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ON CONVERGENCE OF ITERATIVE PROCEDURES FOR ANALYSIS OF COMPLEX LINEAR DYNAMIC SYSTEMS BY MEANS OF PARTIAL MODELS

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Problems of iterative convergence in the analysis of complex dynamic systems with the use of partial models are dealt with. Linear systems are considered. Conditions and ranges of convergence are determined for two basic iteration procedures: the so-called weak and the strong associations of masses in the complete model. Damping in the system is found to accelerate convergence of the procedures.

1. Introduction

Wide use of personal computers has encouraged many programs to be written in various fields of science and technology, among others in the dynamics of mechanical systems such as motor vehicles. For instance, the following programs are available: ADAMS by Mechanical Dynamics Inc. [2], SIMPACK and MEDYNA by MAN Technologie [6,7] and DINAMIKA [3]. The most important problem to create a system is to work out an efficient method of algorithmization of the motion equations for an object to be modelled and to ensure sufficient versatility of the system to deal with various structures with different numbers of degrees of freedom. More detailed information on the method that was the basis for a particular computational system is more often than not impossible to acquire. Even if such an information is available [1], some essential particulars are usually suppressed. Moreover, the programs are often written to analyse very specific types of problems.

The program ADSC, presented in [5] was also prepared to solve a particular class of problems, namely to analyse dynamic loads to which trucks are subjected. To make the system sufficiently flexible and expansive, a method has been worked out to analyse complex dynamic systems with the use of partial models. Theoretical background for the method was described in [4]. Two basic ways to decompose a complete model into partial ones were presented together with corresponding iteration procedures. In the case of the so-called weak associations of masses in the complete model the procedure was shown to converge.

One of the characteristic features of the proposed method is an iterative manner in which the calculations are conducted. That is why its applicability is highly dependent on suitable convergence of the procedures. This problem for a linear system is dealt with in the paper.

2. Convergence of iteration procedures for conservative systems

2.1. Weak associations of masses in the complete model

Relevant relationships between the involved parameters and the convergence of iterative procedures will be analysed with the help of a simple two-mass model. General rules for the model remain valid for more complicated models.

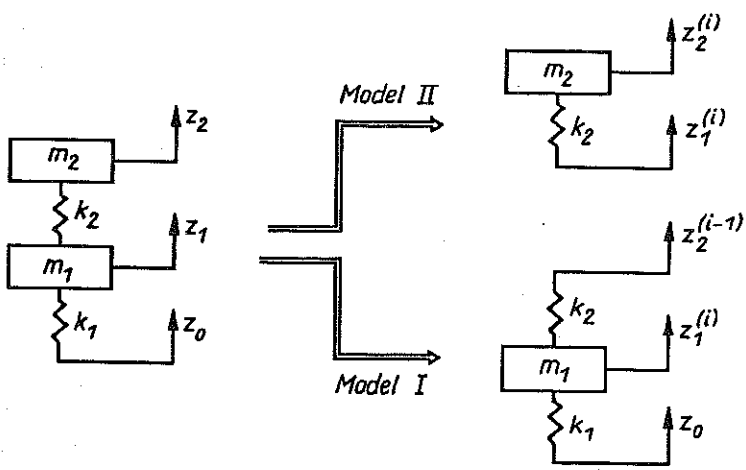

FIG. 1.

According to the decomposition principles presented in [4], in the case of weak associations of masses the complete model can be split up into partial ones as shown in Fig.1. Models I and II are analysed consecutively. For frequency analysis an appropriate convergence condition has the form

$$|H_{12}(\omega) \times H_{21}(\omega)| < 1\,, \tag{2.1}$$

where

$$H_{12}(\omega) = \frac{k_2}{k_1 + k_2 - m_1\omega^2}\,, \tag{2.2}$$

$$H_{21}(\omega) = \frac{k_2}{k_2 - m_2\omega^2}\,. \tag{2.3}$$

Inserting (2.2) and (2.3) into (2.1) and using the following notation for ratios of parameters:

$$S_1 = \frac{k_2}{k_1}\,, \qquad S_2 = \frac{m_1}{k_1}\,, \qquad S_3 = \frac{m_2}{m_1}\,. \tag{2.4}$$

