE LAYER

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Comparison of analytical calculations with results of experimental measurements of stresses and strains occurring in the element with surface layer, subjected to the tension test and then unloaded, was presented. The specimens were thin-walled tubes made of copper, on which a chromium layer was galvanically laid on. The differences between the tension curves for the homogeneous specimen and the specimen with a layer, are found to be considerable even at the macroscopic level. This effect is caused by large differences in mechanical properties of the core and the layer. The comparison of the theoretical model and experimental results has shown a good agreement, up to the strain level above which cracks appear in the chromium layer. The performed experiments confirm the accuracy of the numerical programs prepared for the case of multiaxial state of stress.

1. Introduction

Surface layer of machine parts is an object of many experimental studies, which mostly refer to its effects on such phenomena as fatigue strength, durability and wear. Main macroscopic effects such like crack forming, fracture and degree of wear are considered. It is found that surface layer plays an important role in these phenomena.

Simultaneously, theoretical models of elements with a technological surface layer (TSL) are developed to enable the stress field determination in the whole element, taking into account the influence of TSL, particularly in the surface layer itself [5]. The knowledge of stress and strain fields in the core and layer is essential for theoretical explanation of the experimentally observed phenomena mentioned above. However, there is a lack of experimental studies of these quantities in elements with TSL. It is necessary to continue such studies not only because they would help us to explain surface layer's influence on these quantities, but also because they could be used to verify the theoretical models.

The finite elements method (FEM) is usually applied to model the elements with TSL, in which elasto-plastic strains exist, [1, 2, 3]. In these papers the stress state of directly loaded elements with a surface layer (contact problem) is discussed. There are also other papers dealing with structural elements with a surface layer which are loaded indirectly. In these papers a thin homogeneous film model of the surface layer (e.g. in [4, 5, 6]), is applied. The initial stress and different values of the core and layer material constants are taken into consideration. In the present paper, the results of experimental studies on the deformation process of an element with a surface layer are presented. The influence of the layer on the process is explained. This influence is also analysed by means of the FEM program. The material model (constitutive relations) used in the program is outlined. The purpose of our investigation is the description of experimental results, which confirm the correctness of the theoretical model used in the program.

2. Experiment description

The purpose of the experiment is to verify the theoretical model of the element with a surface layer and the numerical program based on the model. As a result of the computations, the majority of parameters describing the state of specimen after the tension and unloading processes (stresses in the core and layer, total strains, residual strains after unloading etc.) are obtained. The comparison with the experiment is done on selected quantities, which are easy to measure: the components of strain tensor and the load. The specimens are designed so as to achieve the maximum influence of the surface layer on the change of the load-strain characteristic of a specimen with the layer, as compared to a homogeneous specimen. This effect is obtained for the specimens having the form of thin-walled tubes of inner and outer diameters 10 mm and 12 mm, respectively (Fig. 1). The specimens are made of copper, which is covered by a hard chromium layer 40 μm thick. The shape of the specimen enables minimisation of the ratio of core's cross-section area to that of the layer (it should be remembered that core's cross-section area is always much larger than the latter area).

The hardness and thickness of the chromium layer, galvanically laid on copper, has been tested. The diagram presented in Fig. 2 indicates high hardness of the chromium layer, which suggests a high yield point and tendency to brittle cracking.

The difference between the properties of the layer (Cr) and the core (Cu), shown distinctly in Fig. 2, is large enough to expect its considerable influence, even in macroscopic measurements.

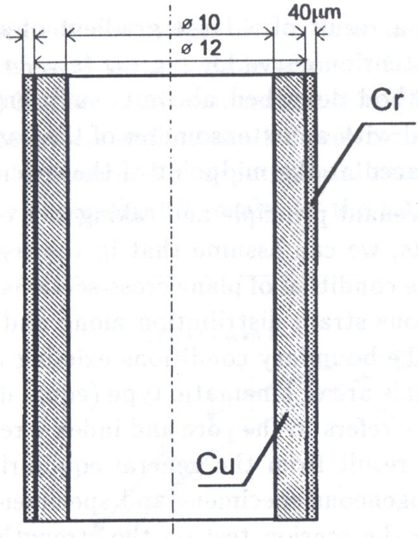

FIG. 1. Longitudinal cross-section of copper specimen with chromium layer.

