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MAGNETOHYDRODYNAMIC FLOW OF VISCOUS FLUID IN A SLOT BETWEEN CURVILINEAR SURFACES OF REVOLUTION

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This paper contains the study of the magnetohydrodynamic, steady, laminar flow of viscous fluid in a slot between curvilinear surfaces of revolution having a common axis of symmetry. The boundary layer equations are expressed in terms of the intrinsic curvilinear coordinate system x, θ, y. The method of perturbation is used to solve the boundary layer equations. As a result, the formulae for parameters of the flow, i.e. the velocity components $\overline{V}_x$, $\overline{V}_\theta$, $\overline{V}_y$ and pressure $\overline{p}$ are obtained.

NOTATIONS

B_k vector of magnetic induction,
E_k vector of electric field intensity,
H_k vector of magnetic field intensity,
Ha Hartmann number,
$2h(x)$ thickness of slot,
h_g, h_d thickness of upper and lower curvilinear conducting surfaces,
I total current,
J_i current density vector,
Ω_c dimensionless parameter corresponding to the effect of centrifugal inertia forces,
Ω_L dimensionless parameter corresponding to the effect of longitudinal inertia forces,
p pressure,
p_i inlet pressure,
p_o outlet pressure,
$R(x)$ radius of the midsurface of the slot,
Re Reynolds number,
V_i velocity vector,
x, θ, y curvilinear coordinates,
X, Y, Z Cartesian coordinates,

ϕ angle determining the position of section between the spherical surfaces,
ϕ_i, ϕ_o angles determining the positions at the inlet and outlet sections,
Φ_d, Φ_g dimensionless parameters determining conductances of the lower and upper surfaces,
μ fluid dynamic viscosity,
σ electrical conductivity,
σ_d, σ_g electrical conductivity of the rotating lower and upper surfaces,
ω_1, ω_2 angular velocities of the lower and upper surfaces.

1. Introduction

Laminar flows of electrically conducting fluids in slots between rotating curvilinear surfaces subject to the action of magnetic and electric fields attracted much attention since many years [1, 2, 3, 4]. New possibilities offered by electrically conducting liquids in solving various problems of construction and operation in the fields of friction, lubrication and wear of machine elements, contributed to the growing interest in the research concerning this type of flows. The present paper is aimed at investigating the effects of electromagnetic fields and inertia forces upon the flows of viscous fluids.

2. Equations of fluid flows

The motion of electrically conducting viscous fluids flowing through a slot between the curvilinear surfaces of revolution, presented in Fig. 1, is assumed to be laminar, stationary and isothermal. The flow occurs in the presence of external magnetic $(0, 0, B_y)$ and electric $(E_x, 0, 0)$ fields in the slot bounded by electrically conducting surfaces. The magnetic Reynolds number Re_M is assumed to be much less than unity, $\mathrm{Re}_M \ll 1$, what makes it possible to disregard the effect of the magnetic field induced by the motion of the fluid.

On the basis of the general laws of conservation of the mass and momentum, the equations of motion written in indicial notation assume the form:

$$V_{i,i} = 0, \tag{2.1}$$

$$\varrho V_j V_{i,j} = -p_{,i} + \tau_{ij,j} + \varepsilon_{ijk} J_j B_k . \tag{2.2}$$

Here

$$\tau_{ij} = \mu(V_{i,j} + V_{j,i}).$$

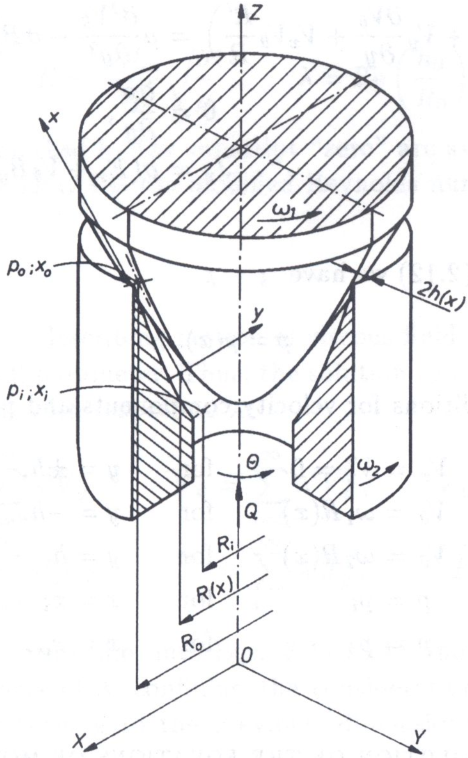


FIG. 1. Flow region of fluid.

"Closure" of the set of Eqs. (2.1), (2.2) necessitates the additional equations of the electromagnetic field:

$$\varepsilon_{ijk}E_{k,j} = 0, \tag{2.3}$$

$$E_{i,i} = 0, \tag{2.4}$$

$$\varepsilon_{ijk}B_{k,j} = \mu_o J_i\,, \tag{2.5}$$

$$B_{i,i} = 0, \tag{2.6}$$

$$J_{i,i} = 0, \tag{2.7}$$

$$J_i = \sigma(E_i + \varepsilon_{ijk}V_jB_k). \tag{2.8}$$

Writing Eqs. (2.1), (2.2), (2.8) in the curvilinear system of coordinates (x,θ,y) (cf. [5]) and applying the estimations typical for flows occurring in thin layers, $h \ll R$, we obtain:

$$\frac{1}{R}\frac{\partial(RV_x)}{\partial x} + \frac{\partial V_y}{\partial y} = 0, \tag{2.9}$$

$$\varrho\left(V_x\frac{\partial V_x}{\partial x} + V_y\frac{\partial V_x}{\partial y} - V_\theta^2\frac{R'}{R}\right) = -\frac{\partial p}{\partial x} + \mu\frac{\partial^2 V_x}{\partial y^2} - \sigma B_y^2 V_x\,, \tag{2.10}$$

$$(2.11)\qquad \varrho\left(V_x\frac{\partial V_\theta}{\partial x}+V_y\frac{\partial V_\theta}{\partial y}+V_xV_\theta\frac{R'}{R}\right)=\mu\frac{\partial^2 V_\theta}{\partial y^2}-\sigma B_y^2V_\theta-\sigma E_xB_y\,,$$

$$(2.12)\qquad 0=\frac{\partial p}{\partial y}\,,$$

$$(2.13)\qquad J_x=\sigma(E_x+V_\theta B_y),$$

where $B_y = B_0$.

Thus, from Eq. (2.12) we have

$$(2.14)\qquad p=p(x).$$

The boundary conditions for velocity components and pressure are:

$$(2.15)\qquad \begin{aligned} V_x = V_y &= 0 && \text{for} && y=\pm h,\\ V_\theta &= \omega_1 R(x) && \text{for} && y=-h,\\ V_\theta &= \omega_2 R(x) && \text{for} && y=h,\\ p &= p_i && \text{for} && x=x_i\,,\\ p &= p_o && \text{for} && x=x_o\,. \end{aligned}$$

3. Solution of the equations of motion

Introducing the following dimensionless quantities:

$$\overline{x}=\frac{x}{R_o}\,,\qquad \overline{R}=\frac{R}{R_o}\,,\qquad \overline{y}=\frac{y}{h_o}\,,$$

$$\overline{V}_x=\frac{V_x}{V_o}\,,\qquad \overline{V}_\theta=\frac{V_\theta}{V_o}\,,\qquad \overline{V}_y=\frac{V_y}{V_o}\frac{R_o}{h_o}\,,\qquad \overline{p}=\frac{ph_o}{\mu V_o}\frac{h_o}{R_o}\,,$$

we can present the equations of motion (2.9)–(2.13) in the form:

$$(3.1)\qquad \frac{1}{\overline{R}}\frac{\partial(\overline{R}\,\overline{V}_x)}{\partial\overline{x}}+\frac{\partial\overline{V}_y}{\partial\overline{y}}=0,$$

$$(3.2)\qquad \lambda\left(\overline{V}_x\frac{\partial\overline{V}_x}{\partial\overline{x}}+\overline{V}_y\frac{\partial\overline{V}_x}{\partial\overline{y}}-\overline{V}_\theta^2\frac{\overline{R}'}{\overline{R}}\right)=-\frac{\partial\overline{p}}{\partial\overline{x}}+\mu\frac{\partial^2\overline{V}_x}{\partial\overline{y}^2}-\mathrm{Ha}^2\overline{V}_x\,,$$

$$(3.3)\qquad \lambda\left(\overline{V}_x\frac{\partial\overline{V}_\theta}{\partial\overline{x}}+\overline{V}_y\frac{\partial\overline{V}_\theta}{\partial\overline{y}}+\overline{V}_x\overline{V}_\theta\frac{\overline{R}'}{\overline{R}}\right)=\mu\frac{\partial^2\overline{V}_\theta}{\partial\overline{y}^2}-\mathrm{Ha}^2\overline{V}_\theta-\mathrm{Ha}\overline{E},$$

$$(3.4)\qquad 0=\frac{\partial\overline{p}}{\partial\overline{y}}\,,$$

where

$$\mathrm{Ha} = B_0 h_0 \sqrt{\frac{\sigma}{\mu}}, \qquad E = \frac{h_0 E_x}{V_0}\sqrt{\frac{\sigma}{\mu}}, \qquad \lambda = \mathrm{Re}\left(\frac{h_0}{R_0}\right), \qquad \mathrm{Re} = \frac{\varrho V_0 h_0}{\mu}.$$

The quantities marked by the subscript "zero" are average values within a discussed flow domain, λ is the modified Reynolds number which satisfies the condition:

$$\lambda < 1. \tag{3.5}$$

In Eqs. (3.2)–(3.4) describing motion of viscous fluid, if condition (3.5) is satisfied, λ is a small parameter. Thus, the solution can be sought for in the form of power series with respect to λ:

$$\begin{aligned} \overline{V}_x &= \sum_{i=0}^{\infty} \lambda^i \overline{V}^i_x, \qquad & \overline{V}_\theta &= \sum_{i=0}^{\infty} \lambda^i \overline{V}^i_\theta, \qquad & \overline{V}_y &= \sum_{i=0}^{\infty} \lambda^i \overline{V}^i_y, \\ \overline{p} &= \sum_{i=0}^{\infty} \lambda^i \overline{p}^i, & \overline{J}_x &= \sum_{i=0}^{\infty} \lambda^i \overline{J}^i_y, & \overline{E} &= \sum_{i=0}^{\infty} \lambda^i \overline{E}^i. \end{aligned} \tag{3.6}$$