The criterion for convergence can be rewritten to become

$$\left|\frac{S_1}{1 + S_1 - S_2\omega^2} \times \frac{S_1}{S_1 - S_2S_3\omega^2}\right| < 1\,. \tag{2.5}$$

Analysis of the above condition leads to the conclusion that it is satisfied for arbitrary values of the parameters S_1, S_2, S_3 outside a frequency interval $(\Omega_{11}, \Omega_{12})$ whose ends are

$$\begin{aligned} \Omega_{11} &= \frac{S_1 + S_3 + S_1S_3 - \sqrt{(S_1 + S_3 + S_1S_3)^2 - 4S_1S_3}}{2S_2S_3}\,, \\ \Omega_{12} &= \frac{S_1 + S_3 + S_1S_3 + \sqrt{(S_1 + S_3 + S_1S_3)^2 - 4S_1S_3}}{2S_2S_3}\,. \end{aligned} \tag{2.6}$$

Let us denote the frequencies of free vibrations of partial models by

$$\begin{aligned} \omega_{01}^2 &= \frac{k_1 + k_2}{m_1} = \frac{1 + S_1}{S_2}\,, \\ \omega_{02}^2 &= \frac{k_2}{m_2} = \frac{S_1}{S_2S_3}\,. \end{aligned} \tag{2.7}$$

The frequencies ω_{01} and ω_{02} can be proved to be related to the frequencies Ω_{11} and Ω_{12} by means of

$$\begin{aligned} \Omega_{11} &< \min(\omega_{01}, \omega_{02})\,, \\ \Omega_{12} &> \max(\omega_{01}, \omega_{02})\,. \end{aligned} \tag{2.8}$$

This means that the range $(\omega_{01}, \omega_{02})$ lies inside the range $(\Omega_{11}, \Omega_{12})$ irrespective of whether ω_{01} is larger or smaller than ω_{02}.

Inside a range $(\Omega_{21}, \Omega_{22})$ whose ends are

$$(2.9)\qquad \begin{aligned} \Omega_{21}^2 &= \frac{S_1 + S_3 + S_1S_3 - \sqrt{(S_1 + S_3 + S_1S_3)^2 - 4S_1S_3(2S_1 + 1)}}{2S_2S_3}, \\ \Omega_{22}^2 &= \frac{S_1 + S_3 + S_1S_3 + \sqrt{(S_1 + S_3 + S_1S_3)^2 - 4S_1S_3(2S_1 + 1)}}{2S_2S_3} \end{aligned}$$

the procedure is conditionally convergent, i.e. only for those cases in which the parameters satisfy the inequalities

$$(2.10)\qquad S_1 < S_{11} \quad \text{or} \quad S_1 > S_{12},$$

where

$$(2.11)\qquad \begin{aligned} S_{11} &= \frac{-S_3 \times (S_3 - 1) - 2S_3\sqrt{S_3}}{S_3^2 - 6S_3 + 1}, \\ S_{12} &= \frac{-S_3 \times (S_3 - 1) + 2S_3\sqrt{S_3}}{S_3^2 - 6S_3 + 1}. \end{aligned}$$

The following relationships among the frequencies $\omega_{01}, \omega_{02}, \Omega_{21}, \Omega_{22}$ can be proved to apply

$$(2.12)\qquad \min(\omega_{01}, \omega_{02}) < \Omega_{21} < \Omega_{22} < \max(\omega_{01}, \omega_{02}).$$

In the intervals $\langle \Omega_{11}, \Omega_{21} \rangle$ and $\langle \Omega_{22}, \Omega_{12} \rangle$ the procedure is always divergent.

