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# THE CHOICE OF FEEDBACK GAINS IN THE COMPUTER MODEL-BASED CONTROL SYSTEMS OF THE DIRECT-DRIVE MANIPULATOR

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The paper presents the results of simulations which were carried out in order to work out the methods of selection of control parameters for two control systems: a) the relatively simple control system with PD compensators of position errors and feedforward compensation of gravitational forces, b) the inverse dynamic control system. For both control systems two procedures of choice of the control parameters are presented: a) the procedure based on the analysis of simplified models of plants and controllers, b) the procedure consisting in numerical optimization of an integral performance index.

## 1. Introduction

Industrial manipulation robots started to be widely installed in many branches of industry at the beginning of the seventies.

Simultaneously with the progress in applications of industrial robots, research works aimed at manipulation robots improvement (in particular, at improvement and development of new drives and control systems) are carried out in many centers around the world.

In the robot control area an important factor stimulating development of new control methods and techniques is the very rapid, present-day progress in microprocessor technology and programming. Microprocessors of great computational power enable the application of model-based control in manipulation robots. In computer model-based control systems, input signals (fed into the actuators) are calculated on-line (or possibly partly on-line, partly off-line) on the basis of a dynamic model of a plant.

In a majority of manipulators every link of a manipulator has an independent drive. Usually the drives are indirect, i.e. they consist of a motor (e.g. DC-motor) and a reductor of a very small ratio ($1/200 \div 1/20$). The first direct-drive manipulator was built according to [2] in 1981. In the direct drive, a link of a manipulator is mounted directly on the motor shaft. Control of the direct-drive manipulator is difficult, but because of important advantages of such a solution it is a subject of investigations and experiments.

A number of publications dealing with technical implementations of direct-drive manipulators is relatively small. It is worth to mention here the article [1] containing a detailed description of the six degrees of freedom manipulator and its control system. On the other hand, some of theoretical aspects of direct-drive manipulators control are considered even in textbooks on robotics [2,4]. Probably this state of the literature on robotics reflects the actual state, i.e. technical difficulties connected with a practical implementation of direct-drive manipulators and justified conviction held by specialists of great advantages of such a solution after overcoming these difficulties.

Control of an indirect-drive manipulator can be in a way easier, because in transmissions of a ratio $1{:}i$, a load torque on a motor shaft is $i$ times smaller than a torque on a load shaft (which is generally nonlinear relative to generalized coordinates and velocities of a manipulator).

However, in indirect-drive manipulators structural resonances of a low frequency occur. They result from the flexibility of gears and are difficult (or almost impossible) to avoid. Structural resonances of a low frequency cause that the realization of fast manipulator motions and the application of high-gain feedback are impossible. Direct-drive manipulators are free from this drawback, because their segments are much more stiff.

The obvious advantages of direct drives are high durability, reliability and easiness of maintenance – due to the absence of mechanical transmissions.

Backlashes, friction, and deformations occur in mechanical transmissions, they are difficult to avoid and also difficult for identification and modeling. For this reason it is practically impossible to develop an accurate mathematical model of an indirect-drive manipulator.

From this standpoint the situation is quite different in the case of a direct-drive manipulator. A model of the direct-drive manipulator is usually very complex, but it is relatively easy for identification. It is an important advantage of such a solution because it enables a succesful application of model-based control.

When designing any control system, one has to take into consideration many various factors, and very often it is not possible until technical implementation is accomplished. For example, let us mention here such factors as: sensitivity and noise of sensors, the type and bounds of disturbances, frequency range of disturbances, and different structural resonances, which may occur but are difficult to foresee.

The factors mentioned above have not been taken into consideration in this paper. The investigations of the control systems of manipulators were first of all oriented towards the properties and limitations of the control systems resulting from the application of digital processing. In particular, quantization of input signals and a time delay in the inverse dynamics control systems, resulting from finite speed of model computations, are in question. The quantization means here that input signals are constant in the intervals: $< t_i, t_i + T)$, where $t_i$ stands for the instant of sampling, and $T$ stands for the sampling period.

A mathematical model of a manipulator consisting of $n$ rigid links has a following form [2]:

$$M(\mathbf{q})\ddot{\mathbf{q}} + V(\mathbf{q}, \dot{\mathbf{q}}) + G(\mathbf{q}) + F(\mathbf{q}, \dot{\mathbf{q}}) = \mathbf{P}, \tag{1.1}$$

where

- $\mathbf{q}$ $n \times 1$ vector of relative displacements of adjacent links in joints,
- $M(\mathbf{q})$ $n \times n$ mass matrix,
- $V(\mathbf{q}, \dot{\mathbf{q}})$ $n \times 1$ vector of centrifugal and Coriolis terms,
- $G(\mathbf{q})$ $n \times 1$ vector of gravity terms,
- $F(\mathbf{q}, \dot{\mathbf{q}})$ $n \times 1$ vector of friction terms,
- $\mathbf{P}$ $n \times 1$ vector of forces (torques) at joints.