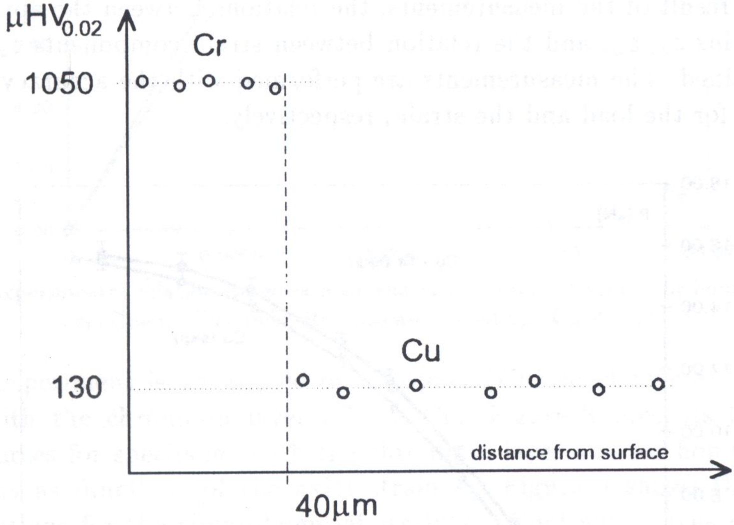

FIG. 2. Relation between microhardness and distance from the surface for a copper specimen with chromium layer.

The cross-sectional area of tested tubes is the same along the whole length, i.e. the tubes have no specifically designed holder sections. At the end of each tube, the steel rods (having the same diameters as the internal tube diameter) are placed to avoid their squeezing in the holders; the specimen length is 250 mm, whereas 50 mm of each specimen at both ends is used for supporting. At this kind of holding, the specimen rupture usually occurs in the holder region as a result of a large gradient of stress. However, if we are interested in the tension curve for $\varepsilon < \varepsilon_{zr}$ (ε_{zr} – the strain at rupture), then the holding method described above is sufficient. In tension process, the strain is measured with an extensometer of the basis equal 12 mm. Both extensometers are placed at the midpoint of the specimen.

Using the Saint - Venant principle and taking into consideration the symmetry of the supports, we can assume that in the region where the extensometers are fixed, the condition of plane cross-sections is satisfied. Therefore we have a homogeneous strain distribution along and across the specimen. In the general case, the boundary conditions existing at the interface of the layer and core materials are of kinematic type (equal displacements $v^c = v^s$, $u^c = u^s$, where index c refers to the core and index s refers to the layer), and the static conditions result from the general equilibrium conditions of the layer and core. Homogeneous specimens and specimens with the chromium layer were subject to the tension test on the strength testing machine INSTRON 1205.

As a result of the measurements, the relations between the applied load and strains ε_z, ε_φ, and the relation between strain components ε_z and ε_φ, are obtained. The measurements are performed with the accuracy of 0.5 % and 2 % for the load and the strain, respectively.

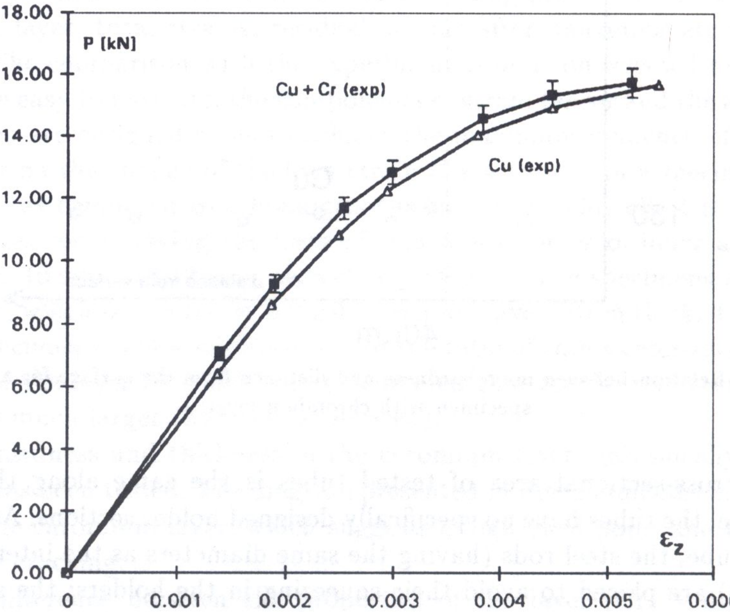

FIG. 3. Experimental relation between load and axial strain for homogeneous specimen (Cu) and specimen with chromium coating (Cu + Cr).

In a case of an element consisting of the core and a surface layer, the notion of the stress referred to the whole element (as in the case of a homogeneous specimen) has no physical meaning. The FEM computation provides the values of the stress evaluated at discrete points of the layer and core, which cannot be determined in experimental tests. Therefore, we have decided to present the experimental results in the form of force-strain diagrams.