Introducing the series (3.6) into Eqs. (3.1)–(3.4), and grouping the terms with the same powers of λ, confining the considerations to the linear approximation and returning to the previous, dimensional form, we get the equations:

$$\frac{1}{R}\frac{\partial(RV^0_x)}{\partial x} + \frac{\partial V^0_y}{\partial y} = 0, \tag{3.7}$$

$$0 = -\frac{\partial p^0}{\partial x} + \mu\frac{\partial^2 V^0_x}{\partial y^2} - \sigma B^2_y V^0_x, \tag{3.8}$$

$$0 = \frac{\partial p^0}{\partial y}, \tag{3.9}$$

$$0 = \mu\frac{\partial^2 V^0_\theta}{\partial y^2} - \sigma B^2_y V^0_\theta - \sigma E^0_x B_y, \tag{3.10}$$

$$\frac{1}{R}\frac{\partial(RV^1_x)}{\partial x} + \frac{\partial V^1_y}{\partial y} = 0, \tag{3.11}$$

$$\varrho\left[V^0_x\frac{\partial V^0_x}{\partial x} + V^0_y\frac{\partial V^0_x}{\partial y} - (V^0_\theta)^2\frac{R'}{R}\right] = -\frac{\partial p^1}{\partial x} + \mu\frac{\partial^2 V^1_x}{\partial y^2} - \sigma B^2_y V^1_x, \tag{3.12}$$

$$0 = \frac{\partial p^1}{\partial y}, \tag{3.13}$$

$$\varrho\left(V^0_x\frac{\partial V^0_\theta}{\partial x} + V^0_y\frac{\partial V^0_\theta}{\partial y} + V^0_x V^0_\theta\frac{R'}{R}\right) = \mu\frac{\partial^2 V^1_\theta}{\partial y^2} - \sigma B^2_y V^1_\theta - \sigma E^1_x B_y. \tag{3.14}$$

The boundary conditions, in accordance with Eqs. (2.15), have the form:

$$
\begin{aligned}
&V_x^0 = V_y^0 = V_x^1 = V_y^1 = V_\theta^1 = 0 && \text{for } y = \pm h,\\
&V_\theta^0 = \omega_1 R(x) && \text{for } y = h,\\
&V_\theta^0 = \omega_2 R(x) && \text{for } y = -h,\\
&p^0 = p_i\,, \qquad p^1 = 0 && \text{for } x = x_i\,,\\
&p^0 = p_o\,, \qquad p^1 = 0 && \text{for } x = x_o\,.
\end{aligned} \tag{3.15}
$$

Integrating Eqs. (3.7)–(3.14) with the boundary conditions (3.15) we have:

$$V_x^0 = \frac{1}{\mu k^2 R}\frac{C_0}{\operatorname{th} kh - kh}\left(\frac{\operatorname{ch} ky}{\operatorname{ch} kh} - 1\right), \tag{3.16}$$

$$V_y^0 = \frac{C_0 h}{\mu k^2 R}\frac{\operatorname{th} kh(ky \operatorname{sh} kh - kh \operatorname{sh} ky)}{\operatorname{ch} kh(\operatorname{th} kh - kh)^2}, \tag{3.17}$$

$$V_\theta^0 = \frac{R}{2}\left[(\omega_1 - \omega_2)\frac{\operatorname{ch} ky}{\operatorname{ch} kh} + (\omega_1 - \omega_2)\frac{\operatorname{sh} ky}{\operatorname{sh} kh}\right] + \frac{E_x^0}{k}\sqrt{\frac{\sigma}{\mu}}\left(\frac{\operatorname{ch} ky}{\operatorname{ch} kh} - 1\right), \tag{3.18}$$

$$p^0 = \frac{[A(x) - A_0]p_i - [A(x) - A_i]p_0}{A_i - A_0}, \tag{3.19}$$

$$
\begin{aligned}
V_x^1 = \frac{1}{6}\Phi_1\Bigg[&2(\operatorname{ch} kh \operatorname{ch} ky + 2)\left(\frac{\operatorname{ch} ky}{\operatorname{ch} kh} - 1\right)\\
&- \frac{(4\operatorname{th} kh - 3kh - 0.5\operatorname{sh} 2kh)}{\operatorname{th} kh - kh}\left(\frac{\operatorname{ch} ky}{\operatorname{ch} kh} - 1\right)\Bigg]\\
+ \frac{1}{4}\Phi_2\Bigg[&2ky \operatorname{sh} ky - 2kh \operatorname{th} kh \operatorname{ch} ky - \frac{(2kh - \operatorname{sh} 2kh)}{\operatorname{ch} kh\,(\operatorname{th} kh - kh)}\left(\frac{\operatorname{ch} ky}{\operatorname{ch} kh} - 1\right)\Bigg]\\
+ \frac{1}{4}\Phi_3\Bigg[&(k^2y^2 - k^2h^2)\operatorname{ch} ky - ky \operatorname{sh} ky + kh \operatorname{th} kh \operatorname{ch} ky\\
&- \frac{(1.5\operatorname{sh} kh - kh \operatorname{ch} 2kh - 2kh)}{\operatorname{ch} kh\,(\operatorname{th} kh - kh)}\left(\frac{\operatorname{ch} ky}{\operatorname{ch} kh} - 1\right)\Bigg]\\
+ \frac{1}{6}\Phi_4\Bigg[&2(\operatorname{ch} ky \operatorname{ch} kh - 1) - \frac{(3kh - 2\operatorname{th} kh - 0.5\operatorname{sh} 2kh)}{\operatorname{th} kh - kh}\Bigg]\left(\frac{\operatorname{ch} ky}{\operatorname{ch} kh} - 1\right)\\
&+ \frac{1}{3}\Phi_5(\operatorname{sh} 2ky - 2\operatorname{sh} ky \operatorname{ch} kh) + \frac{1}{2}\Phi_6(ky \operatorname{ch} ky - kh \operatorname{cth} kh \operatorname{ch} ky),
\end{aligned} \tag{3.20}
$$

$$
\begin{aligned}
V_y^1 = -\frac{1}{R}\frac{\partial}{\partial x}\Bigg\{&\frac{R\Phi_1}{6k}\Bigg[\operatorname{sh} ky \operatorname{ch} ky + ky + 4\left(\frac{\operatorname{sh} ky}{\operatorname{sh} kh} - ky\right)\\
&-2\operatorname{ch} kh \operatorname{sh} ky - \frac{4\operatorname{th} kh - 3\,kh - 0.5\operatorname{sh} 2kh}{\operatorname{th} kh - kh}\left(\frac{\operatorname{sh} ky}{\operatorname{ch} kh} - ky\right)\Bigg]\\
&+ \frac{R\Phi_2}{4k}\Bigg[2ky \operatorname{ch} ky - 2\operatorname{sh} ky - 2kh \operatorname{th} kh\, h \operatorname{sh} ky
\end{aligned} \tag{3.21}
$$

(3.21) [cont.]
$$-\frac{2kh-\operatorname{sh}2kh}{\operatorname{ch}kh\,(\operatorname{th}kh-kh)}\left(\frac{\operatorname{sh}ky}{\operatorname{ch}kh}-ky\right)\Big]$$
$$+\frac{R\Phi_3}{4k}\Big[(k^2y^2-k^2h^2)\operatorname{sh}ky+3\operatorname{sh}ky-3ky\operatorname{ch}ky+kh\operatorname{th}kh\operatorname{sh}ky$$
$$-\frac{1.5\operatorname{sh}2kh-kh-2kh}{\operatorname{ch}kh(\operatorname{th}kh-kh)}\left(\frac{\operatorname{sh}ky}{\operatorname{ch}kh}-ky\right)\Big]$$
$$+\frac{R\Phi_4}{6k}\Big[\operatorname{sh}ky\operatorname{ch}ky+3ky-3(\operatorname{ch}^2kh+1)\frac{\operatorname{sh}ky}{\operatorname{ch}kh}$$
$$-\frac{3kh-2\operatorname{th}kh-0.5\operatorname{sh}2kh}{\operatorname{th}kh-kh}\left(\frac{\operatorname{sh}ky}{\operatorname{ch}kh}-ky\right)\Big]$$
$$+\frac{R\Phi_5}{6k}(\operatorname{ch}2ky-4\operatorname{ch}kh\operatorname{ch}ky-\operatorname{ch}2kh+4\operatorname{ch}^2kh)$$
$$+\frac{R\Phi_6}{2k}(ky\operatorname{sh}ky-\operatorname{ch}ky-kh\operatorname{cth}kh\operatorname{ch}ky$$
$$-kh\operatorname{sh}kh\operatorname{th}ky+kh\operatorname{cth}kh\operatorname{ch}ky\Big\},$$

(3.22)
$$V_\theta^1=\frac{1}{3}\Psi_1(\operatorname{ch}ky\operatorname{ch}kh+2)\left(\frac{\operatorname{ch}ky}{\operatorname{ch}kh}-1\right)$$
$$+\frac{1}{2}\Psi_2(ky\operatorname{sh}ky-kh\operatorname{th}kh\operatorname{ch}ky)+\frac{1}{3}\Psi_3(\operatorname{sh}2ky-2\operatorname{ch}kh\operatorname{sh}ky)$$
$$+\frac{1}{2}\Psi_4(ky\operatorname{ch}ky-kh\operatorname{cth}kh\operatorname{sh}ky)+\frac{1}{3}\Psi_5(\operatorname{ch}ky\operatorname{ch}kh-1)\left(\frac{\operatorname{ch}ky}{\operatorname{ch}kh}-1\right)$$
$$+\frac{1}{4}\Psi_6\left[\operatorname{ch}ky(k^2y^2-k^2h^2)+kh\operatorname{th}kh\operatorname{ch}ky-ky\operatorname{sh}ky\right]$$
$$+\frac{1}{4}\Psi_7\left[\operatorname{sh}ky(k^2y^2-k^2h^2)+kh\operatorname{cth}kh\operatorname{sh}ky-ky\operatorname{ch}ky\right]$$
$$+\Psi_8\left(\frac{\operatorname{ch}ky}{\operatorname{ch}kh}-1\right),$$

(3.23)
$$p^1=B(x)-\frac{[A(x)-A_0]B_i-[A(x)-A_i]B_0}{A_i-A_0}.$$

The complete solution to the fluid flow problem inside the slot between (in general) curvilinear surfaces consists of the sum of partial solutions V^0 and V^1.

Functions C, E_x^0, $A(x)$, $\Phi_1-\Phi_6$, $\Psi_1-\Psi_8$, $B(x)$, k appearing in the solutions (3.16)–(3.23) are presented in explicit forms in Part I of the Appendix. Effective tabulation of the formulae (3.16)–(3.22) will be possible after assuming the forms of the surfaces bounding the slot.