It is characteristic for a majority of manipulators that Eqs. (1.1) describing manipulator motion are strongly nonlinear and strongly coupled. One of the most important trends in a robot control field is searching for model-based control which compensates partly or entirely for the nonlinearities and couplings in Eqs. (1.1) [2, 4].

## 2. Control synthesis on the basis of simplified models

### 2.1. *Control with compensation for gravitational forces*

In the simplest version of model-based control, gravity terms of Eq. (1.1) are compensated for, and components of input signals compensating gravitational forces are calculated off-line (Fig. 1).

In this case input signals are

$$\mathbf{P} = K_p\mathbf{e} + K_v\dot{\mathbf{e}} + \widehat{\mathrm{G}}(\mathbf{q}^d), \tag{2.1}$$

where $\mathbf{q}^d$ – vector of programmed coordinates of a manipulator, $\mathbf{e}$ – vector of errors, i.e. vector of differences between the programmed coordinates and actual coordinates, $K_p$, $K_v$ – matrices of constant factors.

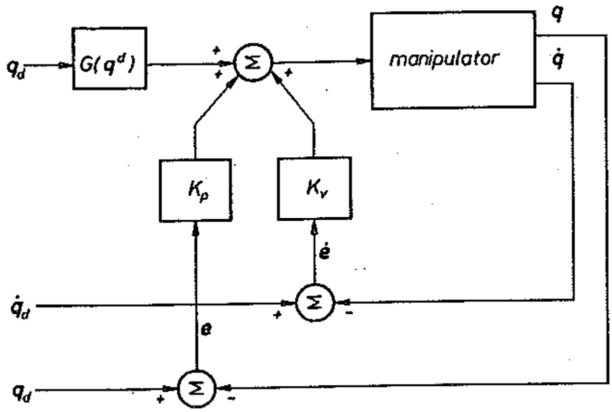


FIG. 1. The control system with compensation of gravitational forces.

In the formula (2.1) and further on, the sign ^ is put above a symbol of a vector or a matrix to distinguish matrices and vectors which refer to a mathematical model from that referring to a real plant.

Even in the case of the simplest manipulators of three or four degrees of freedom, the dynamics of a manipulator is so complex that it is impossible to say anything unquestionable on the dynamical properties of the closed-loop system solely on the basis of the motion equation and the control law.

The system seems to be able to realize effectively the programmed motion provided that inertial forces are small enough compared with the gravitational forces, i.e. in the case when the programmed motion is slow.

Surely, compensation for gravitational forces is advisable, but only under the condition that masses of the manipulator links and locations of their gravity centers are well identified and the errors $\mathbf{e}$ are small, because the component $\widehat{\mathbf{G}}(\mathbf{q}^d)$ of the input signal $\mathbf{P}$ compensates, in fact, for the gravitational forces at the desired position of a manipulator, and not at its actual position.

The drawbacks mentioned above do not exclude the possible applications of such a control system. One can not exclude that dynamical and steady-state properties of the control system will be satisfactory, provided the values of elements of the matrices $K_p$, $K_v$ are chosen properly.

The difficulty of choice of the control parameters $K_p$, $K_v$ is that dynamical properties of a manipulator (simplifying – its inertia and its response to external and driving forces) can be entirely different for different positions $\mathbf{q}$ and velocities $\dot{\mathbf{q}}$.

The report [3] presents the results of simulations which were carried out for the manipulator of three degrees of freedom (with three revolute joints) and for the manipulator of four degrees of freedom (the 1st joint of which was revolute, the 2nd and the 3rd were prismatic, and the 4th was revolute). The investigations revealed that it is possible to achieve good control quality (i.e. small steady-state errors, small tracking errors, good disturbances rejections) when one chooses the control parameters $K_p$, $K_v$ following the reasoning described below.

In most cases it is very important to achieve the desired final position of the manipulator with desired accuracy at the desired time; when the manipulator is moving from the initial to the final position, the deviations of an actual trajectory from the programmed trajectory can be much greater.

Since the selection of control parameters on the basis of a complete, nonlinear and very complex model of a manipulator is impossible, one has to make use of a maximally simplified model.

Let us ignore in the model (1.1) all inertia forces except the forces $m_{ii}\,\ddot{q}_i$ $(i = 1, \ldots, n)$, where $m_{ii}$ denotes values of diagonal elements of the inertia matrix $M$ at the final position of a manipulator. If the motion of a manipulator is stable and really converges to the desired final position and zero values of the velocites $\dot{\mathbf{q}}$, then ignoring these inertia forces which are equal zero for zero values of the velocities $\dot{\mathbf{q}}$ is at least partly justified.