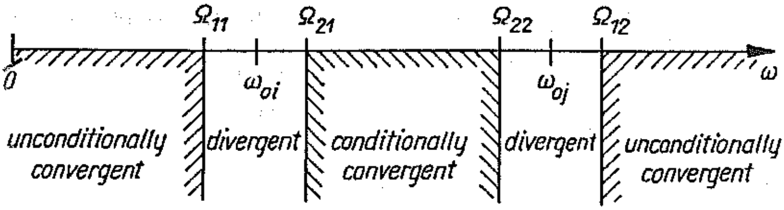

FIG. 2.

The above considerations can be visualized as shown in Fig.2. The conditions (2.11) for convergence within the interval $(\Omega_{21}, \Omega_{22})$ are shown diagrammatically in Fig.3. Shaded area corresponds to those situations in

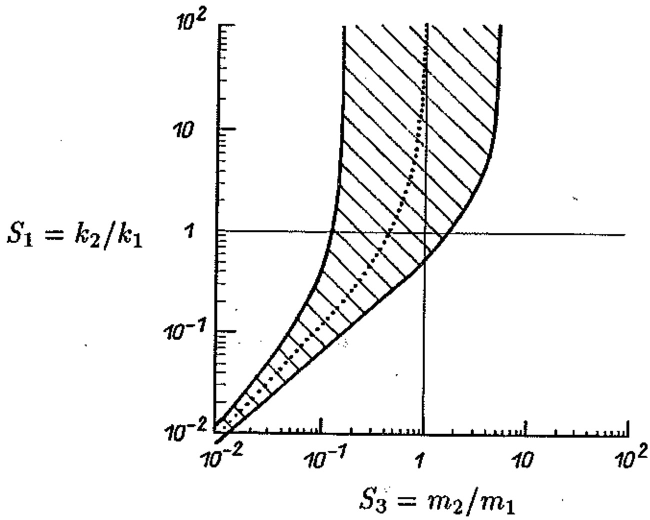

FIG. 3.

which the procedure turns out to be divergent. Skeleton line of this area corresponds to $\omega_{01} = \omega_{02}$. However, the intervals near the free vibration frequencies for which the procedure is divergent are spread to a lesser extent than suggested in Fig.2. From a detailed analysis of the relations (2.6) and (2.9) it follows that for $S_1 < 1$, i.e. for $k_2 < k_1$, their ranges are very small, see Fig.4.

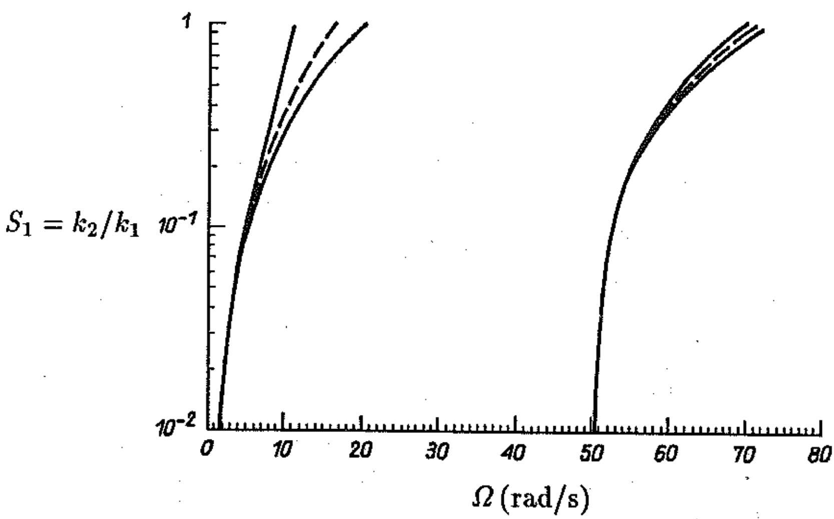

FIG. 4. Divergence regions for: $S_2 = m_1/k_1 = 0.0004(100/250000)$ $S_3 = m_2/m_1 = 10$ ——— regions of divergence, — — — frequencies of free vibrations.