4. Motion of fluid between rotating spherical surfaces

The parameters describing the geometry of the flow region can be written as follows (Fig. 2):

$$
(4.1)\qquad \begin{aligned} &R = R_k \sin(\phi), \qquad && R_i = R_k \sin(\phi_i), \qquad && R_o = R_k \sin(\phi_o),\\ &\phi = \frac{x}{R_k}, && R' = \cos(\phi). && \end{aligned}
$$

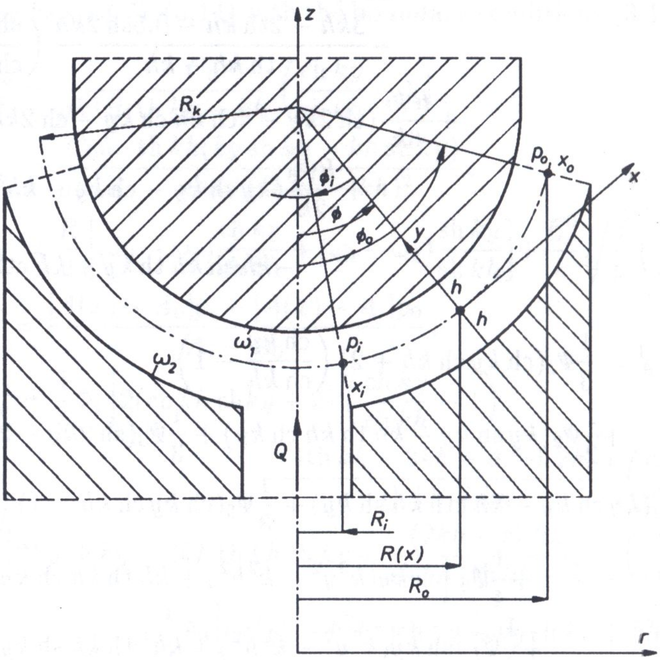


FIG. 2. Spherical clearance geometry.

Introducing Eqs. (4.1) into the solutions (3.16)–(3.23) and using the following dimensionless quantities:

$$
(4.2)\qquad \begin{aligned} &\overline{y} = \frac{y}{h}, \qquad \overline{R} = \frac{R}{R_k}\sin(\phi), \qquad R' = \cos(\phi), \qquad \overline{V}_x = \frac{V_x}{V^0_{x\,\max}},\\ &V_y = \frac{V_y}{V^0_{x\,\max}}\,\frac{R_k \sin(\phi_o)}{h}, \qquad V_\theta = \frac{V_\theta}{\omega_1 R_k \sin(\phi_o)}\,\frac{2}{\sin(\phi)}. \end{aligned}
$$

where

$$
V^0_{x\,\max} = -\frac{1}{2}\,\frac{p_o\, h^2}{\mu R_k \sin(\phi_o)}\,\frac{\overline{p}_i - 1}{a_i - a_o}\,\frac{1}{\sin(\phi)},
$$

we obtain the formulae representing the motion of the fluid inside the clearance between the rotating spherical surfaces:

Approximation of order zero:

$$\overline{V}_x = \frac{2}{\mathrm{Ha}^2}\left(1 - \frac{\mathrm{ch\,Ha}\,\overline{y}}{\mathrm{ch\,Ha}}\right), \tag{4.3}$$

$$\overline{V}_y^0 = 0, \tag{4.4}$$

$$\overline{V}_\theta^0 = \left[(1+K)\frac{\mathrm{ch\,Ha}\,\overline{y}}{\mathrm{ch\,Ha}} + (1-K)\frac{\mathrm{sh\,Ha}\,\overline{y}}{\mathrm{sh\,Ha}}\right] + \frac{2}{\sin(\phi)}\frac{E^0}{\mathrm{Ha}}\left(\frac{\mathrm{ch\,Ha}\,\overline{y}}{\mathrm{ch\,Ha}} - 1\right), \tag{4.5}$$

$$\overline{p}^0 = \frac{[a(\phi) - a_o]p_i - [a(\phi) - a_i]}{a_i - a_o}. \tag{4.6}$$

Linear approximation (sum of partial solutions V^0 and V^1)

$$\begin{aligned}\overline{V}_x = V_x^0 - 2\sin(\phi)\frac{a_i - a_o}{\overline{p}_i - 1}\Bigg\{&\frac{1}{6}\Phi_1\Bigg[2(\mathrm{ch\,Ha\,ch\,Ha}\,y + 2)\left(\frac{\mathrm{ch\,Ha}\,\overline{y}}{\mathrm{ch\,Ha}} - 1\right)\\ &- \frac{4\mathrm{th\,Ha} - 3\mathrm{Ha} - 0.5\mathrm{sh\,2Ha}}{\mathrm{th\,Ha} - \mathrm{Ha}}\left(\frac{\mathrm{ch\,Ha}\,\overline{y}}{\mathrm{ch\,Ha}} - 1\right)\Bigg]\\ &+ \frac{1}{4}\Phi_2\Bigg[2\mathrm{Ha}\,\overline{y}\,\mathrm{sh\,Ha}\,\overline{y} - 2\mathrm{Ha\,th\,Ha\,ch\,Ha}\,y\\ &- \frac{2\mathrm{Ha} - \mathrm{sh\,2Ha}}{\mathrm{ch\,Ha}(\mathrm{th\,Ha} - \mathrm{Ha})}\left(\frac{\mathrm{ch\,Ha}\,\overline{y}}{\mathrm{ch\,Ha}} - 1\right)\Bigg]\\ &+ \frac{1}{4}\Phi_3\Bigg[(\mathrm{Ha}^2\,y^2 - \mathrm{Ha}^2)\frac{\mathrm{ch\,Ha}\,\overline{y}}{\mathrm{ch\,Ha}} - \mathrm{Ha}\,y\,\mathrm{sh\,Ha}\,y + \mathrm{Ha\,th\,Ha\,ch\,Ha}\,y\\ &- \frac{1.5\mathrm{sh\,2Ha} - \mathrm{Ha\,ch\,2Ha} - 2\mathrm{Ha}}{\mathrm{ch\,Ha}(\mathrm{th\,Ha} - \mathrm{Ha})}\left(\frac{\mathrm{ch\,Ha}\overline{y}}{\mathrm{ch\,Ha}} - 1\right)\Bigg]\\ &+ \frac{1}{6}\Phi_4\Bigg[2(\mathrm{ch\,Ha}\,\overline{y}\mathrm{ch;\ Ha} - 1)\left(\frac{\mathrm{ch\,Ha}\,\overline{y}}{\mathrm{ch\,Ha}} - 1\right)\\ &- \frac{3\mathrm{Ha} - 2\mathrm{th\,Ha} - 0.5\mathrm{sh\,2Ha}}{\mathrm{th\,Ha} - \mathrm{Ha}}\left(\frac{\mathrm{ch\,Ha}\,\overline{y}}{\mathrm{ch\,Ha}} - 1\right)\Bigg]\\ &+ \frac{1}{3}\Phi_5(\mathrm{sh\,2Ha}\,\overline{y} - 2\mathrm{sh\,Ha}\,y\,\mathrm{ch\,Ha}) + \frac{1}{2}\Phi_6(\mathrm{Ha}\,\overline{y}\,\mathrm{ch\,Ha}\,\overline{y} - \mathrm{Ha\,th\,Ha\,ch\,Ha}\,y)\Bigg\},\end{aligned} \tag{4.7}$$

$$\begin{aligned}\overline{V}_y = \sin(\phi)\frac{a_i - a_o}{\overline{p}_i - 1}\Bigg\{&\frac{1}{3}\Phi_{11}\Bigg[\mathrm{sh\,Ha}\,\overline{y}\,\mathrm{ch\,Ha}\,\overline{y} + \mathrm{Ha}\overline{y}\\ &- 2\mathrm{Ha\,ch\,Ha}\,\overline{y} + 4\left(\frac{\mathrm{sh\,Ha}\,\overline{y}}{\mathrm{ch\,Ha}} - \mathrm{Ha}\,\overline{y}\right)\\ &- \frac{4\mathrm{th\,Ha} - 3\mathrm{Ha} - 0.5\mathrm{sh\,2Ha}}{\mathrm{th\,Ha} - \mathrm{Ha}}\left(\frac{\mathrm{sh\,Ha}\,\overline{y}}{\mathrm{ch\,Ha}} - \mathrm{Ha}\,\overline{y}\right)\Bigg]\end{aligned} \tag{4.8}$$

(4.8) [cont.]

$$+\frac{1}{2}\Phi_{22}\Big[2\text{Ha}\,\overline{y}\,\text{ch Ha}\,\overline{y} - 2\text{sh Ha}\,\overline{y} - 2\text{Ha th Ha sh Ha}\,y - \frac{2\text{Ha} - \text{sh }2\text{Ha}}{\text{ch Ha}(\text{th Ha} - \text{Ha})}\left(\frac{\text{sh Ha}\,\overline{y}}{\text{ch Ha}} - \text{Ha}\,y\right)\Big]$$

$$+\frac{1}{3}\Phi_{44}\Big(\text{sh Ha}\,\overline{y}\,\text{ch Ha}\,\overline{y} + 3\text{Ha}\,\overline{y} - 2(\text{ch}^2\text{Ha}+1)\frac{\text{sh Ha}\,\overline{y}}{\text{ch Ha}} - \frac{3\text{Ha} - 2\text{th Ha} - 0.5\text{sh }2\text{Ha}}{\text{th Ha} - \text{Ha}}\left(\frac{\text{sh Ha}\,\overline{y}}{\text{ch Ha}} - \text{Ha}\right)\Big]$$

$$+\frac{1}{3}\Phi_{55}\left(\text{ch }2\text{Ha}\,\overline{y} - 4\text{ch Ha ch Ha}\,\overline{y} - \text{ch }2\text{Ha} + 4\text{ch}^2\text{Ha}\right)$$

$$+\Phi_{66}\Big(\text{Ha}\,\overline{y}\,\text{sh Ha}\,\overline{y} - \text{ch Ha}\,\overline{y} - \text{Ha cth Ha ch Ha}\,\overline{y} - \text{Ha sh Ha} + \text{ch Ha} + \text{Ha cth Ha ch Ha}\,\overline{y}\Big)\Big\},$$

(4.9)
$$\overline{V}_\theta = \overline{V}_\theta^0 + \frac{1}{12\sin(\phi)}\Big\{8\Psi_1(\text{ch Ha}\,\overline{y} - \text{ch Ha} + 2)\left(\frac{\text{ch Ha}\,\overline{y}}{\text{ch Ha}} - 1\right) + 12\Psi_2(\text{Ha}\,\overline{y}\,\text{sh Ha}\,\overline{y} - \text{Ha th Ha ch Ha}\,\overline{y}) + 8\Psi_3(\text{sh }2\text{Ha}\,\overline{y} - 2\text{Ha sh Ha}\,\overline{y}) + 12\Psi_4(\text{Ha}\,\overline{y}\,\text{ch Ha}\,\overline{y} - \text{Ha cth Ha sh Ha}\,\overline{y}) + 24\Psi_5\left(\frac{\text{ch Ha}\,\overline{y}}{\text{ch Ha}} - 1\right)\Big\},$$

(4.10)
$$\overline{p} = \overline{B}(\phi) - \frac{[a(\phi) - a_o](\overline{B}_i - \overline{p}_i) - [a(\phi) - a_i](\overline{B}_o - 1)}{a_i - a_o}.$$

Functions $a(\phi)$, $\Phi_1 - \Phi_6$, $\Psi_1 - \Psi_8$, $\overline{B}(\phi)$, Ha, $\Phi_{11} - \Phi_{66}$ are defined in Part II of the Appendix.

Table 1.