Furthermore, let us ignore in the model the gravitational forces, which are partly compensated for by the control system, and let us assume a diagonal form of the matrices $K_p$, $K_v$.

Under the assumptions given above a motion of the $i$-th link of a manipulator around its final position is described by the equation:

$$m_{ii}\ddot{e}_i + k_v^i\dot{e}_i + k_p^i e_i = 0. \tag{2.2}$$

In Eq. (2.2) the coefficients $k_p^i$, $k_v^i$ denote the diagonal elements of the matrices $K_p$, $K_v$.

If the coefficients $k_p^i$, $k_v^i$ are assumed according to the formulae

$$k_v^i = 2m_{ii}\omega, \qquad k_p^i = m_{ii}\omega^2\,, \tag{2.3}$$

then all equations describing the motion of manipulator links have the same form as the equation of a critically damped harmonic oscillator:

$$\ddot{e}_i + 2\omega\dot{e}_i + \omega^2 e_i = 0. \tag{2.4}$$

Critical damping ensures the fastest non-oscillatory reduction of the tracking error $e_i(t)$ to zero. Any oscillations occurring when a robot is in motion are undesirable, because oscillations can excite vibrations of various mechanical sub-systems. For this reason the relationship between the coefficients $k_p^i$, $k_v^i$ determined by the formula (2.3) was accepted.

In such a way the number of control parameters that have to be chosen is reduced to the single parameter $\omega$. The parameter $\omega$ is relatively easy to determine by simulating the motion of the manipulator for various external disturbances, for non-zero initial values of $e_i(0)$, $\dot{e}_i(0)$, and for differences between the model parameters and plant parameters. To evaluate the control quality one can make use of a synthetic performance index. For instance, the performance index can take the form:

$$I = \frac{1}{k_{\max}} \sum_{j=0}^{k_{\max}} \sum_{i=1}^{n} |e_i(j\Delta)| S_i(j\Delta), \tag{2.5}$$

where $\Delta$ – the sampling period, $k_{\max}$ – the number of sampling periods, $k_{\max} = T/\Delta$ where $T$ denotes duration of the programmed motion, $S_i$ – weighting coefficients.

The simulations [3] revealed that in many cases this method of selection of control parameters is effective and good control quality may be achieved.

This method is not infallible, of course. For instance, in the case of strong dependence of the diagonal elements $m_{ii}$ on the configuration $\mathbf{q}$, dynamical properties of a manipulator change significantly when the manipulator motion continues, no matter how slow the motion is. In particular, for such a method of selection of feedback gains, the overall closed-loop system will be strongly overdamped for some configurations and strongly underdamped for other configurations. In such cases, it is worth to consider, whether it would be better to chose the parameters on the basis of average values of the diagonal elements $m_{ii}$ than on the basis of their final values.

### *2.2. Inverse dynamics control*

In the case of the inverse dynamics control, a rational choice of feedback gains is much simpler. In accordance with the scheme in Fig.2, the control law has the form

$$\mathbf{P} = \widehat{M}(\mathbf{q})(\ddot{\mathbf{q}}^d + K_p\mathbf{e} + K_v\dot{\mathbf{e}}) + \widehat{V}(\mathbf{q},\dot{\mathbf{q}}) + \widehat{G}(\mathbf{q}) + \widehat{F}(\mathbf{q},\dot{\mathbf{q}}). \tag{2.6}$$

Contrary to the control system of Fig. 1, calculations of elements of the matrix $\widehat{M}$ and the vectors $\widehat{V}$, $\widehat{G}$, $\widehat{F}$ have to be done on-line.

It is worth to mention here that the control according to the scheme in Fig. 2 is called also in many publications the computed torques control.

The following observations arise from the analysis of the motion equations (1.1) and the control law (2.6). The input signals (2.6) are intended to compensate for all time-dependent inertia forces and moments, and all time-dependent, deterministic, possible to predict external forces and torques – reduced to axes of joints.

Furthermore, it is worth to notice that the components of the input signals which depend on $\mathbf{e}$ and $\dot{\mathbf{e}}$ are equal, respectively, to $M(\mathbf{q})K_p\mathbf{e}$ and $M(\mathbf{q})K_v\dot{\mathbf{e}}$. It can be stated that the block in Fig. 2 described by $M(\mathbf{q})$ adjusts the feedback gains so that for greater inertia the gains are greater.

