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STRESS DISTRIBUTION IN A PIEZOCERAMIC DISC APPLIED IN VIBRATION CONTROL OF HIGH-RISE BUILDING

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The shear strain rate measurement is needed to implement active control on a high-rise building for suppressing its vibration. A piezoceramic disc can be used to measure the shear strain. Shearing stress distribution and displacements in such a disc sensor have been investigated. This study is aimed at proper selection of the type of material for construction of the disc.

1. Introduction

Vibrating control of a high-rise building is essential to increase the safety and integrity of a structure subjected to earthquake ground motion or high winds. The deflections near the top of the building can exceed several feet for a tall building driven by persistent winds. These persistent vibrations can cause considerable discomfort or even illness to the building occupants [1].

Shear walls have been considered to be the most efficient structural components to resist horizontal forces. According to HARRIS *et al.* [2], the shear wall carries 85% of the total shear of the building. Thus, vibration control of the shear walls can be considered the key to reduce the vibration of a building subjected to lateral loads.

Using direct piezoelectric effect, a shear mode piezoceramic can be applied to measure the shear strains of the shear walls. This shear strain rate measurement is needed to implement active control to a large civil structure. The control system provides additional stiffness/damping or generates counteracting forces to the structure for suppressing its vibration. Since piezoelectric materials are dielectric, the electric charge generated due to the external mechanical disturbance will be detected only if the charge is collected through the surface electrode to a measurement device. It should be noted that the surface electrode can be formed by depositing thin metallic layers on the surface of the disc and a current amplifier can be used as the measurement device [1].

The objective of this paper is to study the shearing stress distribution in an inhomogeneous annular piezoceramic disc applied in shear strain measurement. The disc should be placed at the centre of the wall of each story of a multi-storied building (Fig. 1). The shearing stress distribution and the electric displacement in the disc are found by solving the constitutive equations of piezoelectrics and the equations of motion under certain mechanical boundary conditions. From this knowledge of the stress distribution one can guess the position of charge accumulation.

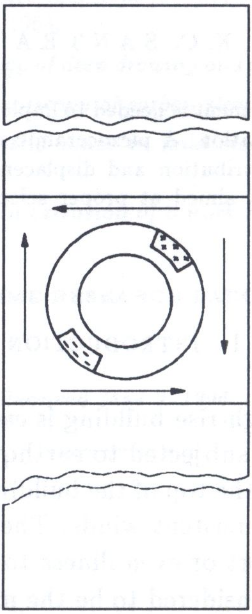

FIG. 1. Configuration of a shear wall with the piezoceramic disc.

2. FUNDAMENTAL EQUATIONS

The annular disc of piezoelectric material under consideration has its inner and outer radii r_1 and r_2, respectively. On taking the centre of the disc to be the origin, the polar coordinates (r, θ) are used as the coordinates of reference. For a plane-stress problem, by using the relevant elastic, piezoelectric and dielectric matrices, the constitutive equations become [1, 3]

$$
\begin{aligned}
T_{r\theta} &= c_6\, s_{r\theta}, \\
D_\theta &= -e_{15}\, s_{r\theta},
\end{aligned}
\tag{2.1}
$$

where $T_{r\theta}$ represent the shearing stress component at (r, θ). $s_{r\theta}$ and D_θ are the corresponding strain component and electric displacement component, respectively, e_{15} is the piezoelectric stress coefficient, c_6 being Voigt's plate parameter. c_6 can be expressed as

$$c_6 = \frac{c_{11} - c_{12}}{2},$$

where c_{11} and c_{12} are the elastic stiffness coefficients.

The shearing stress $s_{r\theta}$ can be represented in terms of circumferential displacement (v) as [3, 4]

$$s_{r\theta} = \frac{\partial v}{\partial r} - \frac{v}{r}. \tag{2.2}$$

Again the equation of motion is [3, 4]

$$\frac{\partial T_{r\theta}}{\partial r} + 2\frac{T_{r\theta}}{r} = \varrho \frac{\partial^2 v}{\partial t^2}, \tag{2.3}$$

ϱ being the mass density of the material.