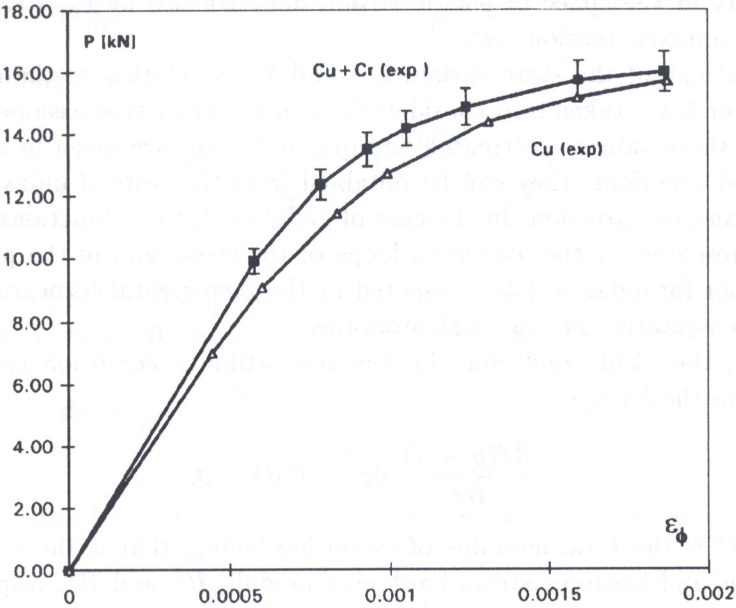

FIG. 4. Experimental relation between load and circumferential strain for homogeneous specimen (Cu) and with chromium coating (Cu + Cr).

The experiment is performed on 7 copper (Cu) specimens and 7 specimens with the chromium layer (Cu + Cr). Figure 3 presents the axial tensile forces for specimens with the chromium layer and for homogeneous specimens as functions of the axial strain ε_z. Figure 4 shows the analogous relations for the circumferential strain ε_φ. Continuous lines represent the mean load values of the same kind of specimens (homogeneous or non-homogeneous). In both figures the scatter of the results is marked. In the case of a homogeneous specimen (Cu), the scatter is negligibly small.

3. THEORETICAL BACKGROUND OF COMPUTATION

The theory of plastic flow with kinematic and isotropic strain hardening [7, 8] is applied for the assumed model. The yield condition is assumed in the form

$$F(\sigma,\alpha,\lambda)=f(\sigma-\alpha)-Y(\lambda)=0, \tag{1}$$

where σ – stress tensor, α – stress tensor determining the translation of the centre of the yield surface (coordinate of the centre of elastic region in uniaxial stress state), λ – parameter of evolution equal to the length of the trajectory in the space of plastic strain, denoted also by $\overline{\varepsilon_p}$, $Y(\lambda)$ – yield point in uniaxial tension test.

Evolutions of the state variables α and Y, in relation to the evolution parameter λ are taken into consideration. In the algorithm assumed, evolutions of these values are treated as input data and are given in a form of tabulated functions; they can be obtained from the tests of uniaxial cyclic tension and compression. In the case of cyclic load, these functions are usually represented by the hysteresis loops of the stress and of the parameter α. Further formulae will be presented in the incremental form, convenient for implementation in the FEM programs.

Using the yield condition (1), the compatibility condition can be expressed in the form

$$\frac{\partial f(\sigma-\alpha)}{\partial\sigma}\cdot d\sigma - H^* d\lambda = 0, \tag{2}$$

where H^* is the total modulus of strain hardening, that is the sum of the kinematic and isotropic strain hardening moduli, H^α and H^I, respectively,

$$H^* = H^\alpha + H^I. \tag{3}$$

In the assumed algorithm moduli H^α and H^I are not constant and depend on the plastic strain (in a multiaxial stress state they depend on the length of the trajectory in the space of plastic strains).

Equation (2) may be rewritten in the form

$$\frac{\partial f(\sigma-\alpha)}{\partial\sigma}\cdot d\sigma + \frac{\partial f(\sigma-\alpha)}{\partial\alpha}\cdot d\alpha - \frac{dY}{d\lambda}d\lambda = 0. \tag{4}$$

By comparing (2) and (4) we have

$$H^* d\lambda = \frac{\partial f(\sigma-\alpha)}{\partial\alpha}\cdot d\alpha - \frac{dY}{d\lambda}d\lambda. \tag{5}$$