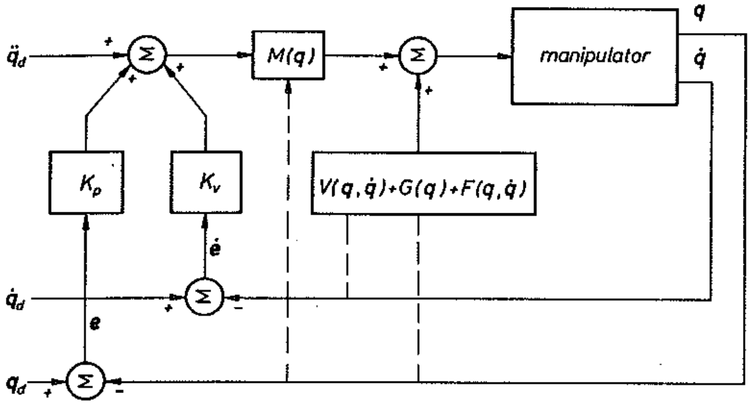


FIG. 2. The inverse dynamics control system.

When it is possible to identify the parameters of a manipulator with good accuracy, one can assume that

$$(2.7)\qquad \widehat{M} = M, \qquad \hat{\mathbf{V}} = \mathbf{V}, \qquad \hat{\mathbf{G}} = \mathbf{G}, \qquad \hat{\mathbf{F}} = \mathbf{F},$$

and then, after substituting Eq. (2.6) into the right-hand side of Eq. (1.1) and after making use of Eq. (2.7), Eq. (1.1) takes the form

$$(2.8)\qquad \ddot{\mathbf{e}} + K_v\dot{\mathbf{e}} + K_p\mathbf{e} = \mathbf{0}.$$

For the diagonal matrices $K_p$, $K_v$ the equations describing the motion of a manipulator with the control system of Fig. 2 are linear and not cross-coupled. The form of Eq. (2.8) is very advantageous with respect to dynamical properties of the overall closed-loop system and also with respect to the synthesis of control. In this case synthesis of control means an appropriate choice of values of elements of the matrices $K_p$, $K_v$.

However, it is necessary to say that the equations describing the motion of a manipulator with the control system of Fig. 2 cease to be linear and become cross-coupled, if unknown external disturbances, constant or time-dependent, act on a manipulator.

Let us denote the vector of external forces and torques acting at the joints by $\mathbf{P}^z$. If the assumption (2.7) is satisfied and the vector $\mathbf{P}^z$ is not zero, the motion equations of the closed-loop system have the form

$$\ddot{\mathbf{e}} + K_v\dot{\mathbf{e}} + K_p\mathbf{e} = M^{-1}(\mathbf{q})\mathbf{P}^z. \tag{2.9}$$

Eq. (2.9) are nonlinear and cross-coupled.

If $\mathbf{P}^z = \mathbf{0}$, but parameters of the model differ from parameters of the manipulator, the motion equations are also nonlinear and cross-coupled and have the following form:

$$\ddot{\mathbf{e}} + K_v\dot{\mathbf{e}} + K_p\mathbf{e} = \widehat{M}^{-1}\left[(M - \widehat{M})\ddot{\mathbf{q}} + (\mathbf{V} - \widehat{\mathbf{V}}) + (\mathbf{G} - \widehat{\mathbf{G}}) + (\mathbf{F} - \widehat{\mathbf{F}})\right]. \tag{2.10}$$

Although Eqs. (2.8) describe the behaviour of the overall closed-loop system with inverse dynamics control only in the ideal case, when disturbances do not occur and the model is entirely conformable to the plant, these equations can be taken initially as the basis for the choice of elements of the matrices $K_p$, $K_v$. It follows from Eqs. (2.8) that it is advisable to assume a diagonal form of the matrices, because in such a case the $i$-th equation of the system (2.8) becomes the second order equation of one variable $e_i$, i.e. the equations of the system (2.8) are not cross-coupled. If disturbances do not act on the mechanism of the manipulator then the tracking error $e_i(t)$ depends only on the initial values $e_i(0)$, $\dot{e}_i(0)$ and on the diagonal elements in the $i$-th rows of the matrices $K_p$, $K_v$, and it does not depend on the trajectory.

Under the assumption of a diagonal form of the matrices $K_p$, $K_v$, and when the diagonal elements $k_p^i$, $k_v^i$ are

$$k_p^i = \omega^2, \qquad k_v^i = 2\omega \qquad \text{for} \quad i = 1, \ldots, n,$$

the motion equation of every link takes the form of the equation of a critically damped harmonic oscillator.

The rate of convergence of the tracking error $e_i(t)$ to zero depends on the pulsatance $\omega$. Assumption of a large value of $\omega$ ensures fast convergence of the tracking errors $e_i(t)$ towards zero, but one has to keep in mind that increasing the pulsatance $\omega$ results in larger driving forces and torques.

