ENGINEERING TRANSACTIONS • *Engng. Trans.* • **42**, 1–2, 85–105, 1994
Polish Academy of Sciences • Institute of Fundamental Technological Research

THE METHOD OF ELIMINATION OF VIBRATIONS AND FORCES EXCITED BY UNBALANCED ROTARY MACHINES WITH INACCESIBLE ROTOR

J. MICHALCZYK and G. CIEPLOK (KRAKÓW)

Possibilities of application of the method of synchronous elimination to the reduction of vibrations and forces in unbalanced rotary machines with inaccessible axes were considered in this paper. The conditions were formulated, which have to be satisfied by the system of inertia vibrators to achieve a stable solution of zero motion of the system in a stationary state. Theoretical considerations have been supported by numerical simulation of motion of the system: rotary machine – off-axial synchronous eliminator, and by experiments. The algorithm of optimization of the real system has been proposed.

1. Introduction

Static or dynamic unbalance of rotary machines is one of the most important industrial sources of vibrations and dynamic forces transferred to the bed. From among all the possible methods of elimination of the results of unbalance, such as the passive or active balancing of rotors, the passive, semi-active and active methods of vibroinsulation or application of the dynamic eliminators of vibrations, the method of synchronous elimination [1] should be distinguished as the method which is effective, of significant adaptation possibilities, relatively simple and dependable in operation.

This method, introduced by Fesce and Thearle [7], involves placing on the rotor axis the unbalanced elements, which can occupy any angular position in relation to the rotor. In the case of static unbalance of an unknown or variable value, it requires two elements (for example balls in drums) to be placed in the rotor axis, in the plane of rotation of the mass centre of the rotor, or as close to this plane as possible.

In the case of dynamic unbalance, the introduction of two assemblies of correction elements in two planes perpendicular to the rotor axis is required. Under definite conditions, among which the most important is the elastic support of the rotor, a spontaneous displacement of the correction elements proceeds, leading to elimination of the rotor unbalance and of the vibrational motion of its axis.

The basic difficulty in application of this method is often a lack of free place for the axial location of the correction elements on the rotor. The possibility of a synchronous elimination in the systems, in which the axes of rotation of the correction elements do not coincide with the rotor axis, has been shown in the paper [4].

2. The off-axial synchronous elimination. The general case

The conditions for attaining the off-axial elimination of vibrations and forces, in the form somewhat more general than in the cited paper, may be formulated as follows. Let us consider the rotary machine, which is unbalanced: A – statically, B – dynamically; the machine is founded in the manner allowing the body to vibrate along at least one generalized coordinate. Moreover, the constraints imposed on the motion of the system are such that forced body vibrations appear, for the defined type of unbalance. Next we shall mount onto the machine body the assembly of inertial vibrators (statically unbalanced rotors), with their axes parallel to the rotor axis and with individual drives, which have soft mechanical characteristics and velocities equal to the rotor rotation velocity, Fig. 1. Let the values of unbalance of the particular vibrators and the coordinates of their pivoting points (we define the pivoting point or centre as the intersection of the rotation plane of the rotor mass centre and its rotation axis) meet the following requirements for the case (A):

a) in the case of a known value of the rotor unbalance:

$$\sum_{i=1}^{n} \chi_i m_i e_i = m_0 e_0; \tag{2.1}$$

$$\sum_{i=1}^{n} \chi_i m_i e_i x_i = 0,\qquad \sum_{i=1}^{n} \chi_i m_i e_i y_i = 0,\qquad \sum_{i=1}^{n} \chi_i m_i e_i z_i = 0, \tag{2.2}$$

where $m_i e_i$ – static moment of the i-th correction vibrator unbalance, $m_0 e_0$ – static moment of the rotor unbalance, x_i, y_i, z_i – coordinates of the

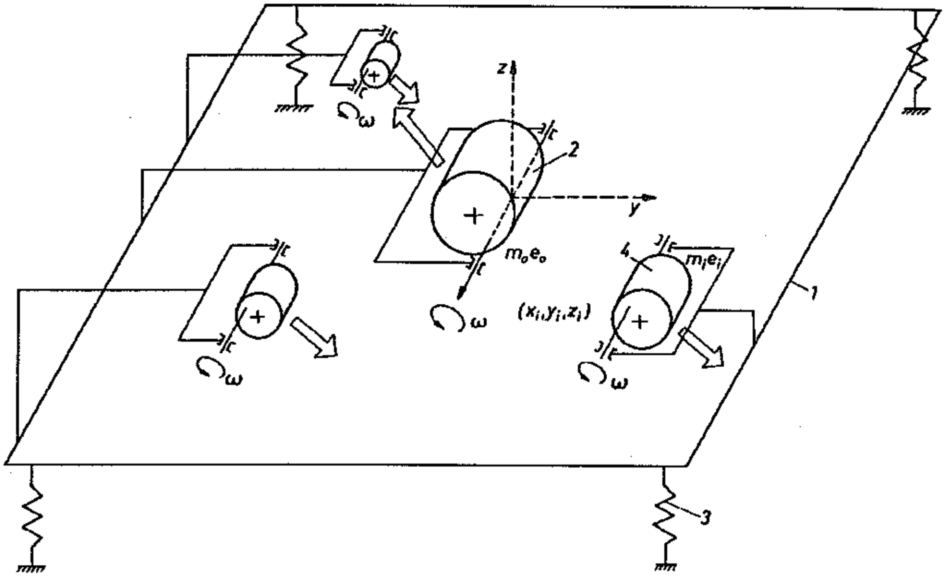

FIG. 1. Diagram of the off-axial synchronous elimination, *1* – body (bed), *2* – unbalanced rotor, *3* – support, *4* – correction vibrator.

i-th vibrator pivoting point, in the coordinate system centered at the rotor pivoting point.

$\chi_i = \pm 1$ the optional constant of the i-th vibrator.

The number and location of the correction vibrators with respect to the machine body may be chosen rather arbitrarily, depending on the accessible place in the surroundings of the rotor, and on the condition that none of the correction vibrators is founded at a point of the machine, which is not forced to vibrate by the unbalance or which vibrates only in the direction of its axis.

In the case of meeting the conditions (2.1) and $(2.2)_1$ to $(2.2)_3$, as well as keeping the identical value of the angular velocity ω of all rotors and keeping the phase angles, assuring the collinearity of the inertia forces in the stationary motion, we obtain the rotating system of parallel inertia forces which senses depend on χ_i ($\chi_i = +1$ in Fig. 1), applied to the machine body, which is equivalent to zero. This leads to elimination of the machine vibrations and to the lack of dynamic loads, transferred to the bed.

The conditions mentioned, together with the assumptions made earlier, concerning the drive of correction vibrators, assure the appearance of the desired type of motion of the system. The physical realization of this motion requires its stability. The factor stabilizing the desired system of phase angles may be, for example, the appropriate mode of control of the correction motors. A more convenient solution seems to be, however, the application of the phenomenon of autosynchronization of mechanical vibrators [2]. This phenomenon, discovered and described for pendulum clocks by Huyghens, is based on generating additional rotation moments, caused by vibrations of vibrator axes in the direction perpendicular to these axes. These motions synchronize the run of vibrators.