Let us now use the Ziegler equation, describing the evolution of variable α:

$$d\alpha = d\mu(\sigma-\alpha). \tag{6}$$

Here μ is such a function of λ that $(d\mu/d\lambda) = c$ (c being a constant or a function of λ), thus

$$(7) \qquad d\alpha = c\,d\lambda(\sigma - \alpha).$$

Taking into account the relations (6) and (7) in Eq. (4) we obtain

$$(8) \qquad \frac{\partial f(\sigma-\alpha)}{\partial\sigma}\cdot d\sigma + \left[c(\sigma-\alpha)\cdot\frac{\partial f(\sigma-\alpha)}{\partial\alpha} - H^I\right] d\lambda = 0.$$

By combining (2), (3), (8) we obtain

$$(9) \qquad H^\alpha = -c(\sigma-\alpha)\cdot\frac{\partial f(\sigma-\alpha)}{\partial\alpha},$$

$$(10) \qquad H^I = \frac{dY(\lambda)}{d\lambda}.$$

If H^α is treated as a given quantity, the parameter c can be eliminated from Eqs. (7) and (10), and $d\mu$ can be expressed by the formula

$$(11) \qquad d\mu = \frac{-H^\alpha d\lambda}{\dfrac{\partial f(\sigma-\alpha)}{\partial\alpha}\cdot(\sigma-\alpha)} = \frac{H^\alpha d\lambda}{\dfrac{\partial f(\sigma-\alpha)}{\partial\sigma}\cdot(\sigma-\alpha)}.$$

It is assumed that the total strain is a sum of plastic and elastic strain, that is

$$(12) \qquad d\varepsilon = d\varepsilon_e + d\varepsilon_p,$$

and from the plastic flow law and from Hooke's law we have

$$(13) \qquad d\varepsilon^p = d\lambda\frac{\partial f(\sigma-\alpha)}{\partial\sigma},$$

$$(14) \qquad d\sigma = D\,d\varepsilon_e,$$

(**D** is the elastic rigidity matrix), and we obtain the governing equation of the incremental calculation procedure:

$$(15) \qquad d\sigma = D(d\varepsilon - d\varepsilon_p) = D\left(d\varepsilon - d\lambda\frac{\partial f(\sigma-\alpha)}{\partial\sigma}\right).$$

Having found H^*, we obtain the expression for $d\lambda$:

$$(16) \qquad d\lambda = \frac{d^T D\,d\varepsilon}{H^* + d^T\cdot\dfrac{\partial f(\sigma-\alpha)}{\partial\sigma}}, \quad \text{where} \quad d^T = \frac{\partial f^T(\sigma-\alpha)}{\partial\sigma}D.$$

In the FEM programs prepared, the algorithm of the procedure of integrating the equations of plasticity is assumed similarly to that of [9], and it is completed by some terms resulting from two types of strain hardening.

In the case of axially-symmetric problem, dealt with in our experiment, the surface layer is modelled as a thin film in a plane stress state, using axial-symmetric three-node membrane elements [5, 6]. Therefore we make a simplifying assumption that the stresses are constant across the whole thickness of the layer. This assumption is reasonable in case of a thin layer [5, 6] and sufficiently far away from the supports. It is worth to mention that in our case, the assumption is valid exactly in this portion of the specimen, in which the measurements are performed (e.g. near the midpoint of the specimen). The core of the specimen is modelled by the known axially symmetric eight-node elements. The model is presented in Fig. 5, where a part of the

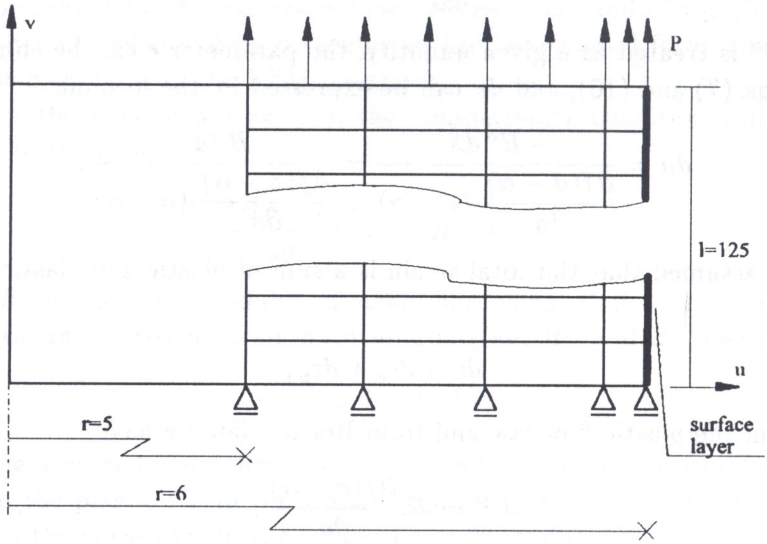

FIG. 5. Model of specimen with surface layer.