$\overline{y}$	$\phi = 20°$		$\phi = 80°$	
	$\Omega_L = \Omega_c = 0$	$\Omega_L = 0.023$, $\Omega_c = 0$	$\Omega_L = \Omega_c = 0$	$\Omega_L = 0.023$, $\Omega_c = 0$
−1.0000	0.0000	0.0000	0.0000	0.0000
−0.8000	0.1575	0.1559	0.1575	0.1573
−0.6000	0.2594	0.2583	0.2594	0.2592
−0.4000	0.3223	0.3226	0.3223	0.3223
−0.2000	0.3563	0.3578	0.3563	0.3565
0.0000	0.3671	0.3691	0.3671	0.3673
0.2000	0.3563	0.3578	0.3563	0.3565
0.4000	0.3223	0.3226	0.3223	0.3223
0.6000	0.2594	0.2583	0.2594	0.2592
0.8000	0.1575	0.1559	0.1575	0.1573
1.0000	0.0000	0.0000	0.0000	0.0000

The above formulae have been illustrated in Figs. 3 – 13, while the influence of longitudinal inertial forces on the longitudinal velocity component profile $\overline{V}_x$ is shown in Table 1.

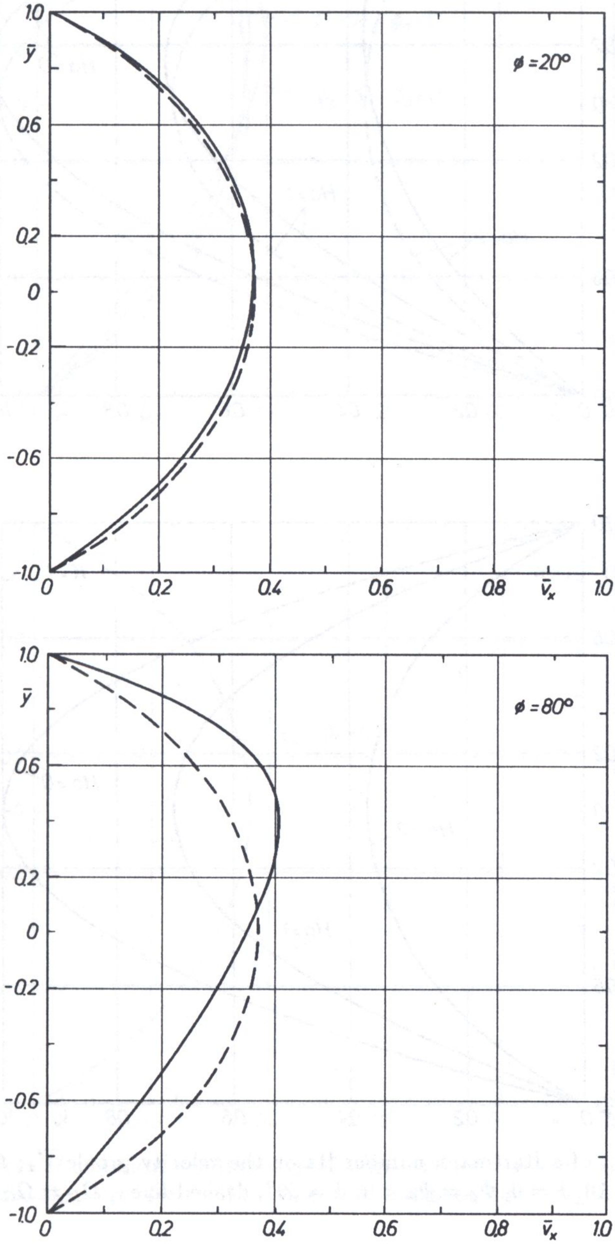


FIG. 3. Effect of centrifugal inertia forces on the velocity profile $\overline{V}_x$; $\Omega_L = 0$, $\Omega_C = 10$, $K = 0$, $\mathrm{Ha} = 2$, $I = 0$, $\Phi_g = \Phi_d = 0$; dashed line – $\Omega_L = \Omega_C = 0$.

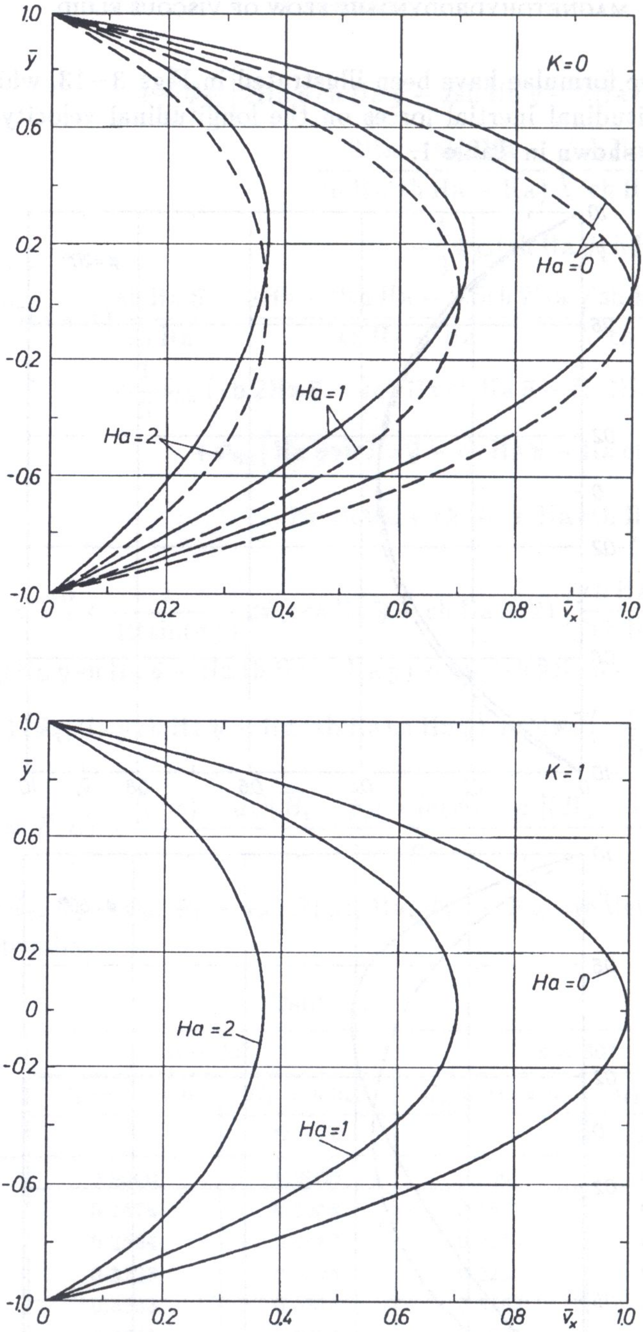


FIG. 4. Effect of a Hartmann number Ha on the velocity profile $\overline{V}_x$; $\Omega_L = 0.023$, $\Omega_C = 10$, $I = 0$, $\Phi_g = \Phi_d = 0$, $\phi = 50°$; dashed line – $\Omega_L = \Omega_C = 0$.

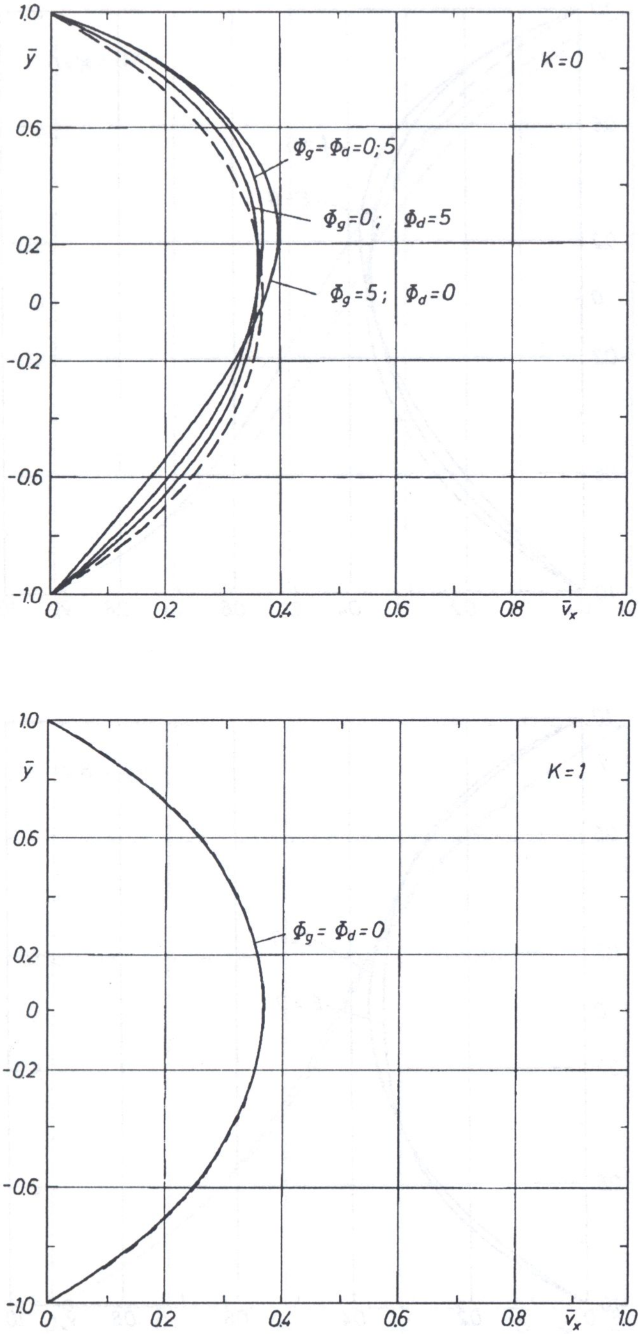


FIG. 5. Effect of electrical conductivity of surfaces on the velocity profile $\overline{V}_x$; $\Omega_L = 0.023$, $\Omega_C = 10$, $I = 0$, $\mathrm{Ha} = 2$, $\phi = 50°$; dashed line – $\Omega_L = \Omega_C = 0$.