Confining our considerations to the systems with eccentrical inertia vibrators, operating outside the resonance region, we use the integral criterion of stability of synchronous motions [2]:

$$(2.3)\qquad D(\alpha_1,\alpha_2,\ldots,\alpha_n)=\frac{1}{T}\left[\int_0^T (E-V)\,dt-\int_0^T (E_w-V_w)\,dt\right]=\min .$$

According to this criterion, the system of phase angles

$$\alpha_1=\phi_1-\phi_0,\qquad \alpha_2=\phi_2-\phi_0,\quad \ldots,\qquad \alpha_n=\phi_n-\phi_0$$

is stable around the values $\alpha_{10},\alpha_{20},\ldots,\alpha_{n0}$, if the function D, defined by the relationship (2.3), takes a local minimum.

Here

$\phi_0,\phi_1,\ldots,\phi_n$ denote the angles of rotation of particular vibrators with respect to their initial locations,

$T=\dfrac{2\pi}{\omega}$ – period of forced vibrations,

E – kinetic energy of the machine body, with rotor masses concentrated in the point of rotation,

V – potential energy of the machine body support system,

E_w, V_w – kinetic and potential energy of constraints between the vibrators, respectively.

Confining our considerations to the systems supported "softly" (that means, to the systems, in which the ratio of ω to the nearest frequency of free vibrations is several times higher than unity), we may neglect V in the relationship (2.3), as being $\ll E$. On the other hand, assuming that the constraints between vibrators are inertialess and rigid (or not introducing these constraints at all), we may neglect E_w, V_w in the relation (2.3).

Then the condition (2.3) takes the form

$$(2.4)\qquad D(\alpha_1,\alpha_2,\ldots,\alpha_n)=\frac{1}{T}\int_0^T E\,dt=\min .$$

Let us note now that, because of the fact that kinetic energy is never negative, existence of the isolated point in the space $(\alpha_1, \alpha_2, \ldots \alpha_n)$, in which $E = 0$, is equivalent to the existence of a minimum of the function (2.3). If the system is in the desired location, in which the rotor inertia forces make the system equivalent to zero and the vibration of the body vanishes, i.e. $E = 0$, then any increase of the i-th phase angle, $d\alpha_i$, is equivalent to the addition of force dF_i to the system of forces in equilibrium, Fig. 2. This violates the equilibrium of the system and leads to nonnegative value of the kinetic energy of the system. This proves that there exists an isolated solution $(\alpha_{10} = \alpha_{20} = \ldots = \alpha_{n0})$, for which function D reaches a minimum, which further proves stability of the desired synchronous solution.

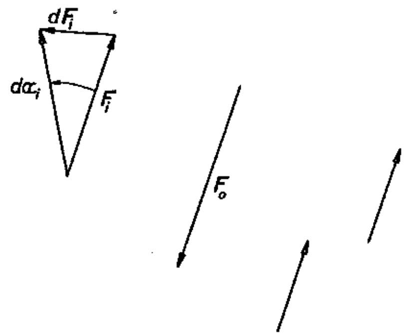

FIG. 2. Diagram of forces.

b) we may easily generalize the above considerations to the case of unbalance of the value which is either unknown or variable. It is sufficient to replace each of the correction vibrators by a system of two vibrators of a common axis and equal static moments of unbalance, $(1/2)m_ie_i$. Depending on the angle of mutual deviation of mass γ_i, this system allows us to obtain any value of the unbalance $me = m_ie_i\sqrt{0.5(1+\cos\gamma_i)}$ belonging to the range $[0, m_ie_i]$.

If the product m_0e_0 denotes the maximal expected value of the main rotor unbalance in condition (2.1), then (because of proportionality between the necessary values of the unbalance of correction vibrators and the values of the unbalance of the main rotor) for each value of the rotor unbalance, belonging to the range $[0, m_0e_0]$, there exists an appropriate value of angle γ_i for each vibrator, which makes it possible to attain the state of equilibrium and also $\gamma_1 = \gamma_2 = \ldots \gamma_n$.

Thus, if there exists a solution for the problem of the off-axial synchronous elimination for each value of the rotor unbalance, from the given range, then this solution is stable, according to the reasoning similar to the previous one.

Let us consider now the problem of elimination for the case (B) – dynamic unbalance of a rotor. When the value of this unbalance is understood as the maximal value of the moment of rotor deviation with respect to the axis x and one of the axes y or z is denoted by D, then this unbalance may be replaced by two opposite static unbalances, of the values me, appearing in the points of the rotor, defined by the arbitrarily chosen coordinates x_1 and x_2.

Then:

$$me = \frac{D}{d}, \tag{2.5}$$

where $d = x_1 - x_2$.

Now we can form the respective assembly of compensating vibrators, meeting the requirements (2.1), (2.2), for each of these unbalances. In the case of an unknown or variable value of the unbalance, the number of vibrators should be doubled, similarly to the case of static unbalance.

In the case of a combined static and dynamic unbalance, the dynamic unbalance should be replaced by two opposite parallel static unbalances, in the points x_1 and x_2, while the static unbalance should be replaced by two static unbalances, consistently parallel, in the same points x_1 and x_2. When the static unbalance is equal $m_s e_s$ and appears in the point x_3, and moreover $x_1 < x_3 < x_2$, then the respective values of the equivalent unbalances, lying in the same plane as $m_s e_s$, passing through the axis x, are equal to:

$$m_1 e_1 = m_s e_s \frac{x_2 - x_3}{x_2 - x_1}, \tag{2.6}$$

$$m_2 e_2 = m_s e_s \frac{x_3 - x_1}{x_2 - x_1}. \tag{2.7}$$

Finally, in each of the planes perpendicular to the x-axis and intersecting it at the points x_1 and x_2, there are two masses, statically unbalanced in the general case, mutually inclined by the angle δ and $180 - \delta$, Fig. 3. Summing them up we obtain the resulting value of the static unbalance in each plane:

$$m_{w1} e_{w1} = \sqrt{(me)^2 + (m_1 e_1)^2 + 2me \cdot m_1 e_1 \cdot \cos(\delta)}, \tag{2.8}$$

$$m_{w2} e_{w2} = \sqrt{(me)^2 + (m_2 e_2)^2 - 2me \cdot m_2 e_2 \cdot \cos(\delta)}. \tag{2.9}$$

We compensate these unbalances similarly as before.

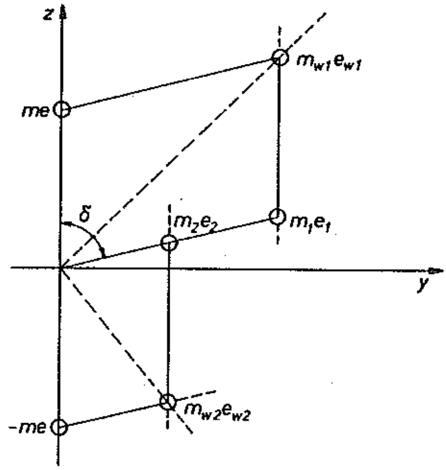

FIG. 3. Summation of statical moments of unbalance.