FEM-mesh (only the quarter of the specimen is considered due to the symmetry) and the methods of loading and supporting are shown. The whole mesh consists of 472 nodes, 132 eight-node elements and 33 film elements. The latter elements are marked by heavy lines in Fig. 5. This model ensures the satisfaction of the boundary conditions $v^c = v^s$, $u^c = u^s$ at the surface and core-layer interface [5, 6]. Figure 5 presents also the boundary conditions taken into account in the specimen model. Tensile force is modelled as a uniformly distributed load **p**, applied only to the cross-section of the core. Close to the simple support, at the bottom of the mesh, the model corresponds exactly to the central part of the real specimen, on which the strain gauge measurements are performed. The calculations based on the model confirm this assumption: the vertical displacements v in the cross-sections located close to the loaded end (Fig. 5) are nonuniform, and they become uniform in the sections located close to the sliding support. In the case of monotonic loading, the knowledge of the evolution parameter α is not necessary, and it is sufficient to know the yield point evolution $Y(\overline{\varepsilon}^p)$ only. Therefore $\alpha = 0$ is assumed, whereas the function $Y(\overline{\varepsilon}^p)$ is determined in the uniaxial tension test of a copper tube without the chromium layer. Figure 3 shows the result of the test.

In the program, the elasto-plastic part of the given stress-strain curve should be presented as a multi-segment broken line, and it represents a function of the plastic strain ($\overline{\varepsilon}^p$). In the calculations we assume the broken line to pass through the points marked in Fig. 6.

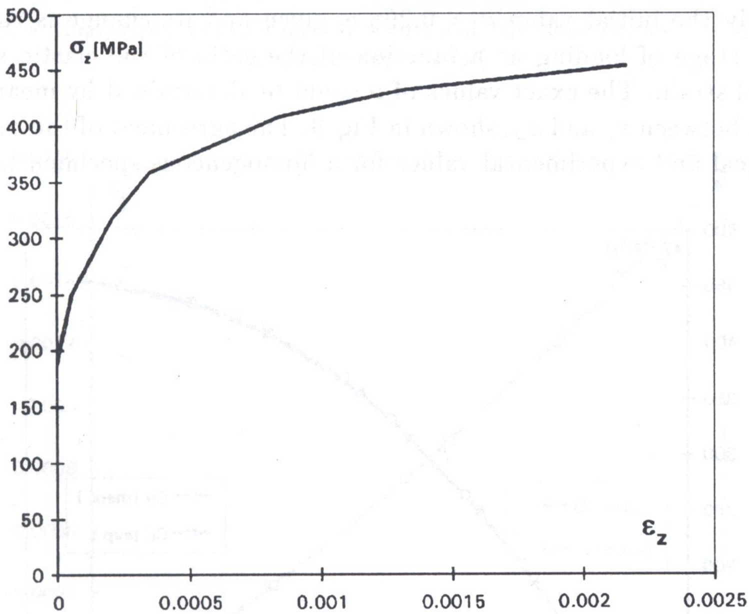

FIG. 6. Strain hardening curve for copper.

For the chromium layer, the following values of material constants have been assumed (c.f. [3]): $E = 250000$ MPa, $\sigma_e = Y(0) = 3600$ MPa, $\nu = 0.21$. In the experiment, the maximum value of tensile force is limited in order to obtain the reduced stress in chromium layer considerably lower than the given yield point, and it is about 900 MPa.

4. The comparison of the theoretical and experimental results

The relations $P - \varepsilon_z$ and $P - \varepsilon_\varphi$ obtained from the experiment were also calculated theoretically. Figures 7 and 8 present diagrams of the relations $P - \varepsilon_z$ and $P - \varepsilon_\varphi$ for a homogeneous copper specimen, calculated by means of the FEM, next to the same diagrams determined experimentally. The theoretical and experimental curves coincide. For the relation $P - \varepsilon_z$, the theoretical curve is a direct repetition of the data taken from the experimental curves. In case of the relation $P - \varepsilon_\varphi$, good agreement of the theoretical and experimental results is the verification of the model of material assumed in the program, and also is a partial verification of the program itself in the elasto-plastic range. It results from the following reasoning. The relation $P - \varepsilon_\varphi$ depends on the Poisson ratio ν which in the case of copper, varies from 0.298 (elastic state) to 0.5 (ideally plastic state). In the program data only the initial value $\nu = 0.298$ is given and its change is calculated at each stage of loading as a function of the ratio of the elastic strain to the total strain. The exact values of ν could be determined by means of the relation between ε_z and ε_φ, shown in Fig. 9. The agreement of the calculated theoretical and experimental values for a homogeneous specimen presented