In addition, it can be proved that

$$(2.13)\qquad \begin{aligned} \lim_{S_1\to 0}(\Omega_{21}-\Omega_{11}) &= 0\,,\\ \lim_{S_1\to 0}(\Omega_{12}-\Omega_{22}) &= 0\,. \end{aligned}$$

The limiting case $S_1 \to 0$ should be here understood as $k_1 \to \infty$ and not as a trivial case $k_2 \to 0$.

2.2. Strong associations of masses in the complete model

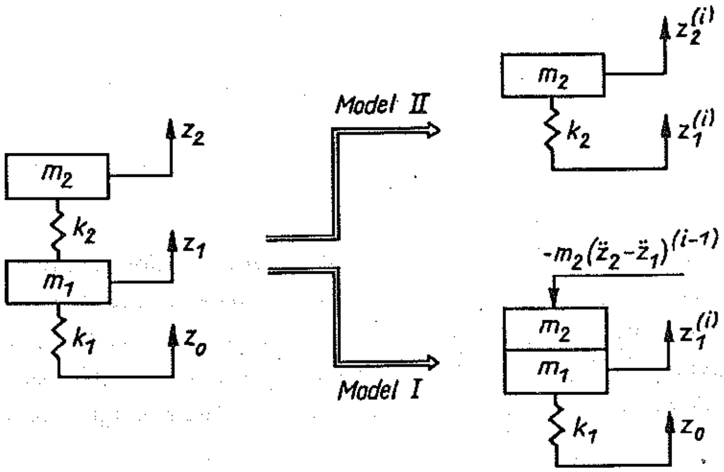

FIG. 5.

In this case the complete model is decomposed into two partial ones as shown in Fig.5. Iteration procedure is similar to that used in the former case. The only two differences are: the partial model I and a type of its coupling. Condition for convergence has the form

$$(2.14)\qquad |H^*_{12}(\omega)\times H_{21}(\omega) - H^*_{12}(\omega)| < 1\,,$$

where

$$(2.15)\qquad H^*_{12}(\omega) = \frac{m_2\times\omega^2}{-(m_1+m_2)\times\omega^2+k_1}$$

and $H_{21}(\omega)$ is determined by means of Eq.(2.3). On inserting Eqs.(2.3) and (2.15) into (2.14) and using notation (2.4), a criterion for convergent procedure assumes the form

$$(2.16)\qquad \left|\frac{S_2^2\times S_3^2\times\omega^4}{[-(S_2+S_2S_3)\omega^2+1]\times[-S_2S_3\omega^2+S_1]}\right| < 1\,.$$

Analysis of this condition leads to similar conclusions as those drawn in the case of weak associations, see Fig.6. It is interesting to note that the ends of the interval $(\Omega_{11}, \Omega_{12})$ outside which the procedure is unconditionally convergent (i.e. stays so far arbitrary combinations of the model parameters), are exactly the same as before, cf. relations (2.6).

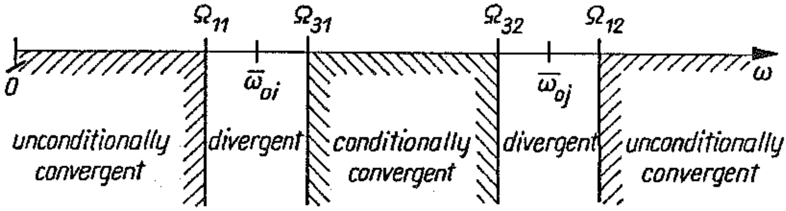

FIG. 6.

Although the first partial model has changed and its frequency of free vibrations now is

$$\bar{\omega}_{01}^2 = \frac{k_1}{m_1 + m_2} = \frac{1}{S_2(1 + S_3)}, \tag{2.17}$$

the following inequalities are still satisfied:

$$\begin{aligned} \Omega_{11} &< \min(\bar{\omega}_{01}, \bar{\omega}_{02}), \\ \Omega_{12} &< \max(\bar{\omega}_{01}, \bar{\omega}_{02}), \end{aligned} \tag{2.18}$$

where $\bar{\omega}_{02} = \omega_{02}$ (relation (2.7)). This means that the free vibration frequencies for partial models are always contained within the interval $(\Omega_{11}, \Omega_{12})$.