3. Particular cases of the off-axial synchronous elimination

3.1. Fixed value unbalance

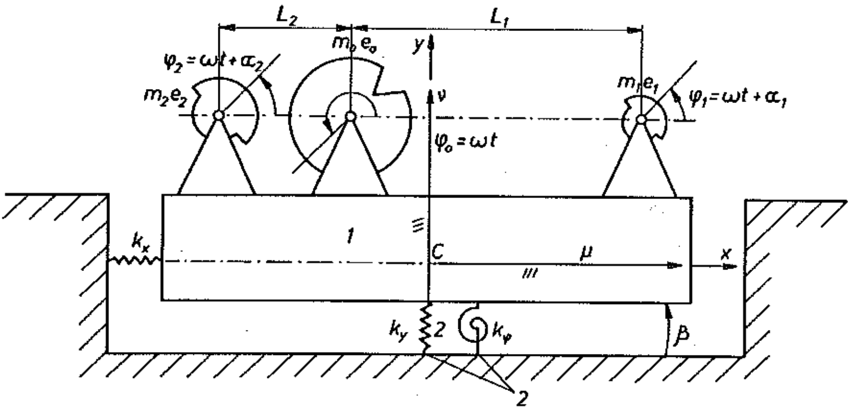

FIG. 4. Diagram of the simple system of off-axial synchronous elimination for the case of unbalance of a known value.

Let us consider the simplest system of the off-axial synchronous elimination, applicable when the unbalance is static, of a fixed value and location with respect to the rotor, or when it is slowly changing its angular orientation with respect to the rotor shaft, Fig. 4. Condition (2.1) leads then to the relationship

$$m_1e_1 + m_2e_2 = m_0e_0, \tag{3.1}$$

while conditions (2.2) impose the location of all pivoting points of the rotors along one line and fulfillment of the relationship

$$m_1e_1L_1 - m_2e_2L_2 = 0, \tag{3.2}$$

where L_1, L_2 denote the distance from the main rotor axis to the axis of the first and second correction vibrator, respectively. Writing the values of the centrifugal forces for the particular rotors, for the stationary motion of the angular velocity ω, in the form:

$$F_0 = m_0 e_0 \omega^2, \qquad F_1 = \xi_1 F_0, \qquad F_2 = \xi_2 F_0, \tag{3.3}$$

where

$$\xi_1 = \frac{L_2}{L_1 + L_2}, \qquad \xi_2 = \frac{L_1}{L_1 + L_2}. \tag{3.4}$$

The equations describing the stationary motion of the system may be written in the following form:

$$M\ddot{x} + k_x x = F_0\left[\cos(\omega t) + \xi_1 \cos(\omega t + \alpha_1) + \xi_2 \cos(\omega t + \alpha_2)\right], \tag{3.5}$$

$$M\ddot{y} + k_y y = F_0\left[\sin(\omega t) + \xi_1 \sin(\omega t + \alpha_1) + \xi_2 \sin(\omega t + \alpha_2)\right], \tag{3.6}$$

$$\begin{aligned} J\ddot{\beta} + k_\beta \beta = F_0 [&-v_0 \cos(\omega t) + \mu_0 \sin(\omega t) - \xi_1 v_1 \cos(\omega t + \alpha_1) \\ &+ \xi_1 \mu_1 \sin(\omega t + \alpha_1) - \xi_2 v_2 \cos(\omega t + \alpha_2) + \xi_2 \mu_2 \sin(\omega t + \alpha_2)]. \end{aligned} \tag{3.7}$$

Here:

M, J – the mass and the central moment of inertia of the machine, with the rotor masses related to their pivoting points,

x, y – the coordinates of the centre of mass C of the machine, in the absolute coordinate system $0xy$,

μ_i, v_i – coordinates of the rotor axis of rotation, in the central, mobile coordinate system $C\mu v$, attached to the machine (the systems $0xy$ and $C\mu v$ coincide in the state of static equilibrium),

α_1, α_2 – phase angles for the correction rotors.

Stationary solution of the equation has the form:

$$x(t) = \frac{F_0[\cos(\omega t) + \xi_1 \cos(\omega t + \alpha_1) + \xi_2 \cos(\omega t + \alpha_2)]}{k_x - M\omega^2}, \tag{3.8}$$

$$y(t) = \frac{F_0[\sin(\omega t) + \xi_1 \sin(\omega t + \alpha_1) + \xi_2 \sin(\omega t + \alpha_2)]}{2}, \tag{3.9}$$

$$\begin{aligned} \beta(t) = &\frac{F_0[-v_0 \cos(\omega t) + \mu_0 \sin(\omega t) - \xi_1 v_1 \cos(\omega t + \alpha_1)}{k_\beta - J\omega^2} \\ &+ \frac{\xi_1 \mu_1 \sin(\omega t + \alpha_1) - \xi_2 v_2 \cos(\omega t + \alpha_2) \xi_2 \mu_2 \sin(\omega)]}{k_\beta - J\omega^2}. \end{aligned} \tag{3.10}$$

By substituting into condition (2.3) the respective relationships for the displacements $x(t)$, $y(t)$, $\beta(t)$ and the relations resulting from those displacements for the velocities we obtain, in the system without constraints between the vibrators,

$$
\begin{aligned}
(3.11)\qquad & D(\alpha_1,\alpha_2) \\
&= \frac{\omega}{2\pi}\int_0^{2\pi/\omega} \frac{1}{2}\left\{M(\dot{x}^2+\dot{y}^2)+J\,\dot{\beta}^2-(k_x x^2+k_y y^2+k_\beta\beta^2)\right\}dt \\
&= \frac{F_0^2}{2}\left\langle\left(\frac{1}{k_x-M\omega^2}+\frac{1}{k_y-M\omega^2}\right)\Big[(\xi_1\xi_2\cos(\alpha_2)+\xi_1)\cos(\alpha_1)\right. \\
&\qquad +\xi_1\xi_2\sin(\alpha_1)\sin(\alpha_2)+\xi_2\cos(\alpha_2)+\frac{1}{2}(1+\xi_1^2+\xi_2^2)\Big] \\
&+\left(\frac{1}{k_\beta-J\omega^2}\right)\Big\{\Big[\xi_1\xi_2(\mu_1\mu_2+v_0^2)\cos(\alpha_2)-\xi_1\xi_2 v_0(\mu_1-\mu_2)\sin(\alpha_2) \\
&\qquad +\xi_1(v_0^2+\mu_1\mu_2)\Big]\cos(\alpha_1)+\Big[\xi_1\xi_2 v_0(\mu_2-\mu_1)\cos(\alpha_2) \\
&\qquad +\xi_1\xi_2(\mu_1\mu_2+v_0^2)\sin(\alpha_2)+\xi_1 v_0(\mu_0-\mu_1)\Big]\sin(\alpha_1) \\
&\qquad +\xi_2(\mu_0\mu_2+v_0^2)\cos(\alpha_2)-\xi_2 v_0(\mu_2-\mu_0)\sin(\beta) \\
&\qquad \left.+\frac{1}{2}\Big[(\mu_0^2+v_0^2)+\xi_1^2(\mu_1^2+v_0^2)+\xi_2^2(\mu_2^2+v_0^2)\Big]\Big\}\right\rangle.
\end{aligned}
$$