The inverse dynamics control does not ensure the zero steady-state error in the case when a constant external disturbance acts on the manipulator, or when parameters of the manipulator (in particular, masses of manipulator's links) differ from the parameters of the model. Let us assume that the manipulator achieves its final configuration $\mathbf{q}^f$ at which $\dot{\mathbf{e}} = \mathbf{0}$, and $\ddot{\mathbf{e}} = \mathbf{0}$.

In accordance with Eq. (2.9), external disturbances acting on the manipulator (which one can reduce to forces and moments $\mathbf{P}^z$ acting at manipulator joints) cause steady-state errors determined by the formula:

$$K_p\mathbf{e} = M^{-1}(\mathbf{q}^f)\mathbf{P}^z .$$

Similarly, if masses of the manipulator links differ from those accepted in the model, then the steady-state errors can be calculated by the formula:

$$K_p\mathbf{e} = \widehat{M}^{-1}(\mathbf{q}^f)\left[\mathbf{G}(\mathbf{q}^f) - \widehat{\mathbf{G}}(\mathbf{q}^f)\right].$$

In the report [3], the results of simulations for two simple manipulators were presented. The 1st manipulator had three degrees of freedom (with three revolute joints), and the 2nd manipulator had four degrees of freedom (the 1st joint was revolute, the 2nd and the 3rd were prismatic, and the 4th was revolute). It was found on the basis of these investigations that it is possible to achieve a good control quality (i.e. small steady-state errors, small tracking errors, good disturbances rejection) using the following procedure of the choice of the parameters $k_p^i$, $k_v^i$.

At first, the smallest value of the pulsatance $\omega$ has to be determined, for which the steady-state errors are less than the permissible errors. The steady-state errors may be calculated by the formulae given above for the estimated constant external disturbances at the final position $\mathbf{q}^f$, or for the assumed maximum differences between the masses of manipulator links and those of the model links.

Next, one can gradually increase $\omega$ (that results in reduction of the tracking errors) checking simultaneously, whether the driving forces do not exceed the permissible values, and whether they do not increase excessively in relation to their nominal values.

It was found, for both the manipulators investigated, that for the sampling frequency 100 Hz and higher it is possible to apply high feedback gains (e.g. $\omega = 15$) without the risk that the system will cease to be stable due to quantization of the input signals.

The procedures of choice of the control parameters described above can provide satisfactory results, but the results are not optimal in any sense (no matter what criterion of optimality is in question), because they are based on simplified models of a plant and control system.

Even though the condition (2.7) and the condition $\mathbf{P}^z = \mathbf{0}$ are satisfied, Eq. (2.8) does not describe exactly the behaviour of the inverse dynamics control system because it does not include the following factors. First, the period of time needed to calculate the model, i.e. to calculate the matrix $\widehat{M}$ and the vectors $\widehat{\mathbf{V}}$, $\widehat{\mathbf{G}}$, $\widehat{\mathbf{F}}$, is not infinitesimal. It equals, in the best case, one sampling period $T$ and therefore some time-delay exists in the system. Second, in digital systems input signals (in the present case driving forces and torques) are constant in the period $< t_i, t_i + T)$ between two samples. Discrete version of the control law (2.6) depends on the method of technical implementation of a particular digital system. For instance, it can take the form:

$$\text{(2.11)} \qquad \mathbf{P}(n\Delta+\tau)=\widehat{\mathbf{M}}(n\Delta-\Delta)\Big[\ddot{\mathbf{q}}_p^d(n\Delta)+K_v\mathbf{e}(n\Delta-\Delta)+K_p\dot{\mathbf{e}}(n\Delta-\Delta)\Big] + \widehat{V}(n\Delta-\Delta)+\widehat{G}(n\Delta-\Delta)+\widehat{F}(n\Delta-\Delta)$$

for $n \in N$ and $\tau \in <0, \Delta)$.

Time-delay in the inverse dynamics control system and quantization of input signals can considerably affect the dynamics of the system. The discrete control system may be unstable, although the system of the same structure but time-continuous has a resonable stability margins. Such a case may happen when the feedback gains and the sampling period are too high.

## 3. Optimization of control parameters

Taking into account the drawbacks and limitations of the described selection procedures of control parameters, attempts were made to find better procedures. It was verified for the manipulator of three degrees of freedom, shown in Fig. 3, what results follow from the optimization of diagonal elements of the matrices $K_p$, $K_v$. At first, the performance index (2.5) was taken as the optimization criterion, and simultaneously it was assumed that

- $\mathbf{e}(0) = \mathbf{0}, \quad \dot{\mathbf{e}}(0) = \mathbf{0}$;
- all parameters of the model implemented into the control system are equal to the respective parameters of the plant (manipulator);
- the weighting coefficients $S_i$ are time-independent and equal to $360/2\pi$, what actually means that the errors $e_i$ expressed in radians are converted into degrees.