An interval within which the procedure is conditionally convergent is now different. Its ends are given by the formulae

$$\begin{aligned} \Omega_{31}^2 &= \frac{S_1 + S_3 + S_1S_3 - \sqrt{(S_1 + S_3 + S_1S_3)^2 - 4S_1S_3(2S_1 + 1)}}{2S_2S_3 \times (1 + 2S_3)}, \\ \Omega_{32}^2 &= \frac{S_1 + S_3 + S_1S_3 + \sqrt{(S_1 + S_3 + S_1S_3)^2 - 4S_1S_3(2S_1 + 1)}}{2S_2S_3 \times (1 + 2S_3)}. \end{aligned} \tag{2.19}$$

The above interval is, as before, contained within an interval whose ends are determined by the frequencies of free vibrations corresponding to partial models, i.e. the following inequalities hold good:

$$\min(\bar{\omega}_{01}, \bar{\omega}_{02}) < \Omega_{31} < \Omega_{32} < \max(\bar{\omega}_{01}, \bar{\omega}_{02}). \tag{2.20}$$

Convergence condition inside the interval $(\Omega_{31}, \Omega_{32})$ has the form

$$S_1 < S_{13} \quad \text{or} \quad S_1 > S_{14}, \tag{2.21}$$

where

$$\begin{aligned} S_{13} &= \frac{S_3 \times (3S_3+1) - 2S_3\sqrt{S_3 \times (2S_3+1)}}{(1+S_3)^2}, \\ S_{14} &= \frac{S_3 \times (3S_3+1) + 2S_3\sqrt{S_3 \times (2S_3+1)}}{(1+S_3)^2} \end{aligned} \tag{2.22}$$

and is shown diagrammatically in Fig.7. Shaded area corresponds to those parameters for which the procedure ceases turns out to be divergent. For remaining values of S_1 and S_3 the condition (2.21) is always satisfied and the procedure remains convergent.

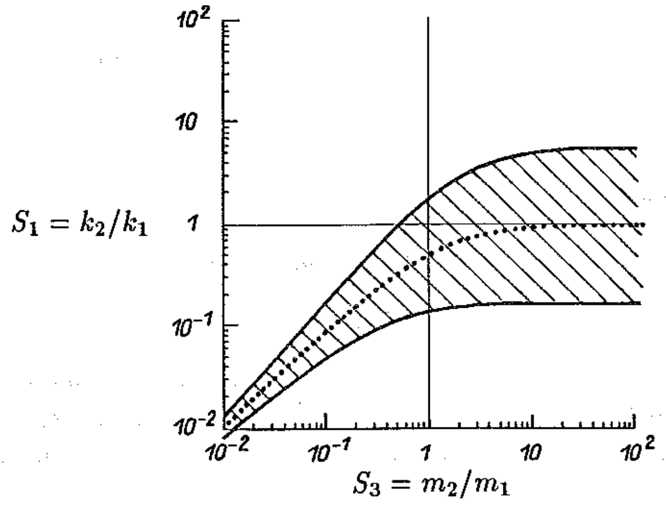

FIG. 7.

The skeleton line of the divergence region is associated with the condition $\bar{\omega}_{01} = \bar{\omega}_{02}$ and is shown as a dashed one.

In the intervals $\langle \Omega_{11}, \Omega_{31} \rangle$ and $\langle \Omega_{32}, \Omega_{12} \rangle$ the procedure is always divergent. These intervals are very narrow and become even narrower with the decreasing value of S_3. It can be readily proved that

$$\begin{aligned} \lim_{S_3 \to 0} (\Omega_{31} - \Omega_{11}) &= 0, \\ \lim_{S_3 \to 0} (\Omega_{12} - \Omega_{32}) &= 0. \end{aligned} \tag{2.23}$$

The limiting case $S_3 \to 0$ should be interpreted as corresponding to $m_1 \to \infty$ and not to $m_2 \to 0$.