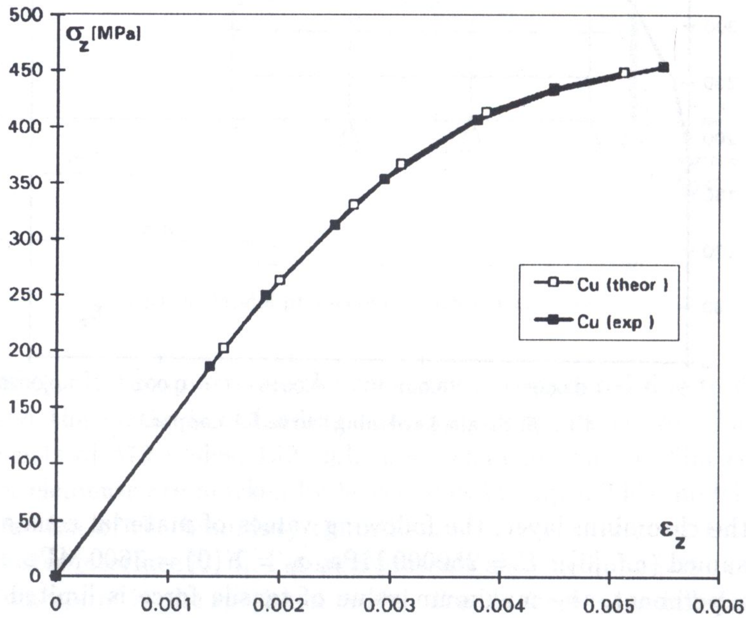

Fig. 7. Comparison of experimental results and calculated curves $P - \varepsilon_z$ for homogeneous specimen (Cu).

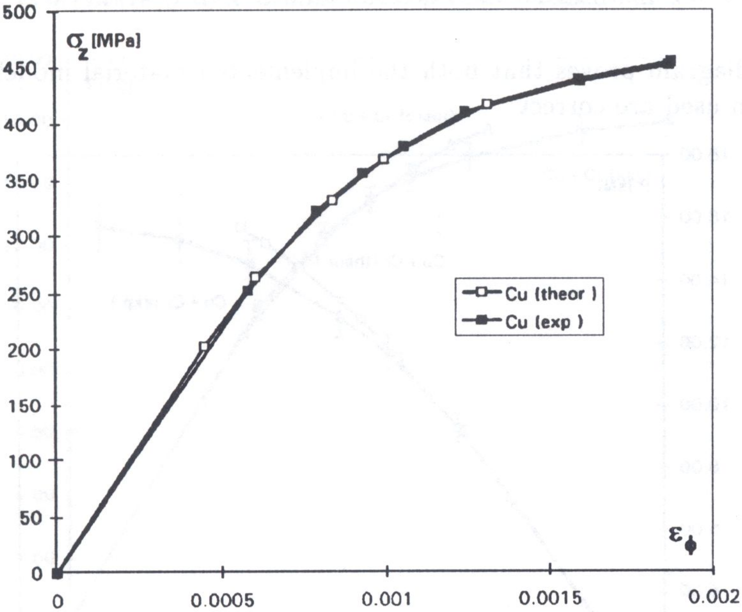

FIG. 8. Comparison of experimental results and calculated curves $P - \varepsilon_\varphi$ for homogeneous specimen (Cu).

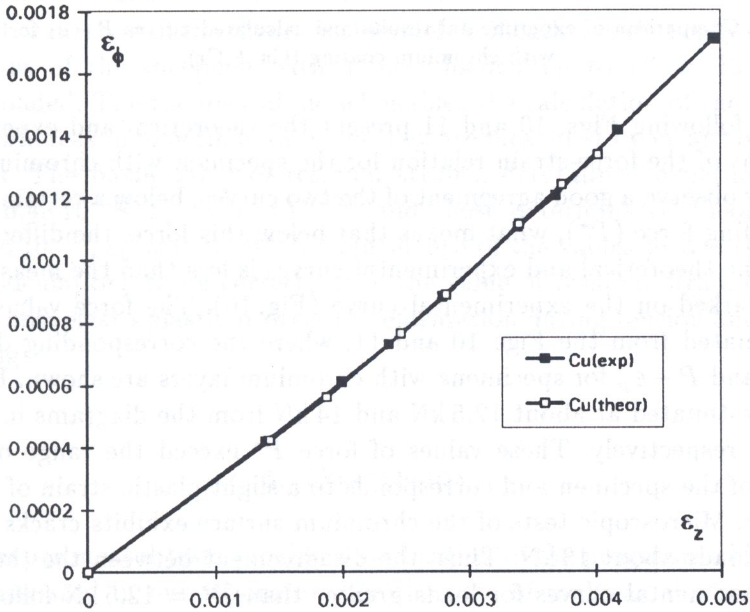

FIG. 9. Comparison of experimental results and calculated curves $\varepsilon_\varphi - \varepsilon_z$ for homogeneous specimen (Cu).

in the diagram proves that both the implemented material model and the program used are correct.