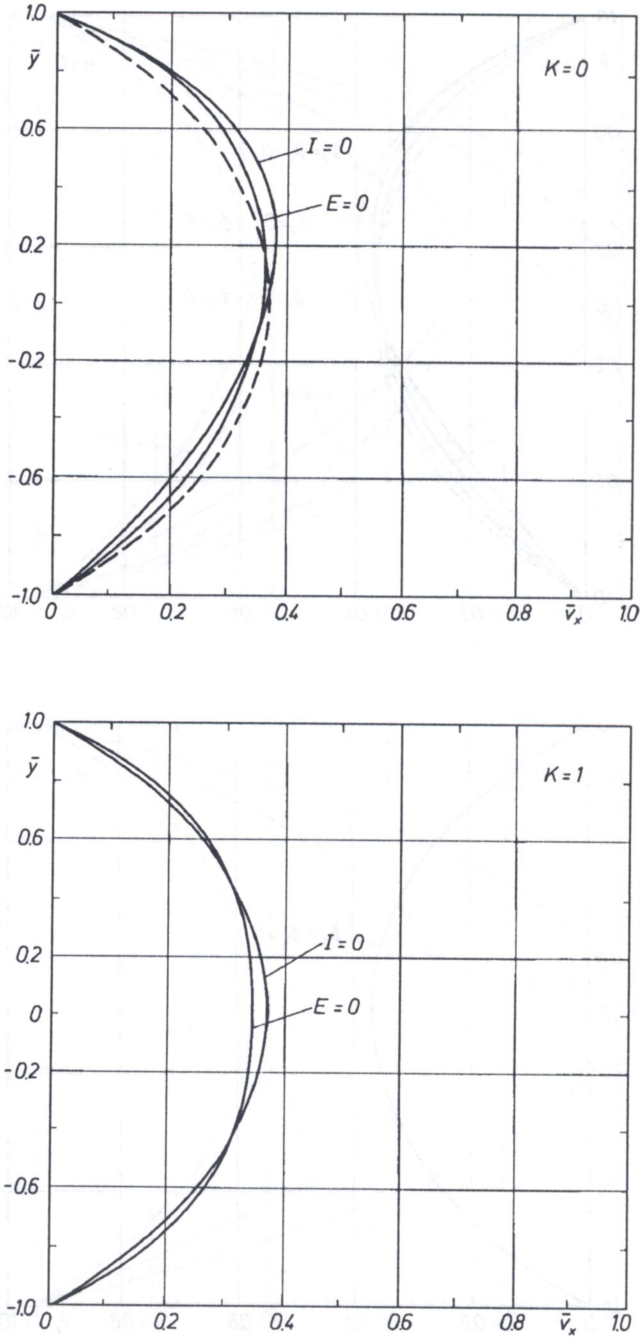
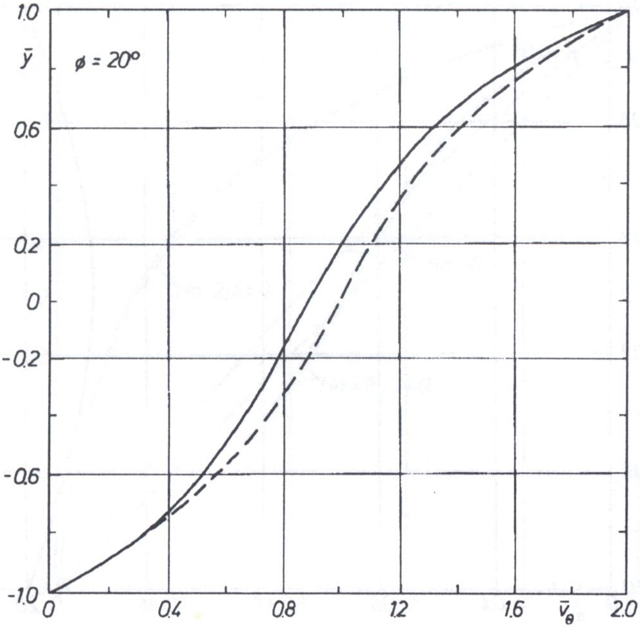


FIG. 6. Profile velocity $\overline{V}_x$; $I = 0$ – open circuit case, $E = 0$ – short circuit case, $\Omega_L = 0.023$, $\Omega_C = 10$, Ha $= 2$, $\Phi_g = \Phi_d = 0$, $\phi = 50°$; dashed line – $\Omega_L = \Omega_C = 0$.



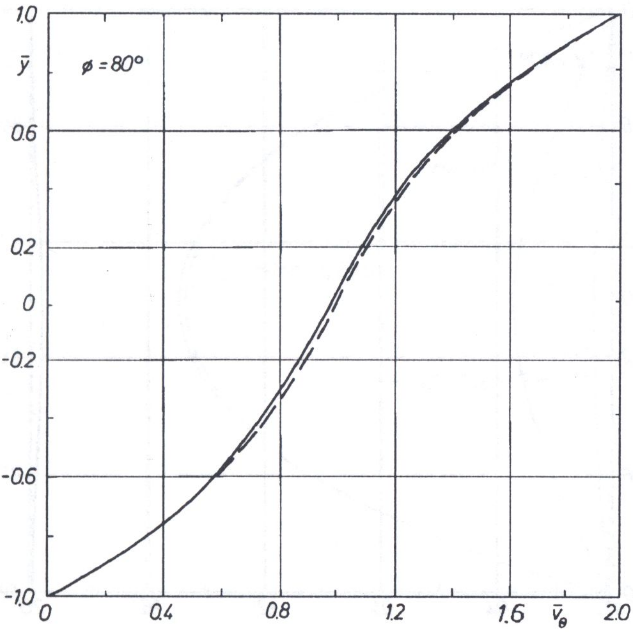


FIG. 7. Effect of longitudinal inertia forces on the velocity profile $\overline{V}_\theta$; $\Omega_L = 0.023$, $\Omega_C = 0$, $K = 0$, $\text{Ha} = 2$, $I = 0$, $\Phi_g = \Phi_d = 0$; dashed line – $\Omega_L = \Omega_C = 0$.

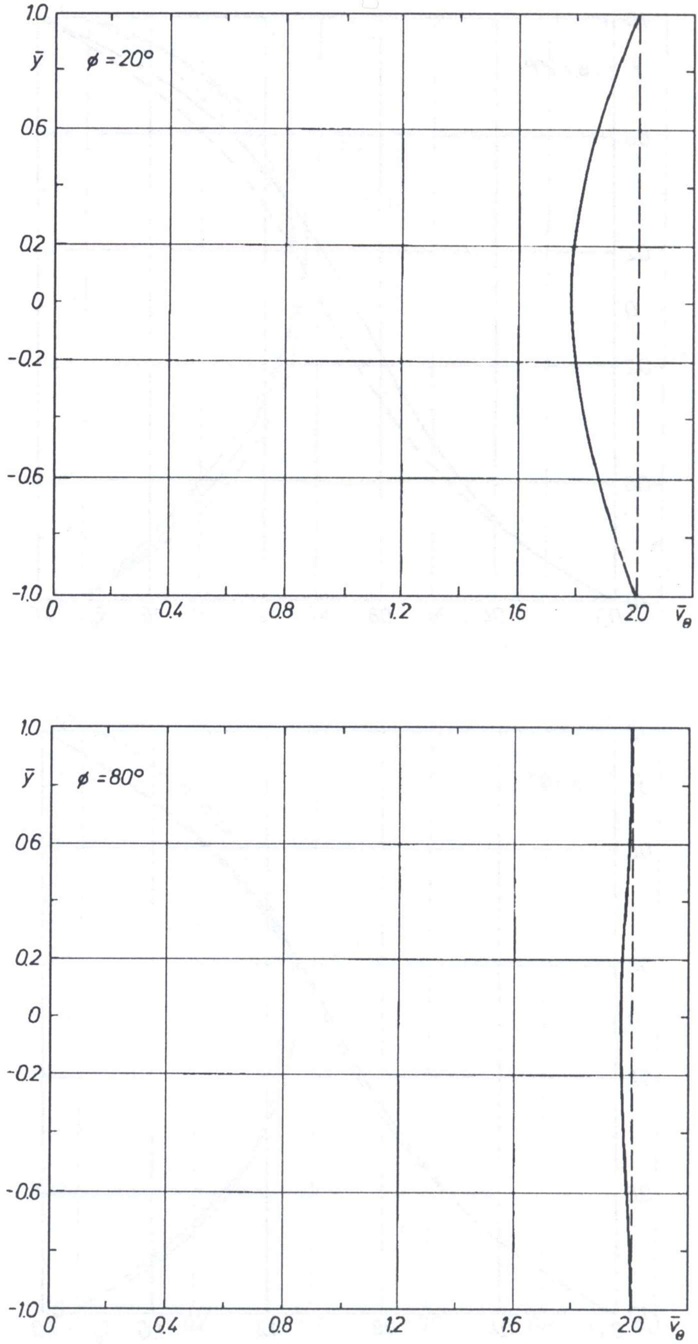


FIG. 8. Effect of longitudinal inertia forces on the velocity profile $\overline{V}_\theta$; $\Omega_L = 0.023$, $\Omega_C = 0$, $K = 1$, $\mathrm{Ha} = 2$, $I = 0$, $\Phi_g = \Phi_d = 0$; dashed line – $\Omega_L = \Omega_C = 0$.

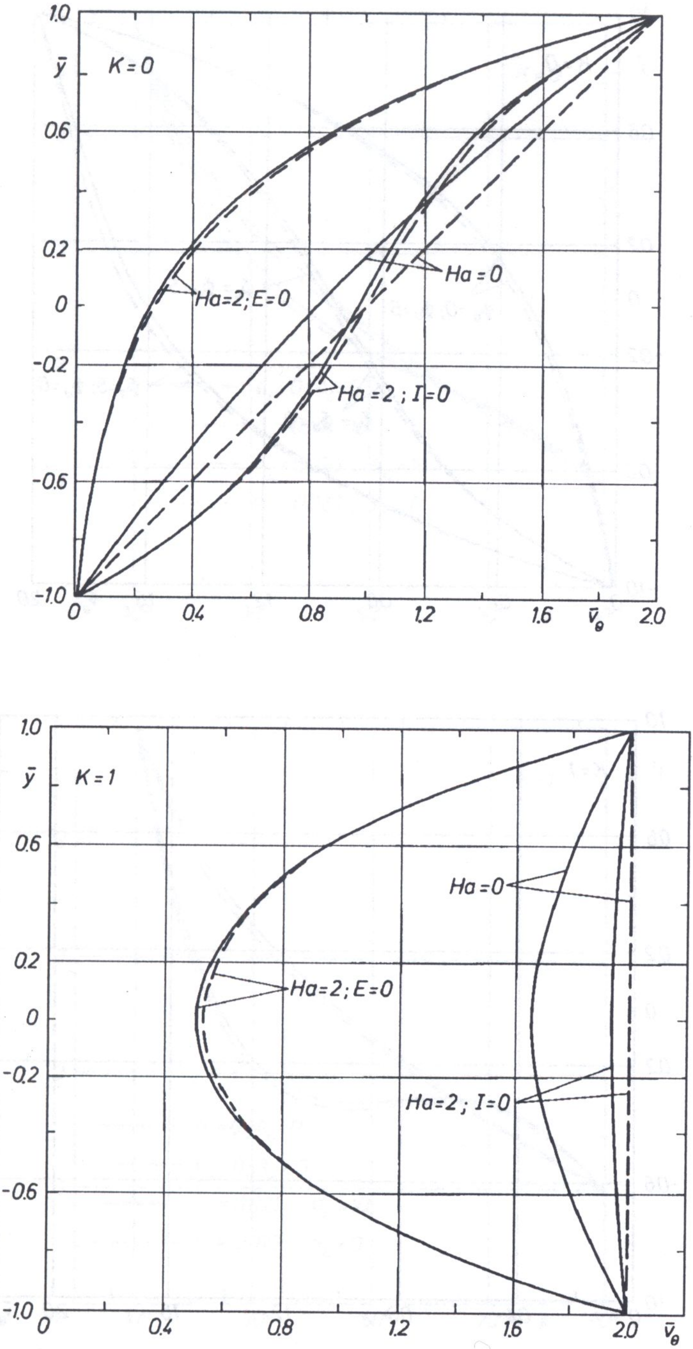


FIG. 9. Effect of a Hartmann number Ha on the velocity profile $\overline{V}_x$; $\Omega_L = 0.023$, $\Omega_C = 0$, $I = 0$, $\Phi_g = \Phi_d = 0$, $\phi = 50°$; dashed line – $\Omega_L = \Omega_C = 0$.