2.3. Comparison of convergence for the two iteration procedures (criteria for their selection)

Analysis of the convergence conditions presented in two previous subsections has shown that both types of procedures (procedure I for weak association of masses and procedure II for strong ones) are convergent in the same regions outside the frequencies Ω_{11} and Ω_{12} (Figs.2 and 6), are divergent in the intervals including the free vibration frequencies for partial models and are conditionally convergent inside the interval determined by the free vibration frequencies. However, in this latter case the convergence conditions and the ends of convergence ranges are different for each of the procedures. Therefore, the decisive factor in choosing the right procedure for the given complete model will be its behaviour inside the interval $(\omega_{0i}, \omega_{0j})$ or $(\bar{\omega}_{0i}, \bar{\omega}_{0j})$ i.e. in between the frequencies of free vibrations for partial models. The relations (2.11) and (2.22) are shown diagrammatically in Fig.8 from which the following conclusions can be easily drawn:

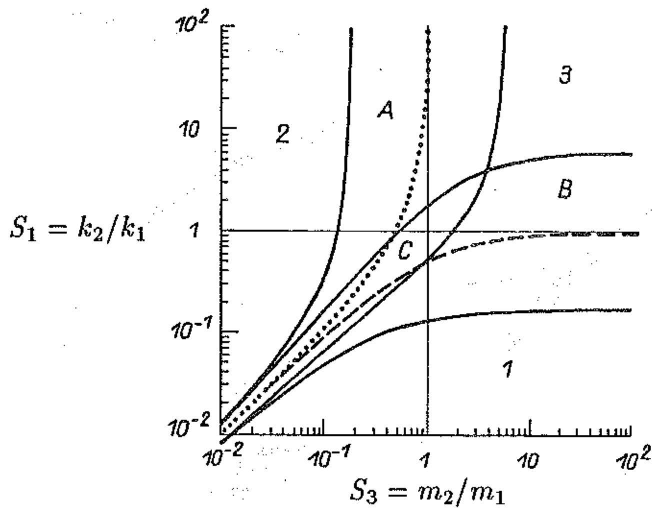

FIG. 8.

in the region A the procedure I is divergent so the other one should be used, in the region B the procedure II is divergent so the first one should be used,

in the region C both procedures appear to be divergent, at least for the conservative systems considered.

Three other regions – 1, 2, 3 – are also seen in Fig.8 for which both procedures stay convergent. For the corresponding mass ratios S_3 and stiffness ratios S_1, any of the two procedures can be employed. However, more advantageous will be this procedure for which the divergence regions around the free vibration frequencies for partial models are smaller. This condition, due to identity of the outer constraints (Ω_{11} and Ω_{12}) for both types of procedures, reduces to the comparison of the differences ($\Omega_{22} - \Omega_{21}$) and ($\Omega_{32} - \Omega_{31}$). Let us denote

$$\begin{aligned} \delta_1 &= \Omega_{22} - \Omega_{21}, \\ \delta_2 &= \Omega_{32} - \Omega_{31}. \end{aligned} \tag{2.24}$$

Inequality $\delta_1 > \delta_2$ indicates that the prucedure I is more effective whereas for $\delta_1 < \delta_2$ procedure II should be used.