The contour diagram for the function $D(\alpha_1,\alpha_2)$ is shown in Fig. 5 obtained for the following values of the parameters:

$$
\begin{aligned}
M &= 222\,\text{kg}, & J &= 43.5\,\text{kgm}^2, & m_0e_0 &= 1\,\text{kgm},\\
\mu_0 &= 0.25\,\text{m}, & \mu_1 &= 3\,\text{m}, & \mu_2 &= -2.5\,\text{m},\\
k_x &= 5000\,\text{N/m}, & k_y &= 20000\,\text{N/m}, & k_\beta &= 4500\,\text{Nm/rad},\\
v_0 &= v_1 = v_2 = 1\,\text{m}, & \omega &= 150\,\text{rad/s}, & m_1 &= m_2 = 1\,\text{kg},\\
 & & & & e_1 &= e_2 = 0.05\,\text{m}.
\end{aligned}
$$

It shows the existence of two minima of the function D. One of them appears for the phase angles $\alpha_1 = \alpha_2 = \pi$ and corresponds to the desired solution, at which full elimination of vibrations occurs; the other minimum corresponds to the partial elimination of vibrations, due to reduction of the rotational vibrations.

The possibility of appearance of two stable states, of which only one corresponds to the desired type of solution (full elimination of vibrations and forces), has been indicated in the paper [4], where a similar system (although rotationally supported) was considered. Both solutions coincide when the system is in translatory motion.

In order to avoid "indeterminacy" of the stable state depending on the initial conditions, i.e. in order to obtain the desirable type of solution, we

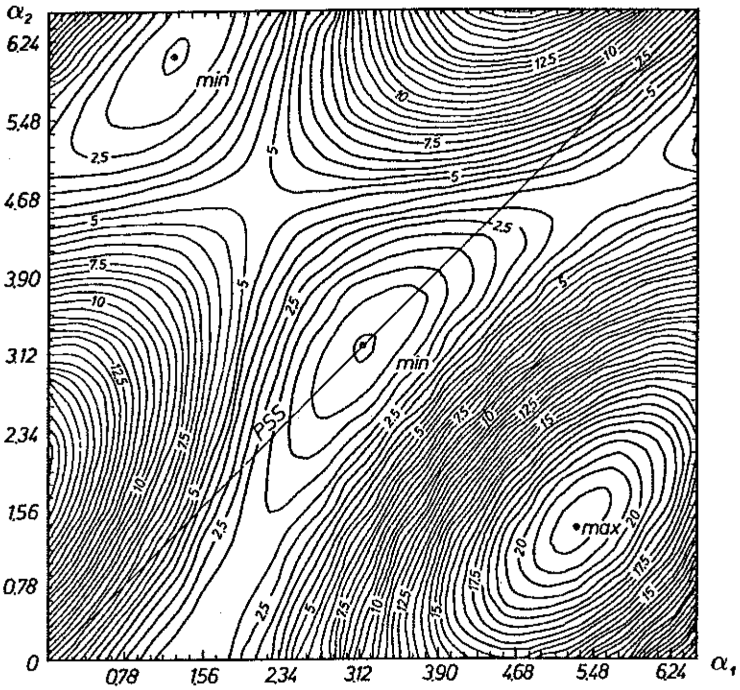

FIG. 5. Plot of the function $D(\alpha_1, \alpha_2)$; PSS – power selsyn system constraints.

can use the power selsyn system (PSS), so that the correction masses are uniform parallel ($\alpha_1 \cong \alpha_2$). This case is represented in Fig. 5 by a solid line PSS, which passes through only one "valley" of the D function.

In order to verify the theoretical considerations and to investigate the course of transient processes, the influence of friction and real characteristics of drive and PSS, simulation studies and experiments were carried out.

The diagram of the laboratory stand used for experimental verification of the system of off-axial synchronous elimination for the case of unbalance of a known value is shown in Fig. 6.

For the values of parameters of the experimental system:

$M = 90\,\mathrm{kg}$,	$J = 15.0\,\mathrm{kgm}^2$,	$m_0e_0 = 0.03\,\mathrm{kgm}$,
$\mu_0 = -0.13\,\mathrm{m}$,	$\mu_1 = 0.49\,\mathrm{m}$,	$\mu_2 = -0.45\,\mathrm{m}$,
$k_x = 20000\,\mathrm{N/m}$,	$k_y = 90000\,\mathrm{N/m}$,	$k_\beta = 44000\,\mathrm{Nm/rad}$,
$v_0 = v_1 = v_2 = 0\,\mathrm{m}$,	$\omega = 150\,\mathrm{rad/s}$,	$m_1 = m_2 = 1\,\mathrm{kg}$,
$e_1 = 0.01\,\mathrm{m}$,	$e_2 = 0.02\,\mathrm{m}$,	

the ratio of reduction of vibration up to 28 times (0.28 mm/0.01 mm) has been obtained, see Fig. 7 where *1* – plot of the vertical vibrations of the mass

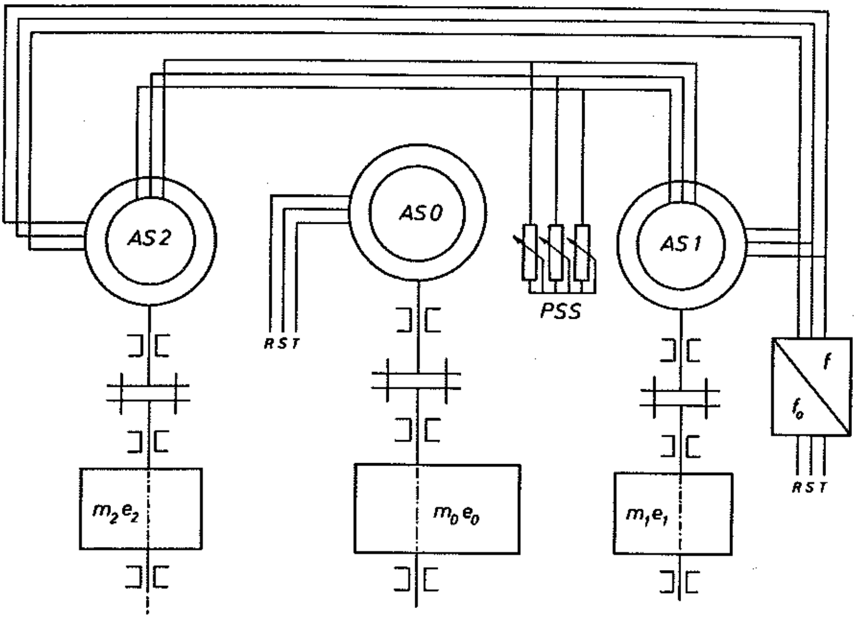

FIG. 6. Diagram of the laboratory stand, AS0 – drive of unbalanced rotor, AS1, AS2 – correction motors, PSS – power selsyn system, f/f_0 – frequency converter.

center $y(t)$ of the machine body without correction system, *2* – the same in the case when a correction system was used.