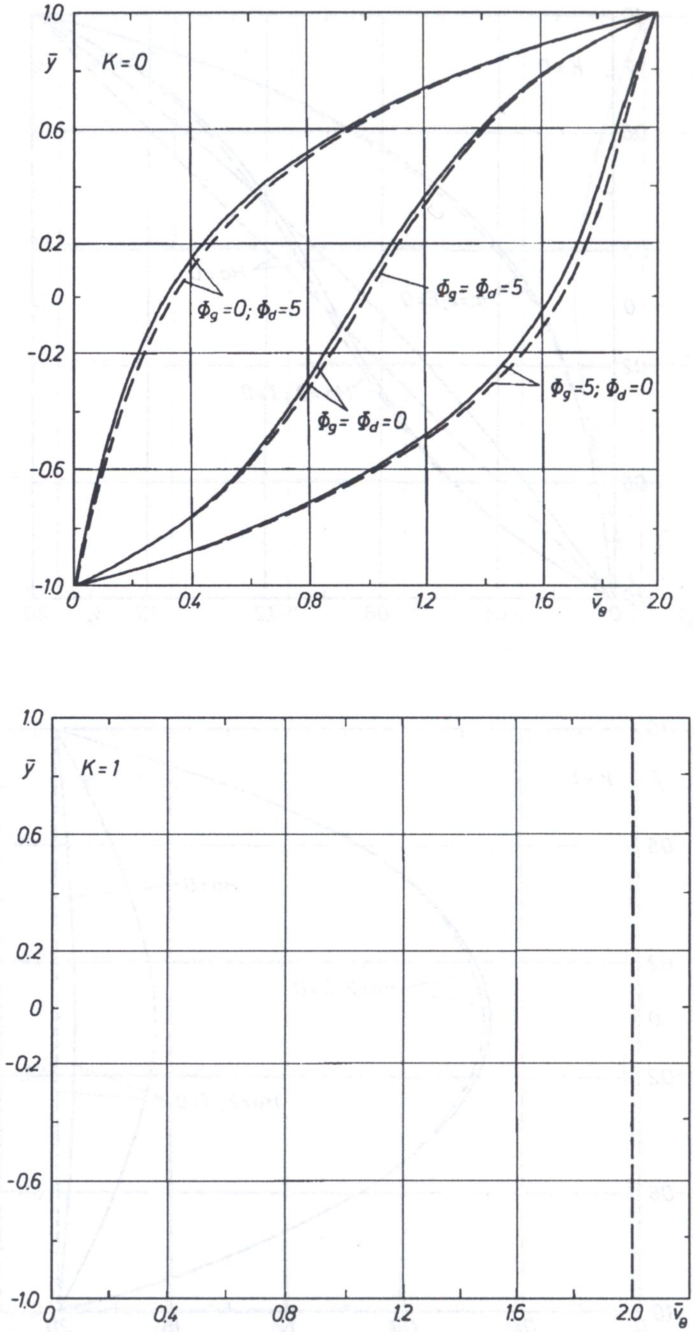


FIG. 10. Effect of electrical conductivity of surfaces on the velocity profile $\overline{V}_\theta$; $\Omega_L = 0.023$, $\Omega_C = 0$, $I = 0$, Ha $= 2$, $\phi = 50°$; dashed line – $\Omega_L = \Omega_C = 0$.

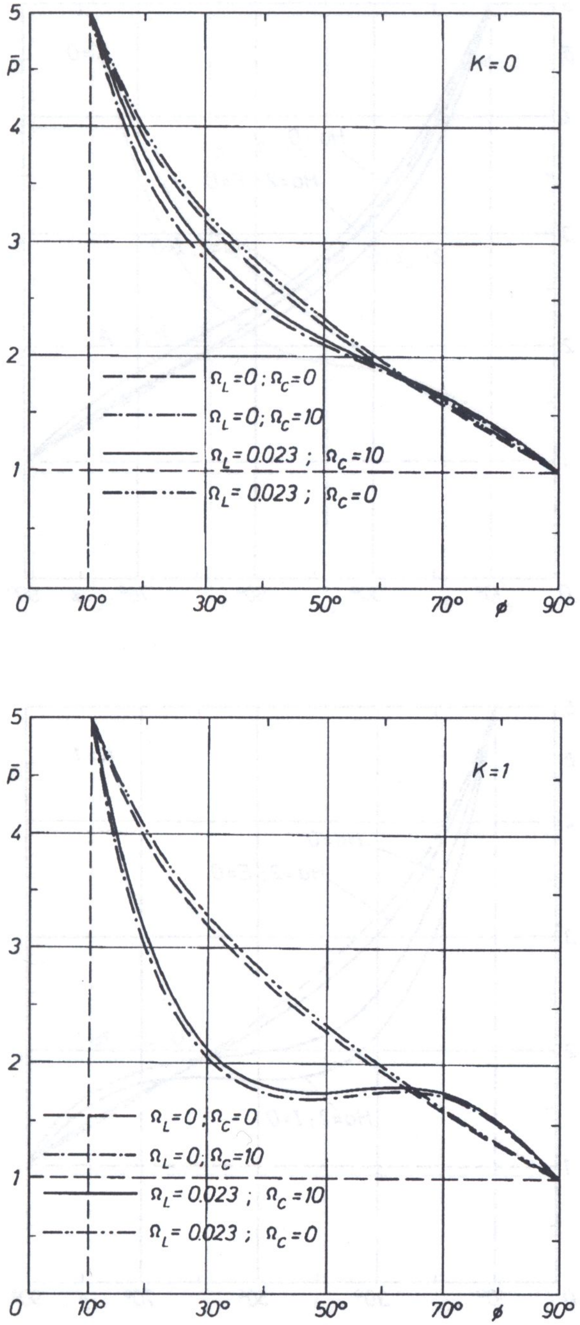


FIG. 11. Effect of inertia forces on the pressure $\overline{p}$ distributions; Ha = 2, $I = 0$, $\Phi_g = \Phi_d = 0$.

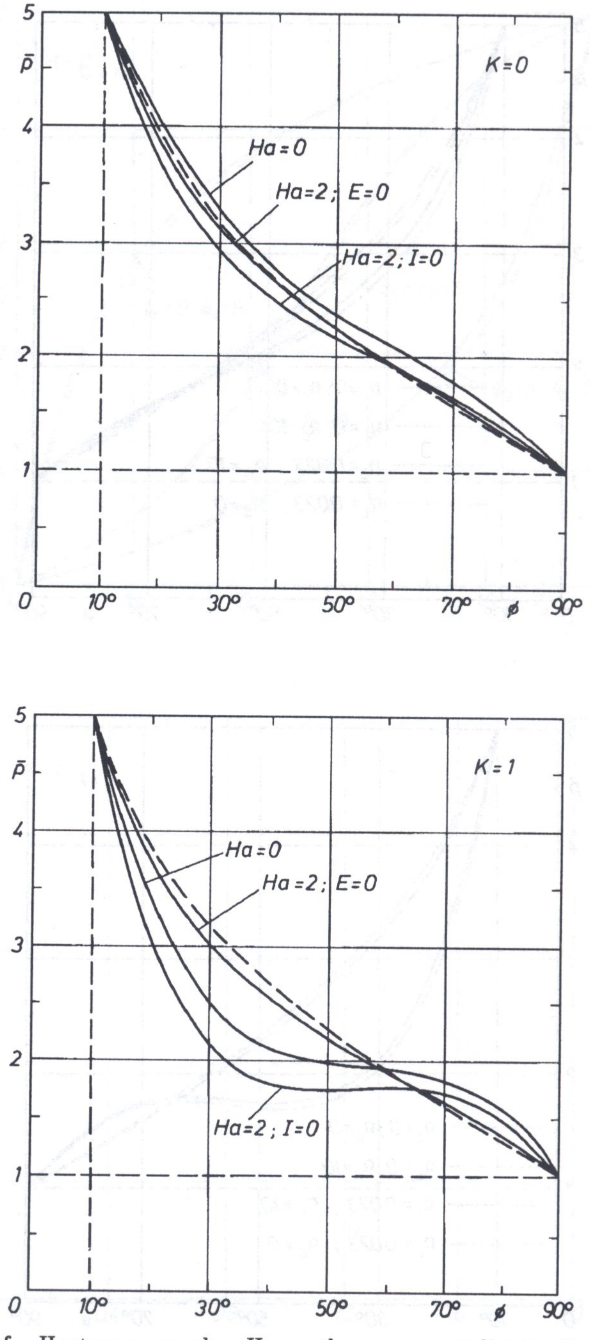


FIG. 12. Effect of a Hartmann number Ha on the pressure $\overline{p}$ distributions; $\Omega_L = 0.023$, $\Omega_C = 10$, $\Phi_g = \Phi_d = 0$, $\phi = 50°$; dashed line – $\Omega_L = \Omega_C = 0$.

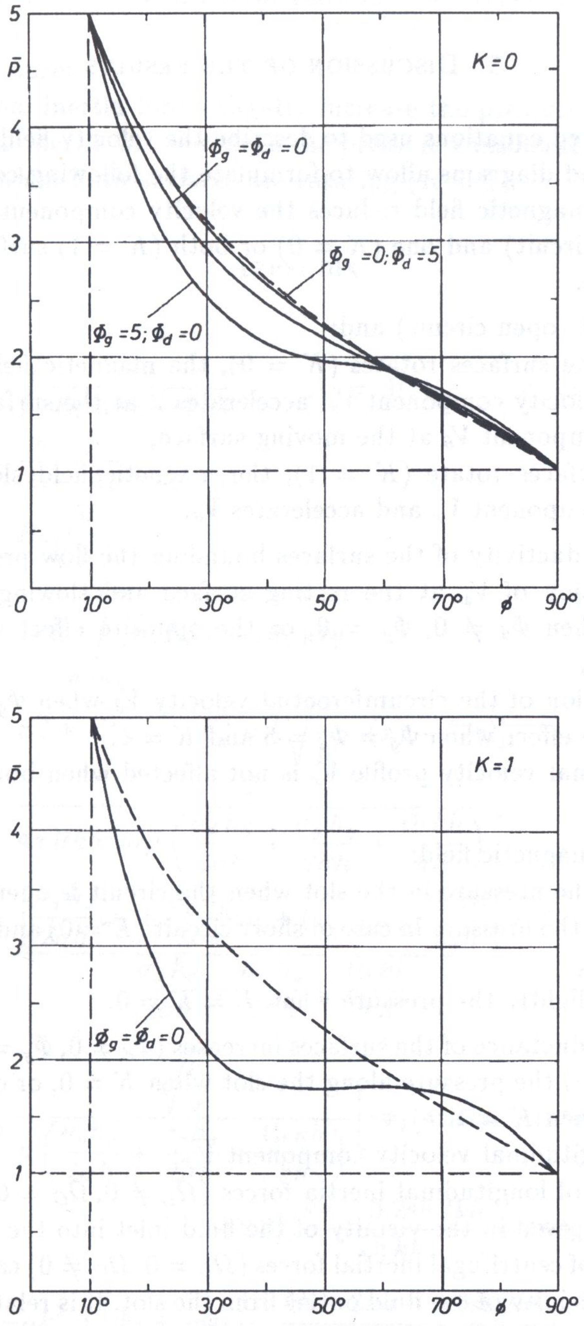


FIG. 13. Effect of electrical conductivity of surfaces on the pressure $\bar{p}$ distributions; $\Omega_L = 0.023$, $\Omega_C = 10$, Ha $= 2$, $I = 0$; dashed line – $\Omega_L = \Omega_C = 0$.