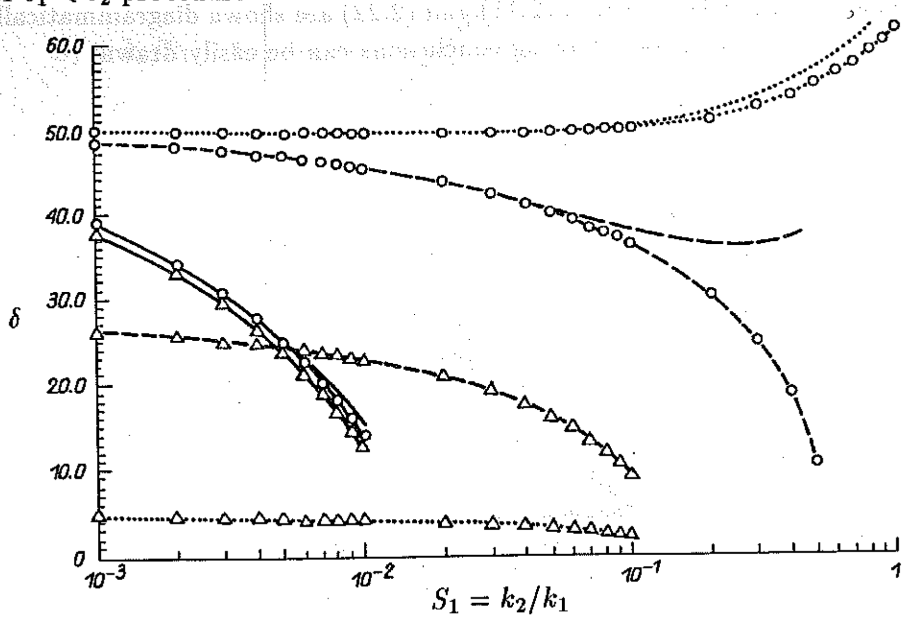

FIG. 9. $S_2 = m_1/k_1 = 0.0004(100/250000)$; —— (solid line) $S_3 = 0.02$; - - - - (dashed line) $S_3 = 1.1$; ······ (dotted line) $S_3 = 50$; lines with no symbols – ($\Omega_{12} - \Omega_{11}$); lines with circles – δ_1 (procedure I); lines with triangles – δ_2 (procedure II).

The values of δ_1 and δ_2 versus S_1 for the pairs S_1, S_3 corresponding to the region 1 (Fig.8) are shown in Fig.9. To emphasize the spreads of divergence intervals, the differences $\Omega_{12} - \Omega_{11}$ are also indicated.

Inspection of Fig.9 and similar relationships for the remaining regions 2 and 3 (Fig.8) leads to the following conclusions:

in the region 1 the procedure I should be used (with increasing S_3 this procedure becomes more and more advantageous),

in the region 2 the procedure II is recommended,

in the region 3 the procedure I should be used (the same remark applies as that for the region 1).

3. Convergence of iteration procedures for dissipative systems

In order to study the effects of damping on the convergence of iteration procedures, let us analyse the same situations as in section 2 under the assumption that the elements supporting each of the masses are now characterized by the Kelvin-Voigt model, i.e. linear elasticity and linear damping in parallel. The condition for convergence of the procedure I (for a two-mass model) has now the form

$$\left| \frac{k_2 + j\omega c_2}{k_1 + k_2 - m_1\omega^2 + j\omega(c_1 + c_2)} \times \frac{k_2 + j\omega c_2}{k_2 - m_2\omega^2 + j\omega c_2} \right| < 1 . \tag{3.1}$$

To simplify the analysis, the damping of supporting elements will be described with the use of dimensionless coefficients $WSP1$ and $WSP2$ that denote ratios of a given damping to the critical damping in a one-degree-of-freedom system:

$$\begin{aligned} WSP1 &= \frac{c_1}{\sqrt{2k_1m_1}} , \\ WSP2 &= \frac{c_2}{\sqrt{2k_2m_2}} . \end{aligned} \tag{3.2}$$

On inserting (2.4) and (3.2) into (3.1) and rearranging, the convergence condition for the procedure I assumes the form

$$\begin{aligned} &\frac{S_1^4 + 4\omega^2 S_1^3 S_2 S_3 (WSP2)^2 + 4\omega^4 (S_1S_2S_3)^2 \times (WSP2)^4}{(1 + S_1 - S_2\omega^2)^2 + 2\omega^2 S_2 (WSP1 + WSP2 \times \sqrt{S_1S_3}\,)^2} \\ &\qquad \times \frac{1}{(S_1 - S_2S_3\omega^2)^2 + 2\omega^2 S_1S_2S_3(WSP2)^2} - 1 < 0 . \end{aligned} \tag{3.3}$$

The present conclusions are qualitatively similar to those drawn for conservative systems and shown in Fig.2. The differences consist in what follows:

with increasing damping the divergence regions around the free vibration frequencies for partial models shrink very rapidly; for most systems they vanish completely at relatively small damping – the procedure becomes convergent in the whole range of frequencies (provided it is convergent inside the interval $(\Omega_{21}, \Omega_{22})$);

the region of divergence, shown in Fig.3 inside the interval $(\Omega_{21}, \Omega_{22})$, diminishes considerably.