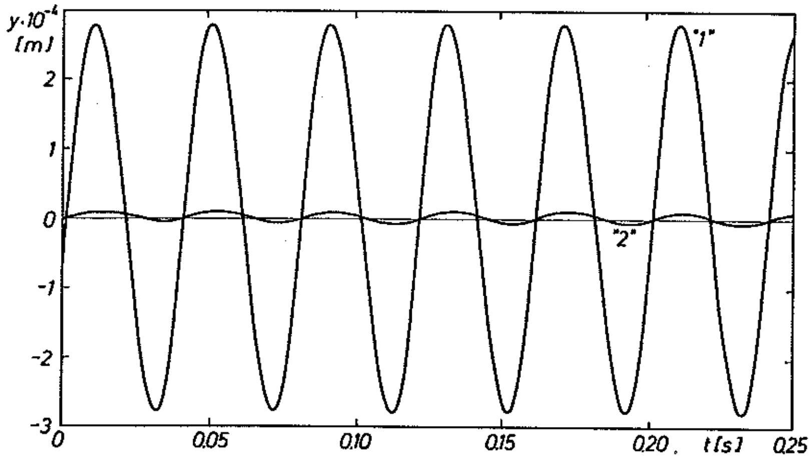

FIG. 7. The time variation of the vertical coordinate of the centre of mass of the machine body during steady state process. *1* – without correction system, *2* – with correction system.

3.2. Unbalance of variable value

Let us consider now the more advanced system, aimed at elimination of the results of unbalance, of an unknown or slowly variable value and the angular position, shown in Fig. 8. It consists of two double-mass vibrators [5], of the type shown in Fig. 9, of the drives joined by a power selsyn system. The electric constraint and the feedback between the motions of masses, introduced by the epicyclic gear of vibrators, assure the coincidental parallelism of the axes of symmetry for the angles of mutual deviations of masses for both vibrators.

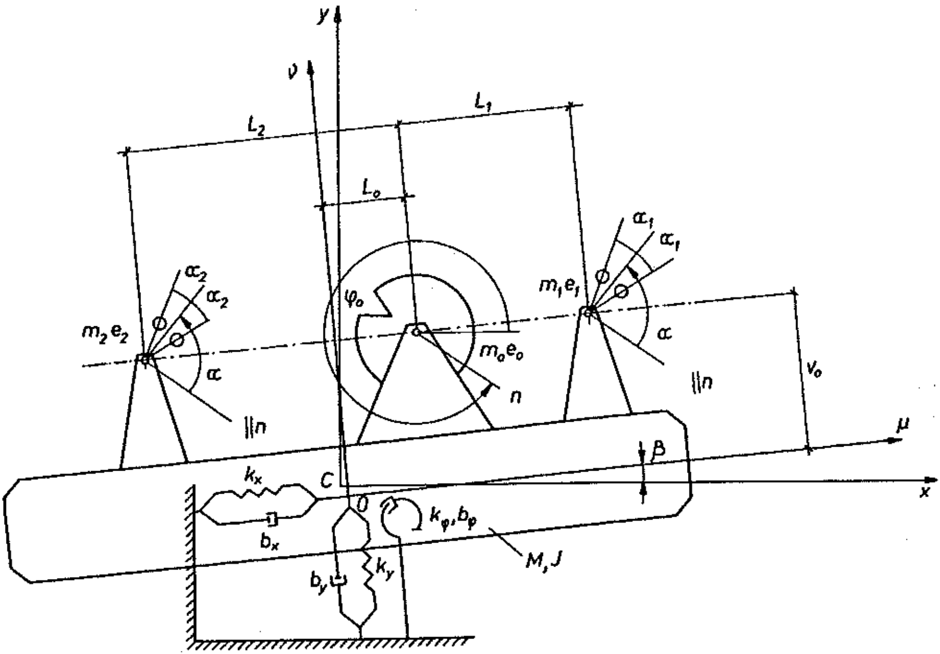

FIG. 8. Diagram of the off-axial synchronous elimination with the coupled two-mass correction vibrators.

Let us denote:

$\phi = \omega t$ – rotational angle of the main rotor unbalanced mass, calculated with respect to the coordinate system $0xy$,

α – the angle between the axis of symmetry for the mutual deviation of vibrator masses and the direction of main rotor unbalance,

$\alpha_{1,2}$ – angles of mutual deviation of the vibrators with respect to the axes of symmetry (due to the vibrator construction – equal and opposite).

The other notations remain unchanged. The equations of motion may

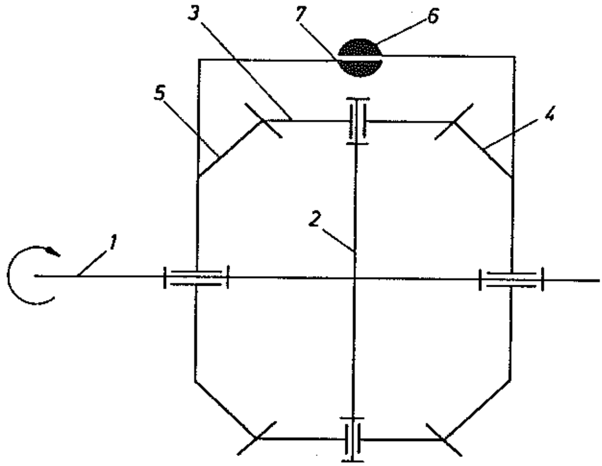

FIG. 9. Diagram of a coupled two-mass correction vibrator, *1* – shaft, *2* – yoke, *3, 4, 5* – epicyclic gear, *6, 7* – correction masses.

then be written as follows [6]:

$$M\ddot{x} + k_x x = F\left\{\cos(\omega t) + \left[\frac{F_1(\alpha_1)}{F} + \frac{F_2(\alpha_2)}{F}\right]\cos(\omega t + \alpha)\right\}, \tag{3.12}$$

$$M\ddot{y} + k_y y = F\left\{\sin(\omega t) + \left[\frac{F_1(\alpha_1)}{F} + \frac{F_2(\alpha_2)}{F}\right]\sin(\omega t + \alpha)\right\}, \tag{3.13}$$

$$J\ddot{\beta} + k_\beta \beta = F\left\{-v_0\cos(\omega t) + \left[\frac{F_1(\alpha_1)}{F} + \frac{F_2(\alpha_2)}{F}\right]\cos(\omega t + \alpha) - \mu_0\sin(\omega t) + \left[\mu_1\frac{F_1(\alpha_1)}{F} + \mu_2\frac{F_2(\alpha_2)}{F}\right]\sin(\omega t + \alpha)\right\}, \tag{3.14}$$

where

$$v_0 = v_1 = v_2 = H, \qquad \mu_0 = L_0, \qquad \mu_1 = L_1 + L_0, \qquad \mu_2 = (L_0 - L_2),$$

$$\frac{F_1(\alpha_1)}{F} = \xi(\alpha_1), \qquad \frac{F_1(\alpha_2)}{F} = \xi(\alpha_2), \qquad F = m_0 e_0 \omega^2,$$