5. Discussion of the results

The presented equations used to describe the velocity fields and pressure components and diagrams allow to formulate the following conclusions:

Increasing magnetic field reduces the velocity components V_x, V_θ when $E = 0$ (short circuit) and one ($K = 0$) or both ($K = 1$) surfaces bounding the flow rotate.

When $I = 1$ (open circuit) and:

a) one of the surfaces rotates ($K = 0$), the magnetic field reduces the longitudinal velocity component V_x, accelerates it at the surface at rest and slows down component V_θ at the moving surface,

b) both surfaces rotate ($K = 1$), the magnetic field slows down the longitudinal component V_x and accelerates V_θ.

Electric conductivity of the surfaces bounding the flow produces:

a) acceleration of V_x at the resting surface and slowing down at the moving one when $\Phi_d \neq 0$, $\Phi_g = 0$, or the opposite effect when $\Phi_d = 0$, $\Phi_g \neq 0$, $K = 0$,

b) acceleration of the circumferential velocity V_θ when $\Phi_g \neq 0$, $\Phi_d = 0$, or the opposite effect when $\Phi_g = \Phi_d = 5$ and $K = 1$,

c) longitudinal velocity profile V_x is not affected when both surfaces rotate.

Increasing magnetic field:

a) reduces the pressure in the slot when the circuit is open ($I = 0$),

b) increases the pressure in case of short circuit ($E = 0$) and both surfaces rotate ($K = 1$),

c) reduces slightly the pressure when $E = K = 0$.

Electric conductance of the surfaces increases ($\Phi_d \neq 0$, $\Phi_g = 0$) or reduces ($\Phi_d = 0$, $\Phi_g \neq 0$) the pressure along the slot when $K = 0$, or does not affect the pressure when $K = 1$.

For the longitudinal velocity component $\overline{V}_x$:

• the effect of longitudinal inertia forces ($\Omega_L \neq 0, \Omega_C = 0$) is negligible and can be observed in the vicinity of the fluid inlet into the slot,

• the effect of centrifugal inertial forces ($\Omega_L = 0, \Omega_C \neq 0$) can be observed mainly in the vicinity of the fluid outlet from the slot. It is related to the case when only one of the surfaces is rotating ($K = 0$). However, the reported effect declines when both the surfaces are rotating ($K = 1$).

For the tangential velocity component $\overline{V}_\theta$:

• the effect of longitudinal inertia forces is important in the vicinity the fluid inlet.

For the pressure:

• longitudinal inertia forces slightly increase the pressure along the slot,

• centrifugal inertia forces reduce the pressure considerably inside a slot in particular, when both surfaces are rotating ($K = 1$).

APPENDIX

PART I

$$A(x) = \int \frac{dx}{R(\operatorname{th} kh - kh)}, \qquad A_i = A(x_i), \qquad A_o = A(x_o),$$

$$C = \frac{p_i - p_o}{A_i - A_o},$$

$$k = B_o\sqrt{\frac{\sigma}{\mu}},$$

$$J = 2\pi R \int_{h-h_d}^{h+h_g} \sigma(E_x + V_\theta B_y)\, dy,$$

$$E_x = E_x^0 + E_x^1, \qquad E_x^0 = k\sqrt{\frac{\sigma}{\mu}}[\alpha(x) - \beta(x)],$$

$$\alpha(x) = \frac{J}{4\pi Rkh\sqrt{\sigma\mu}\left(\dfrac{\sigma_d h_d}{2\sigma h} + \dfrac{\sigma_g h_g}{2\sigma h} + \dfrac{\operatorname{th} kh}{kh}\right)},$$

$$\beta(x) = \frac{\left[\dfrac{\sigma_d h_d}{2\sigma h}\omega_2 + \dfrac{\sigma_g h_g}{2\sigma h}\omega_1 + (\omega_1 + \omega_2)\dfrac{\operatorname{th} kh}{2kh}\right] R}{\dfrac{\sigma_d h_d}{2\sigma h} + \dfrac{\sigma_g h_g}{2\sigma h} + \dfrac{\operatorname{th} kh}{kh}},$$

$$E_x^1 = \frac{1}{48}\,\frac{\sqrt{\dfrac{\mu}{\sigma}}}{h\left(\dfrac{\sigma_d h_d}{2\sigma h} + \dfrac{\sigma_g h_g}{2\sigma h} + \dfrac{\operatorname{th} kh}{kh}\right)}\Big[8\Psi_1(4\operatorname{th} kh - 0.5\operatorname{sh} 2kh + kh)$$
$$+24\Psi_2\frac{kh - 0.2\operatorname{sh} 2kh}{\operatorname{ch} kh} + 48\Phi_8(\operatorname{th} kh - kh)\Big],$$

$$B(x) = -\mu k^2 \int \Big[\Phi_7 + \frac{1}{6}\Phi_1\frac{4\operatorname{th} kh - 3kh - 0.5\operatorname{sh} 2kh}{\operatorname{th} kh - kh}$$
$$+\frac{1}{2}\Phi_2\frac{kh - \operatorname{sh} kh \operatorname{ch} kh}{\operatorname{ch} kh(\operatorname{th} kh - kh)} + \frac{1}{4}\Phi_3\frac{1.5\operatorname{sh} 2kh - kh \operatorname{ch} 2kh - 2kh}{\operatorname{ch} kh\,(\operatorname{th} kh - kh)}$$
$$+\frac{1}{6}\Phi_4\frac{3kh - 2\operatorname{th} kh - 0.5\operatorname{sh} 2kh}{\operatorname{th} kh - kh}\Big] dx,$$

$$B_i = B(x_i), \qquad B_o = B(x_o),$$

$$\Phi_1 = \frac{\varrho}{\mu}\Big[\frac{C^2h'}{\mu^2R^2k^5}\frac{kh\,\mathrm{th}\,kh}{\mathrm{ch}^2kh(\mathrm{th}\,kh-kh)^3} - \frac{C^2R'}{\mu^2k^6R^3}\frac{1}{\mathrm{ch}^2kh(\mathrm{th}\,kh-kh)^2} + \frac{RR'}{4k^2}\frac{(\omega_1+\omega_2)^2}{\mathrm{ch}^2kh} - \frac{E_x^0}{k^3}\sqrt{\frac{\sigma}{\mu}}\frac{R'(\omega_1-\omega_2)}{\mathrm{ch}\,kh} - \frac{(E_x^0)^2}{k^4}\frac{\sigma}{\mu}\frac{R'}{R\,\mathrm{ch}^2kh}\Big],$$

$$\Phi_2 = \frac{\varrho}{\mu}\Big[\frac{C^2R'}{\mu^2R^3k^6}\frac{2\mathrm{ch}\,kh}{\mathrm{ch}^2kh(\mathrm{th}\,kh-kh)^2} - \frac{C^2h'}{\mu^2R^2k^5}\frac{\mathrm{th}\,kh(kh\,\mathrm{ch}\,kh+\mathrm{sh}\,kh)}{\mathrm{ch}^2kh(\mathrm{th}\,kh-kh)^3} + \frac{E_x^0}{k^3}\sqrt{\frac{\sigma}{\mu}}\frac{R'(\omega_1+\omega_2)}{\mathrm{ch}\,kh} + \frac{(E_x^0)^2}{k^4}\frac{\sigma}{\mu}\frac{2R'}{R\,\mathrm{ch}\,kh}\Big],$$

$$\Phi_3 = \frac{\varrho}{\mu}\frac{C^2h'}{\mu^2R^2k^5}\frac{\mathrm{sh}\,kh\,\mathrm{th}\,kh}{\mathrm{ch}^2kh(\mathrm{th}\,kh-kh)^3},$$

$$\Phi_4 = \frac{\varrho}{\mu}\Big[-\frac{C^2h'}{\mu^2R^2k^5}\frac{kh\,\mathrm{th}\,kh}{\mathrm{ch}^2kh(\mathrm{th}\,kh-kh)^3} - \frac{RR'}{4k^2}\frac{(\omega_1-\omega_2)^2}{\mathrm{sh}^2kh}\Big],$$

$$\Phi_5 = \frac{\varrho}{\mu}\Big[-\frac{RR'}{2k^2}\frac{(\omega_1^2-\omega_2^2)}{\mathrm{sh}\,2kh} - \frac{E_x^0}{k^3}\sqrt{\frac{\sigma}{\mu}}\frac{R'(\omega_1-\omega_2)}{\mathrm{sh}\,2kh}\Big],$$

$$\Phi_6 = \frac{\varrho}{\mu}\frac{E_x^0}{k^3}\sqrt{\frac{\sigma}{\mu}}\frac{R'(\omega_1-\omega_2)}{\mathrm{sh}\,kh},$$

$$\Phi_7 = \frac{\varrho}{\mu}\Big[\frac{C^2h'}{\mu^2R^2k^5}\frac{\mathrm{sh}^2kh}{\mathrm{ch}^2kh(\mathrm{th}\,kh-kh)^3} - \frac{C^2R'}{\mu^2R^3k^6}\frac{1}{(\mathrm{th}\,kh-kh)^2} - \frac{(E_x^0)^2}{k^4}\frac{\sigma}{\mu}\frac{R'}{R}\Big],$$

$$\Psi_1 = \frac{\varrho C}{2\mu^2k^4R^2}\frac{1}{\mathrm{ch}^2kh\,(\mathrm{th}\,kh-kh)}\Big\{2RR'(\omega_1+\omega_2) - kh'\mathrm{th}\,kh(\omega_1+\omega_2)R^2 - 2kh'\,\mathrm{th}\,kh[\alpha(x)-\beta(x)]R + 2[\alpha(x)-\beta(x)]R + 2[\alpha'(x)-\beta'(x)]R + 2[\alpha(x)-\beta(x)]R'\Big\},$$

$$\Psi_2 = \frac{\varrho C}{2\mu^2k^4R^2}\frac{1}{\mathrm{ch}^2kh\,(\mathrm{th}\,kh-kh)}\Big\{kh'\mathrm{th}\,kh\,(\omega_1+\omega_2)R^2 - 2RR'(\omega_1+\omega_2) + 2kh'\mathrm{th}\,kh\,[\alpha(x)-\beta(x)] - 4\,[\alpha'(x)-\beta'(x)]\,R - 4R'\,[\alpha(x)-\beta(x)]\Big\},$$