The masses and mass moments of inertia with respect to the central principal axes are given in Table 1. Important geometrical dimensions are given in Fig. 3.

The results of optimization and simulation of the manipulator motion for two programmed trajectories are given in the paper. In the case of the 1st trajectory, the programmed motion begins from the intial configuration presented in Fig. 4, and the duration $T$ of the programmed motion is 1.0 s or 0.5 s.

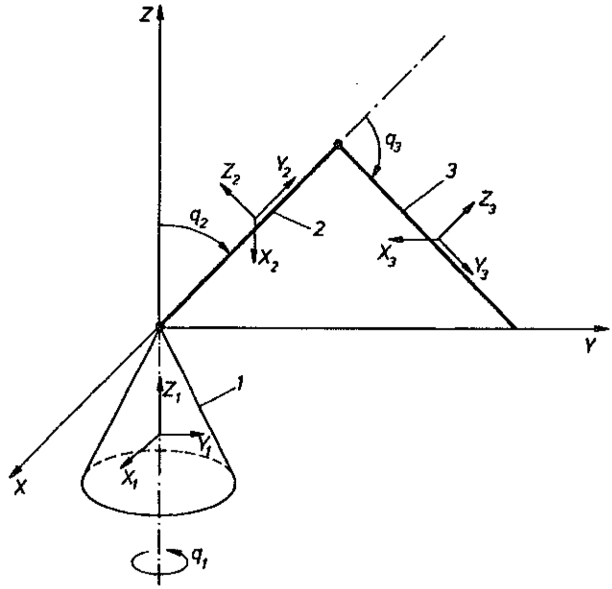


FIG. 3. The scheme of the manipulator.

**Table 1. The parameters of the manipulator.**

| No of link | mass kg | mass moments of inertia kgm$^2$ | | |
|---|---|---|---|---|
| | | $I_{xi}$ | $I_{yi}$ | $I_{zi}$ |
| 1 | – | – | – | 1.0 |
| 2 | 10 | 1.0 | 0 | 1.0 |
| 3 | 10 | 1.0 | 0 | 1.0 |

During the time $T$, the first manipulator link rotates together with the entire mechanism anticlockwise by 90 degrees, the 2nd and the 3rd link move in such a way that the free end of the 3rd link remains on the $0xy$ plane of the fixed coordinate frame $0xyz$, and at the final position the longitudinal axes of the 2nd and the 3rd link coincide with the axis $0y$. The motion is programmed in such a way that in the time interval from 0 to $0.5\,T$ all links of the manipulator move with constant accelerations and in the time period $0.5T \div 1.0T$ they are slowed down. In the second phase of the motion the accelerations of links have the same values as in the first phase, but with opposite signs.

In the case of the trajectory No 2, the programmed motion consists also of two phases, i.e. of accelerating and slowing down of the manipulator

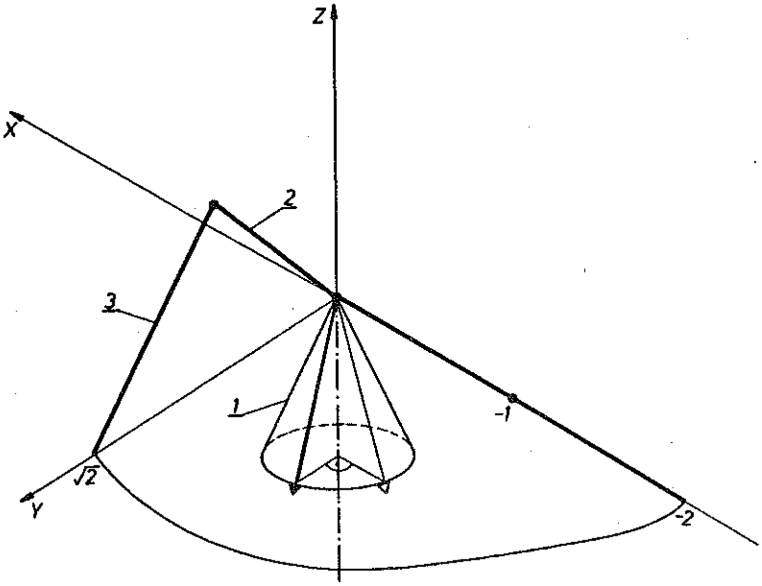


FIG. 4. The initial and the final configuration of the manipulator for the first trajectory.

links with constant accelerations in the equal time periods from 0 to $0.5T$ and from $0.5T$ to $1.0T$. At the initial position (Fig. 5) longitudinal axes of the 2nd and the 3rd link coincide with the axis $0y$ of the fixed coordinate frame $0xyz$. The free end of the 3rd link has the coordinates (0,2,0). The mechanism of manipulator rotates anticlockwise by 90 degrees. The motions of the 2nd and the 3rd links are synchronized so that the free end of the 3rd member lies on the plane $0xy$, and at the final position it has the coordinates (0,0,0).