$$F_1 = 2m_i e_i \omega^2 \cos(\alpha_i), \qquad i = 1, 2.$$

Denoting

$$\mu_1\xi(\alpha_1) + \mu_2\xi(\alpha_2) = W_{\xi\mu}(\alpha_1, \alpha_2), \tag{3.15}$$

$$\xi(\alpha_1) + \xi(\alpha_2) = W_\xi(\alpha_1, \alpha_2), \tag{3.16}$$

we may write the stationary solution for these equations, in the form:

$$x(t) = \frac{F}{k_x - M\omega^2}\left[\cos(\omega t) + W_\xi(\alpha_1, \alpha_2)\cos(\omega t + \alpha)\right], \tag{3.17}$$

$$y(t) = \frac{F}{k_y - M\omega^2}\left[\sin(\omega t) + W_\xi(\alpha_1,\alpha_2)\sin(\omega t + \alpha)\right], \tag{3.18}$$

$$\beta(t) = \frac{F}{k_\beta - J\omega^2}\left[-v_0\cos(\omega t) - v_0 W_\xi(\alpha_1,\alpha_2)\cos(\omega t+\alpha) + \mu_0\sin(\omega t) + W_{\xi\mu}(\alpha_1,\alpha_2)\sin(\omega t+\alpha)\right]. \tag{3.19}$$

For the analyzed case, the function D may be written in the form

$$D = \frac{\omega}{2\pi}\int_0^{2\pi/\omega} \frac{1}{2}\left[M(\dot{x}^2 + \dot{y}^2) + J\dot{\beta}^2 - (k_x x^2 + k_y y^2 + k_\beta \beta^2)\right] dt. \tag{3.20}$$

Substituting the respective values for the displacement and velocity, after integration and reduction, we finally obtain

$$\begin{aligned} D(\alpha,\alpha_1,\alpha_2) = \frac{-F^2}{4}\Big\{&\left[1 + 2W_\xi(\alpha_1,\alpha_2)\cos(\alpha) + W_\xi^2(\alpha_1,\alpha_2)\right] \\ &\times\left[\frac{1}{k_x - M\omega^2} + \frac{1}{k_y - M\omega^2} + \frac{1}{k_\beta - J\omega^2}\right] + \left[2v_0\mu_0 W_\xi(\alpha_1,\alpha_2)\sin(\alpha)\right. \\ &\left.+ W_{\xi\mu}(\alpha_1,\alpha_2)\cdot 2(\mu_0\cos(\alpha) - v_0\sin(\alpha)) + W_{\xi\mu}^2(\alpha_1,\alpha_2) + \mu_0^2\right]\frac{1}{k_\beta - J\omega^2}\Big\}. \end{aligned} \tag{3.21}$$

The conditions necessary and sufficient for the function (3.20) to have an extremum lead [6] to the relationship:

$$\alpha = \pm n\pi, \qquad n = 0,1,2,3\ldots, \tag{3.22}$$

$$W_\xi(\alpha_1,\alpha_2) = 1, \tag{3.23}$$

$$W_{\xi\mu}(\alpha_1,\alpha_2) = \mu_0, \tag{3.24}$$

$$\left[\frac{1}{k_x - M\omega^2} + \frac{1}{k_y - M\omega^2} + \frac{\mu_0^2 + v_0^2}{k_\beta - J\omega^2}\right]\cos(\alpha) > 0. \tag{3.25}$$

Condition (3.22) includes the desired solution $\alpha = \pm\pi(1+2n)$, which corresponds to the synchronous elimination. Conditions (3.23) and (3.24) impose the necessity of assuming the sufficiently high values of unbalance of the correction vibrators unbalance at such spatial position with respect to the rotor that there exist angles α_{10} nd α_{20} satisfying the above conditions. In particular, if $L_1 > 0$, $L_2 > 0$, it is sufficient to assume, for each ratio of lengths L_1, L_2, the following value of unbalance of the vibrators:

$$m_i e_i = \frac{1}{2} m_0 e_0, \qquad i = 1,2, \tag{3.26}$$

where $m_i e_i$ denotes the static moment of unbalance of each of the masses for the i-th double-mass vibrator.

From the form of inequality (3.25) it follows that:

1. When the angle α corresponds to the desired type of synchronization $\alpha = \pm\pi(1+2n)$, i.e. the opposite direction of the resultant forces of the correction rotors with respect to the main rotor, then the condition of stability of this solution is the negative sign of the expression in square brackets.

2. When the angle α assumes the value $\alpha = \pm 2n\pi$, corresponding to the enhancement of vibrations, then the condition for stability of this state is the positive sign of the expression in square brackets.

In particular, stability of the desired solution of the synchronous elimination may be obtained:

a) in the case when all frequencies of free vibrations of the system are lower than the rotation frequency of the main rotor,

b) in case of a system without possible motions along one or two generalized coordinates, when the frequencies of free vibrations along the other coordinates are lower than the rotation frequency of the main rotor.

4. Problems of optimization and adaptation of the system to the variable angular velocity of the main rotor

The natural angular velocities of the main rotor and the correction motors are usually different. Thus, the vibrational torques [1] must counterbalance the resistances appearing in the system when all the forced velocities are the same. The greater are the differences of mechanical characteristics of the driving motors and the steeper are these characteristics, the greater will be the resistances and the phase angle deviations, and the poorer will be the vibration abatement of the system. So, in order to minimize the vibration, we have to match the mechanical characteristics of correction motors to their movement resistances in order to obtain equal rotary velocities of the main rotor and the correction motors. Similar situation occurs when velocity of the main rotor is variable. In order to equalize the velocities of the main and correction rotors, the system of thyristor controller of supply voltage for correction motors can be used. Optimization and adaptation to the variable value of velocity of the main rotor can be carried out by means of the algorithm shown in Fig. 10. The amplitude A of vibration of the machine body was used as the measure of quality of choice of the supply voltage U_{cor}.