$$\Psi_3 = \frac{\varrho C}{2\mu^2 k^4 R}\,\frac{\omega_1 - \omega_2}{\operatorname{sh} 2kh\,(\operatorname{th} kh - kh)^2}\Big[2R'(\operatorname{th} kh - kh) - kh'\operatorname{cth} kh(\operatorname{th} kh - kh)R - kh' R\, kh \operatorname{th} kh\Big],$$

$$\Psi_4 = \frac{\varrho C}{2\mu^2 k^4 R}\,\frac{\omega_1 - \omega_2}{\operatorname{sh} kh\,(\operatorname{th} kh - kh)^2}(Rkh'\operatorname{cth} kh - 2R'),$$

$$\Psi_5 = -\frac{\varrho C}{2\mu^2 k^4 R}\,\frac{kh' kh}{\operatorname{ch}^2 kh\,(\operatorname{th} kh - kh)^2}\Big\{\operatorname{th} kh(\omega_1+\omega_2)R - 2[\alpha(x)-\beta(x)]\Big\},$$

$$\Psi_6 = \frac{\varrho C}{2\mu^2 k^4 R}\,\frac{kh'\operatorname{sh} kh}{\operatorname{ch}^2 kh\,(\operatorname{th} kh - kh)^2}\Big\{\operatorname{th} kh\,(\omega_1-\omega_2)R - 2[\alpha(x)-\beta(x)]\Big\},$$

$$\Psi_7 = \frac{\varrho C}{2\mu^2 k^4 R}\,\frac{kh'\operatorname{th} kh}{\operatorname{ch} kh\,(\operatorname{th} kh - kh)^2}(\omega_1 - \omega_2)R,$$

$$\Psi_8 = \frac{\varrho C}{\mu^2 k^4 R}\,\frac{1}{(\operatorname{th} kh - kh)}\Big\{R[\alpha'(x) - \beta'(x)] + R'[\alpha(x) - \beta(x)]\Big\},$$

PART II

$$a(\phi) = \ln\left(\operatorname{tg}\frac{\phi}{2}\right), \qquad a_i = a(\phi_i), \qquad a_o = a(\phi_o),$$

$$\alpha = \frac{I}{\mathrm{Ha}\left(\Phi_d + \Phi_g + \dfrac{\operatorname{th}\mathrm{Ha}}{\mathrm{Ha}}\right)},$$

$$\beta = \frac{\Phi_g + \Phi_d K + (1+K)\dfrac{\operatorname{th}\mathrm{Ha}}{2\mathrm{Ha}}}{\Phi_d + \Phi_g + \dfrac{\operatorname{th}\mathrm{Ha}}{\mathrm{Ha}}},$$

$$I = \frac{J}{4\pi R_o^2 \omega_1 \sqrt{\sigma\mu}}, \qquad E = \frac{E_x h}{\omega_1 R_o}\sqrt{\frac{\sigma}{\mu}}, \qquad \mathrm{Ha} = kh,$$

$$\Phi_d = \frac{\sigma_d h_d}{2\sigma h}, \qquad \Phi_g = \frac{\sigma_g h_g}{2\sigma h},$$

$$\Phi_1 = \Omega_L \frac{\cos\phi}{\sin^3\phi}\,\frac{(p_i - 1)^2}{\mathrm{Ha}^2 \operatorname{ch}\mathrm{Ha}} - \Omega_C \frac{1}{\mathrm{Ha}^2\operatorname{ch}^2\mathrm{Ha}}\Big[0.25(1+K)^2\sin\phi\cos\phi + \left(\frac{\alpha}{\sin\phi} - \beta\sin\phi\right)(1+K)\cos\phi + \left(\frac{\alpha}{\sin\phi} - \beta\sin\phi\right)^2 \operatorname{ctg}\phi\Big],$$

$$\Phi_2 = \Omega_L \frac{\cos\phi}{\sin^3\phi}\frac{2(p_i-1)^2}{\mathrm{Ha}^2\mathrm{ch\,Ha}} + \Omega_C \frac{1}{\mathrm{Ha}^2\mathrm{ch\,Ha}}\left[\left(\frac{\alpha}{\sin\phi} - \beta\sin\phi\right)(1+K)\cos\phi + \left(\frac{\alpha}{\sin\phi} - \beta\sin\phi\right)^2 2\,\mathrm{ctg}\,\phi\right],$$

$$\Phi_3 = 0,$$

$$\Phi_4 = -\frac{1}{8}\Omega_C(1-K)^2\frac{\sin 2\phi}{\mathrm{Ha}^2\mathrm{sh}^2\mathrm{Ha}},$$

$$\Phi_5 = -\Omega_C\frac{1}{\mathrm{Ha}^2\mathrm{sh\,2Ha}}\left[0.25(1-K^2)\sin 2\phi + \left(\frac{\alpha}{\sin\phi} - \beta\sin\phi\right)(1-K)\cos\phi\right],$$

$$\Phi_6 = \Omega_C\frac{1}{\mathrm{Ha}^2\mathrm{sh\,Ha}}\left(\frac{\alpha}{\sin\phi} - \beta\sin\phi\right)(1-K)\cos\phi,$$

$$\Phi_{11} = \Omega_L(p_i-1)^2\frac{1+\cos^2\phi}{\sin^4\phi\mathrm{Ha}^7\mathrm{ch\,Ha}} - \Omega_C\frac{1}{\mathrm{Ha}^3\mathrm{ch}^2\mathrm{Ha}}\Big[0.25(1+K)^2(\cos 2\phi + \cos^2\phi) - \alpha(1+K) - \beta(1+K)(\cos 2\phi + \cos^2\phi) + \alpha^2\frac{1+\cos^2\phi}{\sin^4\phi} + 2\alpha\beta + \beta^2(\cos 2\phi + \cos^2\phi)\Big],$$

$$\Phi_{22} = -\Omega_L(p_i-1)^2\frac{2(1+\cos^2\phi)}{\sin^4\phi\mathrm{Ha}^7\mathrm{ch\,Ha}} + \Omega_C\frac{1}{\mathrm{Ha}^3\mathrm{ch\,Ha}}\Big[-\alpha(1+K) - \beta(1+K)(\cos 2\phi + \cos^2\phi) + 2\alpha^2\frac{1+\cos^2\phi}{\sin^4\phi} + 4\alpha\beta + 2\beta^2(\cos 2\phi + \cos^2\phi)\Big],$$

$$\Phi_{33} = 0,$$

$$\Phi_{44} = -0.25\Omega_C(1-K)^2(\cos 2\phi - \cos^2\phi)\frac{1}{\mathrm{Ha}^3\mathrm{sh}^2\mathrm{Ha}},$$

$$\Phi_{55} = -\Omega_C\frac{1}{\mathrm{Ha}^3\mathrm{sh\,Ha}}\left[-\alpha(1-K) - \beta(1-K)(\cos 2\phi + \cos^2\phi)\right],$$

$$\Phi_{66} = \Omega_C \frac{1}{\mathrm{Ha}^3 \mathrm{sh\,Ha}} \left[-\alpha(1-K) - \beta(1-K)(\cos 2\phi + \cos^2\phi)\right],$$

$$\Omega_{Lp} = \Omega_L(p_i - 1)(a_i - a_o),$$

$$\Psi_1 = \Omega_{Lp} \frac{\mathrm{ctg}\,\phi}{\mathrm{Ha}^4 \mathrm{ch}^2 \mathrm{Ha}}(1 + K - 2\beta),$$

$$\Psi_2 = \Omega_{Lp} \frac{\mathrm{ctg}\,\phi}{\mathrm{Ha}^4 \mathrm{ch\,Ha}}[4\beta - (1-K)],$$

$$\Psi_3 = \Omega_{Lp} \frac{\mathrm{ctg}\,\phi}{\mathrm{Ha}^4 \mathrm{sh\,2Ha}}(1-K),$$

$$\Psi_4 = -\Omega_{Lp} \frac{\mathrm{ctg}\,\phi}{\mathrm{Ha}^4 \mathrm{sh\,Ha}}(1-K),$$

$$\Psi_5 = \Psi_6 = \Psi_7 = 0,$$

$$\Psi_8 = -\Omega_{Lp} \frac{\mathrm{ctg}\,\phi}{\mathrm{Ha}^6} 2\beta,$$

$$\overline{B}(\phi) = f_L \Omega_L \frac{1}{\sin^2\phi} + f_C \Omega_C \sin^2\phi, \qquad B_i = \overline{B}(\phi_i), \qquad B_o = \overline{B}(\phi_o),$$

$$f_L = (\overline{p}_i - 1)^2 \frac{\gamma_1 + 6\mathrm{ch\,Ha} - 6\gamma_2 \mathrm{ch\,Ha}}{12\mathrm{Ha}^4 \mathrm{ch}^2\mathrm{Ha}},$$

$$f_C = \left(\frac{\alpha^2}{2\sin^4\phi} + 2\alpha\beta\frac{\ln(\sin\phi)}{\sin^2\phi} - \frac{1}{2}\beta^2\right) \frac{6\gamma_2 \mathrm{ch\,Ha} - 6\mathrm{ch}^2\mathrm{Ha} - \gamma_1}{6\mathrm{ch}^2\mathrm{Ha}} + \left(\alpha\frac{\ln(\sin\phi)}{\sin^2\phi} - \frac{1}{2}\beta(1+K)\right)\frac{\gamma_1 - 3\gamma_2 \mathrm{ch\,Ha}}{6ch^2\mathrm{Ha}} + \frac{\gamma_1(1+K)^2}{48\mathrm{ch}^2\mathrm{Ha}} + \frac{\gamma_4(1-K)^2}{48\mathrm{sh}^2\mathrm{Ha}},$$

$$E_o = \mathrm{Ha}\left(\frac{\alpha}{\sin\phi} - \beta\sin\phi\right),$$

$$E = E^0 + E^1 = E^0 + \frac{1}{48}\frac{1}{\Phi_d + \Phi_g + \dfrac{\mathrm{th\,Ha}}{\mathrm{Ha}}}\left[8\Psi_1(4\mathrm{th\,Ha} - 0.5\mathrm{sh\,2Ha} - \mathrm{Ha}) + 24\Psi_2\frac{\mathrm{Ha} - 0.5\mathrm{sh\,2Ha}}{\mathrm{ch\,Ha}} + 48\Psi_8(\mathrm{th\,Ha} - \mathrm{Ha})\right],$$

$$\gamma_1 = \frac{4\text{th Ha} - 3\text{Ha} - 0.5\text{sh } 2\text{Ha}}{\text{th Ha} - \text{Ha}},$$

$$\gamma_2 = \frac{\text{Ha} - 0.5\text{sh } 2\text{Ha}}{\text{ch Ha}(\text{th Ha} - \text{Ha})},$$

$$\gamma_3 = \frac{3\text{Ha} - 2\text{th Ha} - 0.5\text{sh } 2\text{Ha}}{\text{th Ha} - \text{Ha}},$$

$$\Omega_L = \frac{\varrho p_o h^4}{\mu^2 R_k^2 \sin^2\phi_o (a_i - a_o)^2}, \qquad \Omega_C = \frac{\varrho \omega_1^2 R_k^2 \sin^2\phi_o}{p_o}, \qquad K = \frac{\omega_2}{\omega_1}.$$

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