A pioezoelectric body becomes inhomogeneous when the body is composed of stratified media of piezoelectric aggregate using Bimorph principle [3, 5]. Again, by doping or mixing $BaZrO_3$ or $BaHfO_3$ with $BaTiO_3$, the material parameters of the inhomogeneous structure can be varied point by point [6, 7]. The inhomogeneity of the body is characterized by the variations of material density, elastic and piezoelectric parameters from point to point. To suit the practical purposes of describing exponential, linear or parabolic variations, they may be of the form [3, 8]

$$\begin{aligned} \varrho &= \overline{\varrho}(r/r_1)^{2p} \exp(-qr), \\ c_6 &= \overline{c}_6(r/r_1)^{2p} \exp(-qr), \\ e_{15} &= \overline{e}_{15}(r/r_1)^{2p} \exp(-qr). \end{aligned} \tag{2.4}$$

Here the parameters under bar (−) sign are for the homogeneous material, and p, q are termed as inhomogeneity parameter.

3. Boundary conditions and method of solutions

The mechanical boundary condition follows from the fact that the inner edge of the disc suffers no shearing stress, and the outer edge is under the action of a time-dependent shearing stress, and may be formulated according to [3] as

$$\begin{aligned} T_{r\theta} &= 0 \qquad \text{at} \qquad r = r_1, \\ T_{r\theta} &= -T_0 \exp(-kt) = T \qquad \text{at} \qquad r = r_2, \end{aligned} \tag{3.1}$$

where T_0 and k are given constants.

The above set of equations may be satisfied if one chooses

$$v = v_s(r)\exp(-kt). \tag{3.2}$$

By virtue of Eq. (2.1)–(2.4), $v_s(r)$ of Eq. (3.2) would have to satisfy the following equation

$$r^2\frac{d^2v_s}{dr^2} + [(2p+1) - qr]\,r\frac{dv_s}{dr} + (n^2r^2 + qr - 2p - 1)v_s = 0, \tag{3.3}$$

where

$$n^2 = \frac{-\overline{\varrho}k^2}{\overline{c}_6}.$$

The solution of Eq. (3.3) can be expressed as [9, 10]

$$v_s(r) = r^l\,\exp(mr)\Big[P_1\,{}_1F_1(A_2, A_1; gr) + P_2\,(gr)^{1-A_1}\,{}_1F_1(1 + A_2 - A_1, 2 - A_1; gr)\Big], \tag{3.4}$$

where

$$l = -p \pm (p+1),$$
$$m = \frac{q \pm (q^2 - 4n^2)^{1/2}}{2},$$
$$g = (q - 2m),$$
$$A_1 = 2p + 2l + 1,$$
$$A_2 = -[A_1\,(m+q) - ql]/g.$$

P_1 and P_2 are arbitrary constants and ${}_1F_1(a, c; x)$ is the confluent hypergeometric function.

Using Eq. (2.1), (2.2), (2.3) and (3.4), the expression for $T_{r\theta}$ can be presented as

$$T_{r\theta} = \overline{c}_6\,[P_1\,M_{11}(r) + P_2\,M_{12}(r)]\exp(-kt). \tag{3.5}$$

$M_{11}(r)$ and $M_{12}(r)$ are defined in the Appendix.

P_1 and P_2 can be determined from the boundary conditions (3.1). Using Gauss' law of electrostatics, the closed circuit charge signal $Q(t)$ measured from the surface electrodes can be expressed as [1]

$$Q(t) = \iint D_\theta(t)\,r\,d\theta\,dr = -\iint e_{15}s_{r\theta}(t)\,r\,d\theta\,dr. \tag{3.6}$$

Furthermore, since the current equals the rate of change of the charge signal, the current output from a piezoceramic sensor can be expressed as

$$i(t) = \frac{dQ(t)}{dt} = -\iint e_{15}\,\frac{d}{dt}S_{r\theta}(t)\,r\,d\theta\,dr. \tag{3.7}$$

The signal is therefore proportional to the shear strain rate at the position of the piezoceramic disc sensor.

4. Numerical results and discussions

The shearing stress distribution in an annular piezoceramic disc made of $BaTiO_3$ aggregate has been numerically found in this section. The numerical constants for Barium Titanate in S.I. units are [10] $\bar{c}_6 = 4.49 \times 10^{10}\,\mathrm{N/m^2}$, $\bar{\varrho} = 5.5 \times 10^3\,\mathrm{kg/m^3}$, $\bar{e}_{15} = 0.1419\,\mathrm{Coul/m^2}$. Choosing $k = \sqrt{-1}\cdot 20$ cycles/sec, $r_1 = 1.5$ m, $r_2 = 3$ m, the shearing stress distribution for the homogeneous case as well as for the inhomogenoeus case characterized by $p = 0.45$ and $q = 2/r_1$ are depicted in Fig. 2.