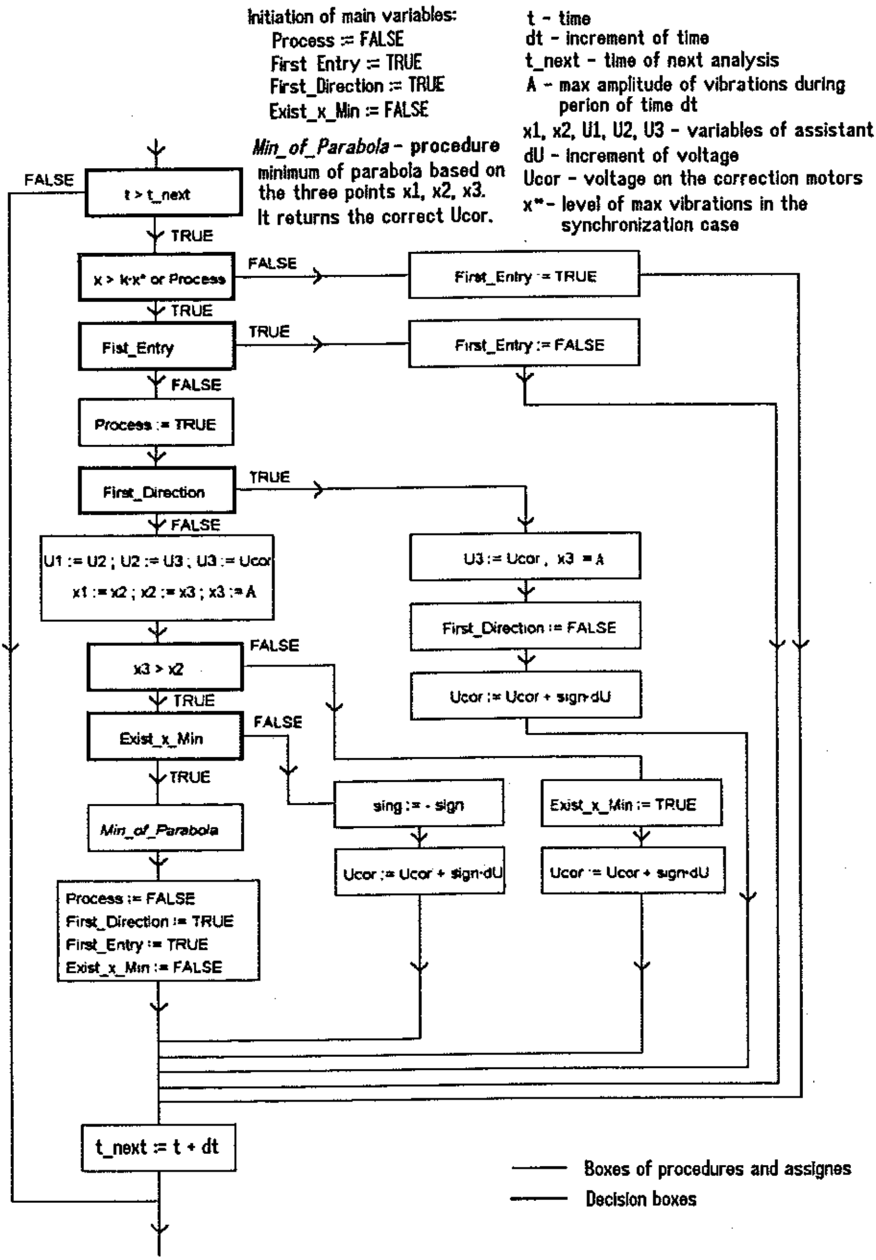

FIG. 10. Algorithm of supply voltage U_{cor} control assuring minimum of the amplitude A of vibrations.

4.1. Simulation studies

Numerical simulation of the behaviour of the system shown in Figs. 8, 9, 10, based on its suitably extended mathematical model (4.1) has been carried out.

$$[\mathbf{A}]\frac{d}{dt}\{\mathbf{q}\} = \{\mathbf{M}\}, \tag{4.1}$$

where:

$$\{\mathbf{q}\} = \mathrm{col}(\nu_x, \nu_y, \omega_\beta, \omega_1, \omega_2, \omega_3, \omega_4),$$

$$\{\mathbf{M}\} = \mathrm{col}(M_1, M_2, M_3, M_4, M_5, M_6, M_7),$$

$$M_1 = me\omega_1^2\cos(\varphi_1) + me\omega_2^2\cos(\varphi_2) + me\omega_3^2\cos(\varphi_3) + me\omega_4^2\cos(\varphi_4) + m_0e_0\left[\frac{d^2\varphi_0}{dt^2}\sin(\varphi_0) + \left(\frac{d\varphi_0}{dt}\right)^2\cos(\varphi_0)\right] - k_x x - b_x\nu_x,$$

$$M_2 = me\omega_1^2\sin(\varphi_1) + me\omega_2^2\sin(\varphi_2) + me\omega_3^2\sin(\varphi_3) + me\omega_4^2\sin(\varphi_4) + m_0e_0\left[-\frac{d^2\varphi_0}{dt^2}\cos(\varphi_0) + \left(\frac{d\varphi_0}{dt}\right)^2\sin(\varphi_0)\right] - k_y y - b_y\nu_y,$$

$$\begin{aligned} M_3 = \Big\langle -m_0e_0\Big\{&[v_0\sin(\varphi_0) + \mu_0\cos(\varphi_0)]\frac{d^2\varphi_0}{dt^2} \\ &+ [v_0\cos(\varphi_0) - \mu_0\sin(\varphi_0)]\left(\frac{d}{dt}\varphi_0\right)^2\Big\} \\ &- me\Big\{\omega_1^2[v_0\cos(\varphi_0) - \mu_1\sin(\varphi_0)] \\ &+ \omega_2^2[v_0\cos(\varphi_2) - \mu_1\sin(\varphi_2)] + \omega_3^2[v_0\cos(\varphi_3) - \mu_2\sin(\varphi_3)] \\ &+ \omega_4^2[v_0\cos(\varphi_1) - \mu_2\sin(\varphi_4)]\Big\} - b\omega_\beta - k\beta\Big\rangle, \end{aligned}$$

$$M_4 = \frac{1}{2}M_A - \frac{1}{4}(b_0 + 2b_w)\omega_1 - \frac{1}{4}(b_0 - 2b_w)\omega_2 + \frac{1}{2}b_0\omega_\beta,$$

$$M_5 = \frac{1}{2}M_A - \frac{1}{4}(b_0 + 2b_w)\omega_2 - \frac{1}{4}(b_0 - 2b_w)\omega_1 + \frac{1}{2}b_0\omega_\beta,$$

$$M_6 = \frac{1}{2}M_B - \frac{1}{4}(b_0 + 2b_w)\omega_3 - \frac{1}{4}(b_0 - 2b_w)\omega_4 + \frac{1}{2}b_0\omega_\beta,$$

$$M_7 = \frac{1}{2}M_B - \frac{1}{4}(b_0 + 2b_w)\omega_4 - \frac{1}{4}(b_0 - 2b_w)\omega_3 + \frac{1}{2}b_0\omega_\beta.$$

$$
[A] = \mathrm{SYM}\begin{bmatrix}
m_s + m_0 + 4m & & & & & & \\
0 & m_s + m_0 + 4m & & & & & \\
-v_0(m_0 + 4m) & m_0\mu_0 + 2m(\mu_1 + \mu_2) & J_s + m_0(4v_0^2 + 2\mu_1^2 + 2\mu_2^2) & & & & \\
-me\sin(\varphi_1) & me\cos(\varphi_1) & me[v_0\sin(\varphi_1) + \mu_1\cos(\varphi_1)] & J + \frac{1}{4}J_w + me^2 & & & \\
-me\sin(\varphi_2) & me\cos(\varphi_2) & me[v_0\sin(\varphi_2) + \mu_1\cos(\varphi_2)] & \frac{1}{4}J_w & J + \frac{1}{4}J_w + me^2 & & \\
-me\sin(\varphi_3) & me\cos(\varphi_3) & me[v_0\sin(\varphi_3) + \mu_2\cos(\varphi_3)] & 0 & 0 & J + \frac{1}{4}J_w + me^2 & \\
-me\sin(\varphi_4) & me\cos(\varphi_4) & me[v_0\sin(\varphi_4) + \mu_2\cos(\varphi_4)] & 0 & 0 & \frac{1}{4}J_w & J + \frac{1}{4}J_w + me^2
\end{bmatrix} \tag{102}
$$