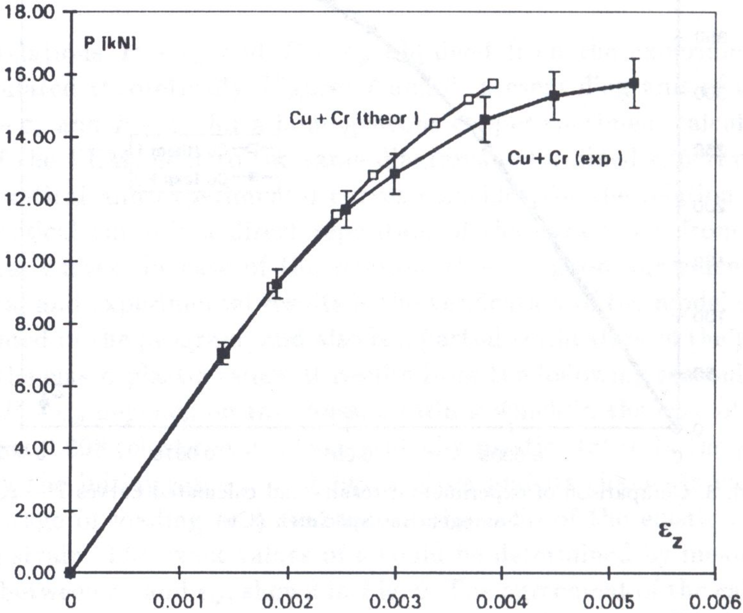

FIG. 10. Comparison of experimental results and calculated curves $P - \varepsilon_z$ for specimen with chromium coating (Cu + Cr).

The following Figs. 10 and 11 present the theoretical and experimental diagrams of the force-strain relation for the specimen with chromium layer. We may observe a good agreement of the two curves, below a certain value of the loading force (P^*), what means that below this force, the difference between the theoretical and experimental curves is less than the measurement error marked on the experimental curve (Fig. 10). The force value P^* can be estimated from the Figs. 10 and 11, where the corresponding diagrams $P - \varepsilon_z$ and $P - \varepsilon_\varphi$ for specimens with chromium layers are shown. Force P^* can be estimated at about 12.5 kN and 14 kN from the diagrams in Figs. 10 and 11, respectively. These values of force P^* exceed the range of elastic strains of the specimen and corresponds to a slight plastic strain of the copper core. Microscopic tests of the chromium surface exhibits cracks forming for the loads about 13 kN. Thus, the disagreement between the theoretical and experimental curves for loads greater than $P^* = 12.5$ kN follows from the fact that in the theoretical considerations, the chromium layer is capable of carrying the applied load, while in the experimental tests it gradually ceases to carry the load because of cracking. However, in the whole range of measurements the discrepancy does not exceed a few percent.

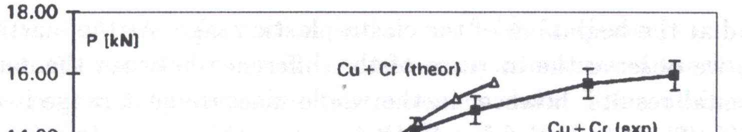

FIG. 11. Comparison of experimental results and calculated curves $P - \varepsilon_\varphi$ for specimen with chromium coating (Cu + Cr).

Another method of the theoretical model verification is the study of residual strains of the specimen with a layer loaded up to $P^* = 13\,\mathrm{kN}$ and then unloaded. The theoretical model enables the calculation of the residual strains, whereas in experimental tests they are measured by means of extensometers. The mean value of residual strains, determined experimentally, is $\varepsilon_z = 0.48\,10^{-03}$, $\varepsilon_\varphi = 0.23\,10^{-03}$, and those determinated theoretically $\varepsilon_z = 0.46\,10^{-03}$, $\varepsilon_\varphi = 0.21\,10^{-02}$. Coincidence of the values proves the theoretical calculations to be correct, since the value of residual strain depends on the whole elasto-plastic process of deformation during tension and elastic unloading.