The smallest values of the performance index (2.5) obtained by means of numerical minimization for the 1st trajectory, for the duration of motion $T = 1\,\mathrm{s}$, and for the sampling period $\Delta = 0,01\,\mathrm{s}$ are equal to:

1. 0.086 – for the control system of Fig. 1,
2. 0.046 – for the control system of Fig. 2.

The minimal values of the performance index $I$ are very small. One can interprete the value of this index as the average value of a sum of the tracking errors $e_i$ $(i = 1, 2, 3)$, expressed in degrees.

However, one has to keep in mind that the minimal values of the performance index were calculated for the case when all the conditions given above were satisfied. That is for the case, when quantization of the input signals (and time-delay in the inverse dynamics control system) are the only disturbances.

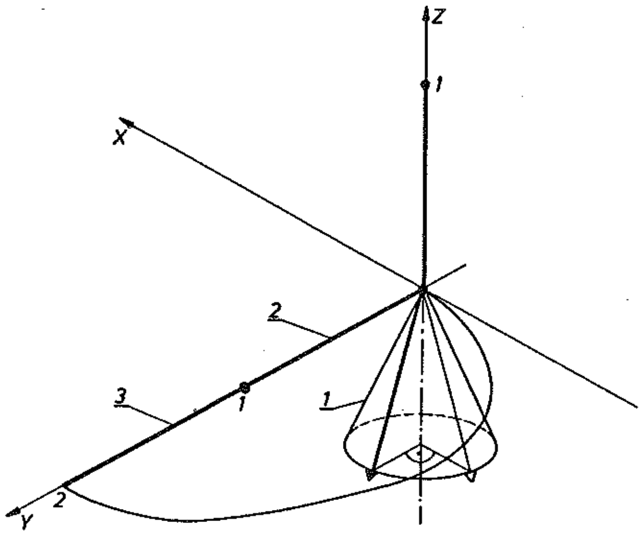


FIG. 5. The initial and the final configuration of the manipulator for the second trajectory.

Apart from the small tracking errors, every control system should ensure low sensitivity to disturbances, random or deterministic. It was investigated, how the initial values $e_i(0)$ different from zero influence value of the performance index $I$ calculated for the control parameters $k_p^i$, $k_v^i$, which optimize the index $I$ in the case, when $\mathbf{e}(0) = \mathbf{0}$, $\dot{\mathbf{e}}(0) = \mathbf{0}$.

The results of investigations on the influence of the initial values $e_i(0)$ on a value of the performance index $I$ are given in Table 2. The time $\tau$ expressed in seconds, given in the first column of Table 2, is a measure of initial disturbances of the velocities which are determined by the formula:

$$(3.1) \qquad \dot{e}_i(0) = -\dot{q}_i^d(t)|_{t=\tau}, \qquad i = 1, 2, 3.$$

The initial disturbances with signs opposite to the programmed velocities in the first phase of the motion can be interpreted as the result of impulses of torques (of the Dirac delta-function type) applied at time $t = 0$ at the manipulator joints and directed against the motion of the manipulator links.

The values of the performance index were calculated for two cases:

1. The case, when driving forces can be arbitrarily large.

2. The case, when driving torques can not be larger than the assumed limiting values. The maximum nominal values of driving torques (necessary to realize the programmed motion by the system without feedback) multiplied by 2 were accepted as the limiting values.

**Table 2. The values of the performance index (2.5) for disturbances of initial velocities determined by the formula (3.1), for the feedback gains established by optimization with respect to the criterion (2.5).**

| $\tau$ | control system of Fig. 1 | | control system of Fig. 2 | |
|---|---|---|---|---|
| $s$ | inputs unlimited | inputs limited | inputs unlimited | inputs limited |
| 0 | 0.086 | | 0.046 | |
| 0.01 | 0.089 | | 0.073 | |
| 0.02 | 0.095 | | 0.12 | |
| 0.03 | 0.107 | | 0.16 | |
| 0.04 | 0.106 | 0.772 | 0.21 | |
| 0.05 | 0.113 | 0.628 | 0.255 | |
| 0.06 | 0.122 | 2.04 | 0.301 | 1.60 |
| 0.07 | 0.129 | 3.32 | 0.384 | 1.95 |
| 0.08 | 0.138 | 3.96 | 0.396 | 1.90 |
| 0.09 | 0.149 | 4.92 | 0.443 | 1.67 |
| 0.10 | 0.156 | 5.57 | 0.491 | 2.01 |