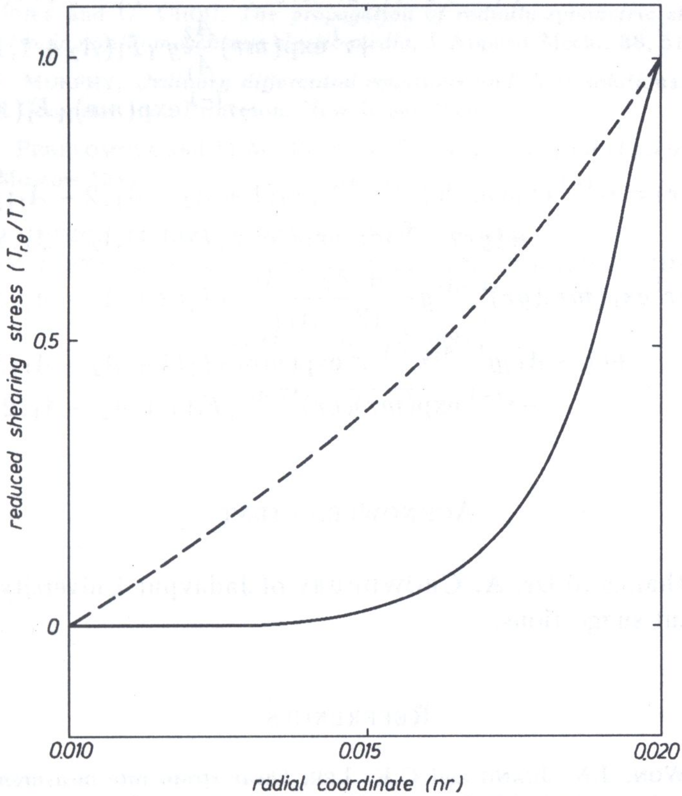

Fig. 2. Shearing stress distribution due to applied stress. Dashed curve for homogeneous case, solid curve for inhomogeneous case.

From the figure it is concluded that the shearing stress increases uniformly for homogeneous disc but it is nearly zero at the inner side and rapidly increases in the outer zone of the inhomogeneous disc. As a matter of fact, considerable charge will develop in the outer zone of the inhomogeneous disc. So it is preferable to use inhomogeneous piezoceramic disc for the purpose of shearing strain measurement.

By using a current amplifier to collect charges accumulated on the electrodes, the shear strain rate can be measured. This measurement is applied to implement active control (e.g. hydraulic tendon) on the shear walls of a high-rise building to supress its vibration.

APPENDIX

$$\begin{aligned} M_{11}(r) = {} & l r^{l-1} \exp(mr)\, {}_1F_1(A_2, A_1; gr) + r^l m \exp(mr)\, {}_1F_1(A_2, A_1; gr) \\ & + r^l \exp(mr) \frac{A_2}{A_1} g\, {}_1F_1(A_2+1, A_1+1; gr) \\ & - r^{l-1} \exp(mr)\, {}_1F_1(A_2, A_1; gr), \end{aligned}$$

$$\begin{aligned} M_{12}(r) = {} & l r^{l-1} \exp(mr)(gr)^{1-A)_1}\, {}_1F_1(1+A_2-A_1; 2-A_1; gr) \\ & + (gr)^{1-A_1} m r^l \exp(mr)\, {}_1F_1(1+A_2-A_1, 2-A_1; gr) \\ & + r^l \exp(mr)(gr)^{1-A_1} g \frac{(1+A_2-A_1)}{(2-A_1)}\, {}_1F_1(2+A_2-A_1, 3-A_1; gr) \\ & + (1-A_1) g^{1-A_1} r^{-A_1} r^l \exp(mr)\, {}_1F_1(1+A_2-A_1, 2-A_1; gr) \\ & - r^{l-1} \exp(mr)(gr)^{1-A_1}\, {}_1F_1(1+A_2-A_1, 2-A_1; gr). \end{aligned}$$

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