5. CONCLUSIONS

The experimental tests provide a verification of the programs which were earlier prepared in our laboratory for the case of multi-axial stress state. For the homogeneous specimen, the theoretical and experimental curves show satisfactory agreement in the whole measurement range (elastic and elasto-plastic). For the specimens with a chromium layer, a good agreement between the theoretical and experimental curves is seen in the whole elastic range and at the beginning of the elasto-plastic range. As the plastic strains increase, we observe the increase of the difference between theoretical and experimental results, however in the whole measurement range it does not exceed 7%. This is a satisfying result in view of the accuracy of evaluation of the material constants (for chromium they are taken from tables) and the cracking phenomena occurring in the chromium layer. The studies described above verify the applied method of layer modelling. In the tested specimens with the layer, two-dimensional stress state (in spite of uniaxial tension applied) exists and elasto-plastic strains occur. The calculated values of strains ε_φ and ε_z are a consequence of this state.

The results of studies indicate that the surface layer, even in case of relatively small thickness, increases the stiffness of the element. The specimens with a surface layer exhibit smaller strains than the homogeneous specimens under the same forces. This effect could be easily predicted qualitatively; however its determination in the elasto-plastic range needs complicated calculations based on the FEM programs. In the considerations presented we observe the influence of surface layer only below a certain value of the loading force P^+ which is greater than P^* (defined earlier). Above this force P^+, the experimental tension curves for specimens with chromium layers gradually tend to coincide with the curves for homogeneous specimens. It follows from the specific features of the material of the tested layer. Chromium is a brittle material and thus it can be deformed only in a small range. The macroscopic tests of specimen under tension show that the limit of this range approximately coincides with force P^*. For the loads exceeding this force, layer fracture phenomena are observed which lead to a gradual strength reduction.

The verification presented in this paper is limited to a problem in which the strain field is homogeneous. Applying the easily available and inexpensive measurement set consisting of the testing machine and the extensometers, we may study the multiaxial stress state in relatively simple cases. The studies presented are treated as the first approach to further verification in the cases of large strain gradients. This kind of problems occur for instance in notch analysis or even in the specimen considered here in the direct neighbourhood of the holder. However, the experimental tests in such cases require a much more expensive and complicated measuring apparatus.

The elasto-plastic model of the element with surface layer described here, which was verified experimentally for one case of a composite core-surface layer (copper-chromium), needs further verification in experimental tests.

Nevertheless, the results of the studies performed make us believe that the model will prove to be useful in analyzing the influence of different surface layers on the deformation and stress fields in elements subjected to static load. We may analyse more geometrically complicated specimens with the same program by changing the necessary data. The obtained results indicate that this type of model could also be applied to determine the material constants of technological layers of unknown properties (by solving the inverse problem). The simulation studies concerning the strain and stress field in the surface layer and the core of a specimen subjected to cyclic loading are advanced as well.

References

1. K. Komvopoulos, *Finite element analysis of a layered elastic solid in normal contact with a rigid surface*, ASME J. Tribology, 110, pp. 477–485, 1988.
2. A.K. Bhattacharya and W.D. Nix, *Analysis of elastic and plastic deformation associated with indentation testing of thin films on substrates*, Int. J. Solid Struct., **24**, 12, pp. 1287–1298, 1988.
3. P. Montmitonnet, M.L. Edlinger and E. Felder, *Finite element analysis of elastoplastic indentation. Part II. Application to hard coatings*, J. Tribology, 115, pp. 15–19, 1993.
4. Z. Mróz and G. Starzyński, *An elasto-plastic analysis of a cylinder with surface layer under cyclic loading*, Arch. Machine Design, 1994 [in press].
5. Z. Handzel-Powierża, S. Kucharski, G. Starzyński and M. Dąbrowski, *Methods of determination of stress fields in elements with technological surface layer strained in elasto-plastic range* [in Polish], Zesz. Nauk. PW, Warszawa 1990.
6. Z. Handzel-Powierża and S. Kucharski, *Determination of stresses in structures with TSL by the method of finite elements* [in Polish], Rozpr. Inż., **38**, 3-4, pp. 635–647, 1990.
7. J. Skrzypek, *Plasticity and creep – theory, applications, problems*, PWN, Warszawa 1986.
8. Z. Mróz, *Phenomenological constitutive models for metals*, [in:] Modelling of Small Deformations of Polycrystals, J. Zarka [Ed], Elsevier Appl.Sci., 1986.
9. D.R.J. Owen and E. Hinton, *Finite element in plasticity. Theory and practice*, Pineridge Press Ltd., Swansea 1980.

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