On the basis of Table 2 one can come to the following conclusions. The inverse dynamics control system is more sensitive to the initial disturbances than the control system with PD compensators and feedforward compensation for gravitational forces (Fig. 1) in the 1st case, when the values of driving torques are unbounded. In the numerical experiments, large maximum values of driving torques were assumed, nevertheless in the case of the control system of Fig. 1 these maximum values were exceeded, even for the initial values $\dot{e}_i(0)$ equal to only three hundredths of the maximum nominal velocities $\dot{q}_i(T/2)$. It was found that for initial disturbances $\dot{e}_i(0) = 0.1(\dot{q}_i^d)_{\max}$ driving torques in the control system of Fig. 1 are a dozen times greater than the nominal driving torques in the time-continuous, open system (i.e. the system without feedback). In practice, it is impossible to accept so large driving torques, because of actuator sizes and energy consumption limitations.

When the control parameters are optimized with respect to the criterion (2.5), high values of the gains $k_p^i$, $k_v^i$ are obtained because the driving torques necessary to realize the optimal control are not taken into account in the criterion (2.5).

The feedback gains calculated in such a way may be too high, when it appears that external disturbances are more hazardous than the disturbances resulting from quantization of input signals.

It was supposed that including the input signals into the optimization criterion can increase the tracking errors but will also decrease the sensitivity of the control system to external disturbances.

For both control systems, the control parameters were optimized with respect to the following criterion:

$$I = I_1 + I_2 = \frac{1}{k_{\max}} \sum_{j=0}^{k_{\max}} \sum_{i=1}^{n} |e_i(j\Delta)| S_i(j\Delta) + \frac{1}{k_{\max}} \sum_{j=0}^{k_{\max}} \sum_{i=1}^{n} |p_i(j\Delta)| R_i(j\Delta), \tag{3.2}$$

where $p_i$ – the difference between an actual value of the torque $P_i$ and its nominal value in an open system without feedback, $R_i$ – the weighting coefficients.

The weighting coefficients $R_i$ were chosen so that the influence of $p_i$ equal to 10% of the maximum nominal torque $P_i$ on the criterion $I$ should be the same as the influence of the tracking error $e_i$ equal to one degree.

It was tested how the control systems with parameters assumed in such a way behave when there exist disturbances of initial velocities determined by the formula (3.1). The results of calculations are given in Table 3.

**Table 3. The values of the performance index (2.5) for disturbances of initial velocities determined by the formula (3.1), for the feedback gains established by optimization with respect to the criterion (3.1).**

| | system of Fig. 1 | system of Fig. 2 |
|---|---|---|
| 0 | 0.274 | 0.072 |
| 0.01 | 0.276 | 0.083 |
| 0.02 | 0.278 | 0.093 |
| 0.03 | 0.278 | 0.109 |
| 0.04 | 0.280 | 0.122 |
| 0.05 | 0.283 | 0.135 |
| 0.06 | 0.285 | 0.148 |
| 0.07 | 0.281 | 0.144 |
| 0.08 | 0.292 | 0.177 |
| 0.09 | 0.296 | 0.195 |
| 0.1 | 0.303 | 0.215 |

For the control parameters chosen by optimizing the criterion (3.2), the driving torques do not exceed their limiting values in any of the cases considered. The tracking errors are relatively small in both control systems, they are from 1.5 to 4 times smaller in the inverse dynamics control system.

The numerous numerical experiments, which were carried out, show clearly that the inverse dynamics control system has better dynamical properties in comparison with the control system of Fig. 1. The examples of optimization with respect to the criterion (3.2) for four different cases of manipulator motion are given in Table 4.

**Table 4. The minimal values of the performance index (3.2).**

| | control system of Fig. 1 | | | control system of Fig. 2 | | |
|---|---|---|---|---|---|---|
| | $I$ | $I_1$ | $I_2$ | $I$ | $I_1$ | $I_2$ |
| 1st trajectory $T = 1\,\text{s}$, $\Delta = 0.01\,\text{s}$ | 1.07 | 0.27 | 0.80 | 0.30 | 0.07 | 0.23 |
| 1st trajectory $T = 1\,\text{s}$, $\Delta = 0.02\,\text{s}$ | 2.6 | 0.64 | 1.96 | 1.18 | 0.53 | 0.66 |
| 1st trajectory $T = 0.5\,\text{s}$, $\Delta = 0.01\,\text{s}$ | 4.4 | 0.76 | 3.6 | 1.6 | 0.63 | 1.03 |
| 2nd trajectory $T = 1\,\text{s}$, $\Delta = 0.01\,\text{s}$ | 3.3 | 0.96 | 2.3 | 0.62 | 0.17 | 0.45 |

The procedure for the choice of control parameters based on minimization of the performace index in the form (3.2) appeared to be efficient – better than the procedure based on the analysis of simplified models – although the weighting coefficients $R_i$ were chosen by intuition.

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