The variation of the rotational velocity of the main rotor was given in the form

$$\omega = \omega_0[1 - \exp(-t/T)]. \tag{4.2}$$

The torqe of the correction motors was assumed according to the Kloss formula

$$M_{A,B} = 2M_u \left[(\omega_s - \omega_{A,B})(\omega_s - \omega_u)\right] / \left[(\omega_s - \omega_u)^2 + (\omega_s - \omega_{A,B})^2\right], \tag{4.3}$$

where M_u is a function of the voltage U_{cor}:

$$M_u = M_{u\,zn}(U_{\mathrm{cor}}/220)^2. \tag{3.30}$$

The simulation studies were performed using the parameters:

$$\begin{array}{lll}
m = 1\,[\mathrm{kg}], & J = 0.002\,[\mathrm{kgm^2}], & k_x = 5000\,[\mathrm{N/m}], \\
L_1 = 2.75\,[\mathrm{m}], & e = 0.05\,[\mathrm{m}], & J_0 = 0.05\,[\mathrm{kgm^2}], \\
k_y = 20000\,[\mathrm{N/m}], & L_0 = 0.25\,[\mathrm{m}], & m_0 = 20\,[\mathrm{kg}], \\
J_s = 20\,[\mathrm{kgm^2}], & k_\beta = 4500\,[\mathrm{N/m}], & L_2 = 2.25\,[\mathrm{m}], \\
m_s = 50\,[\mathrm{kg}], & b_0 = 0.002\,[\mathrm{Nms/rad}], & H = 1\,[\mathrm{m}], \\
M_{u\,zn} = 5\,[\mathrm{Nm}], & b_W = 0.005\,[\mathrm{Nms/rad}], & \omega_0 = 150\,[\mathrm{rad/s}], \\
\omega_s = 50\Pi, & J_w = 0.009\,[\mathrm{kgm^2}], & T = 0.25\,[\mathrm{s}], \\
\omega_u = 0\,[\mathrm{rad/s}]. & &
\end{array}$$

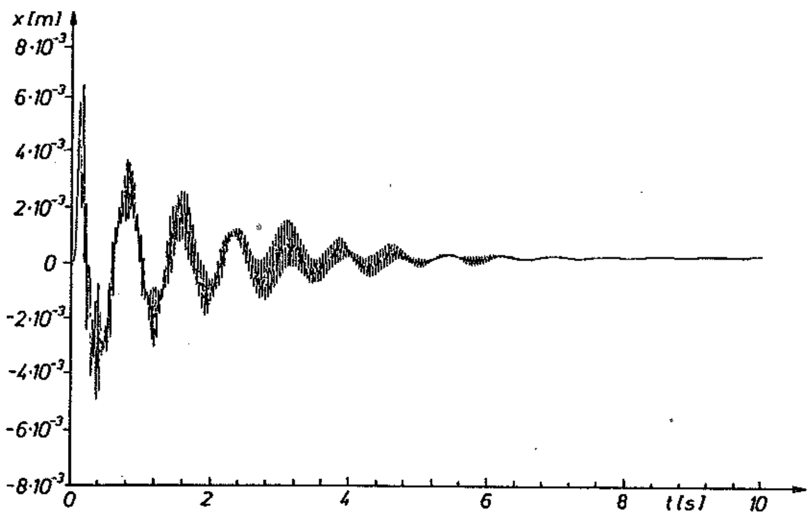

FIG. 11. Variation of the horizontal coordinate $x(t)$ of the correction system with the supply voltage control system (simulation).

The simulation studies confirmed the expectations of possibility of synchronous elimination of the system. In the case analyzed, the vibrations along all three coordinates x, y, β were reduced by an average of 1.35 mm/1.55 µm ≅875 times for x, y (Fig. 11), and by 2.1 10^{-3} rad/0.8 10^{-6} rad≅2625 times for rotations.

Variation of the voltage $U_{\text{cor}}(t)$ and phase relations between the rotors are shown in Fig. 12 and Fig. 13.

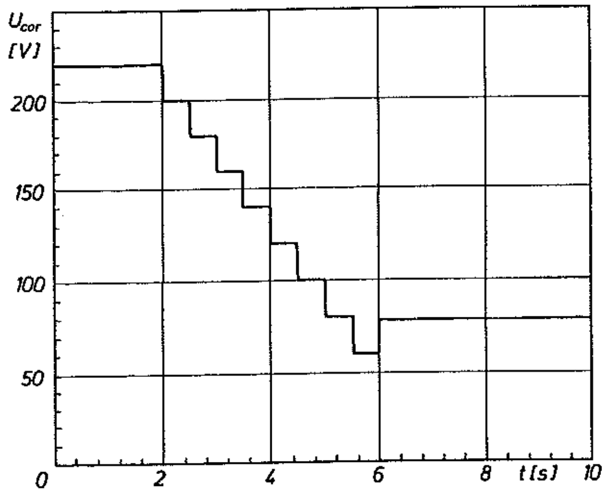

FIG. 12. The time variation of the supply voltage.

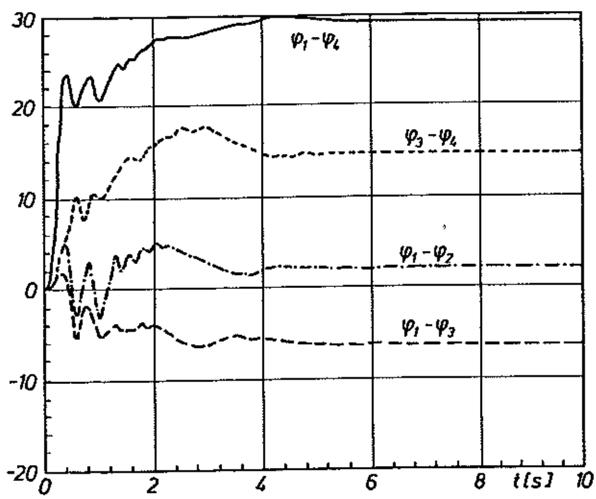

FIG. 13. Phase relations between rotors.

References

1. I.I. Blechman, *What can vibration do*, Science, Moscow 1988.
2. I.I. Blechman, *Synchronization of dynamic systems*, Science, Moscow 1971.
3. T. Majewski, *Self-balance of the rotor supported elastically in two directions*, Theoretical and Applied Mech., **16**, 1, 1978.
4. J. Michalczyk, *Off-axial synchronous elimination of vibration and forces in unbalanced rotary machines*, Theoretical and Applied Mech., **3**, 2, 1993.
5. J. Michalczyk, *Synchronous eliminator*, Patent Claim P-297 703/1993.
6. J. Michalczyk, G. Cieplok, *Generalized problem of synchronous elimination*, [in:] Machine Vibration, Springer-Verlag Ltd, London 1994.
7. A. Fesce, A. Thearle, *Automatic balancers*, Machine Design, no 9, 10, 11, 1950.

STANISŁAW STASZIC ACADEMY OF MINING AND METALLURGY, KRAKÓW.

Received October 8, 1993.