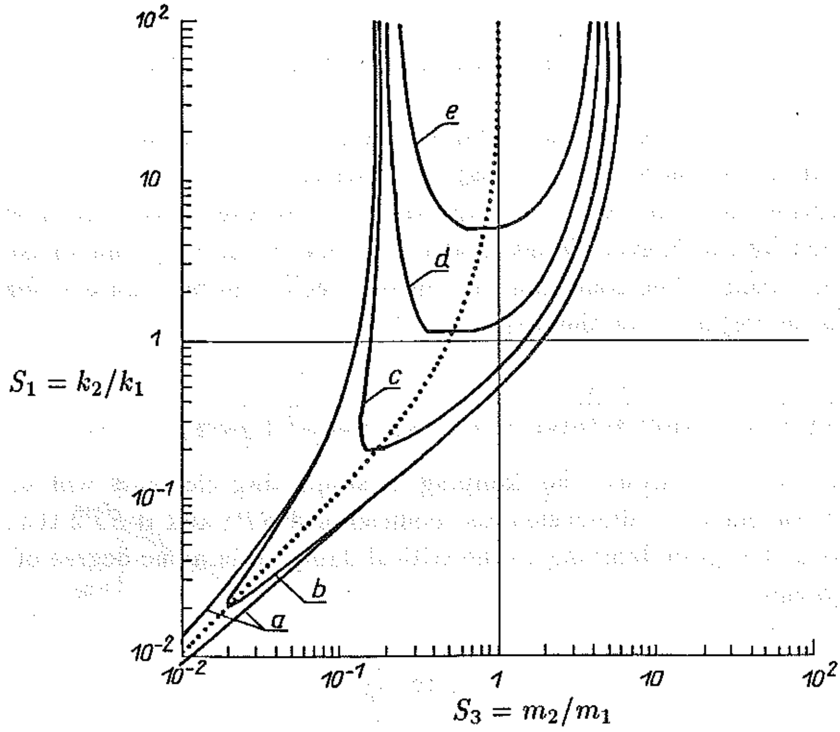

FIG. 10. Procedure I. Divergence regions for: $S_2 = m_1/k_1 = 0.0004(100/250000)$; a – system without damping, b – $WSP1 = WSP2 = 0.1$, c – $WSP1 = WSP2 = 0.3$, d – $WSP1 = WSP2 = 0.6$, e – $WSP1 = WSP2 = 1.0$.

Effects of damping on the decreasing regions of divergence inside the interval $(\Omega_{21}, \Omega_{22})$ are shown in Fig.10. However, for conservative systems the convergence condition (2.10) and (2.11) depended solely on the values of coefficients S_1 and S_3. For dissipative systems those conditions depend also on the frequencies ω and the coefficient S_2. The diagrams in Fig.10

are prepared for an average value ω_a taken from ω_{01} and ω_{02}. According to (2.7), ω_a can be calculated from the formula

$$\omega_a = \frac{\omega_{01} + \omega_{02}}{2} = \frac{\sqrt{S_3(1+S_1)} + \sqrt{S_1}}{2\sqrt{S_2 S_3}} \,. \tag{3.4}$$

In the calculations that led to the curves given in Fig.10 the damping in the supports of both masses was increased in proportion. Another question was the sensitivity of the system to the global amount of damping and to its location. To investigate this problem damping at one support was kept very small whereas at the other it was varied. The results are shown in Figs.11 and 12 (calculations were made for ω_a determined from (3.4)).

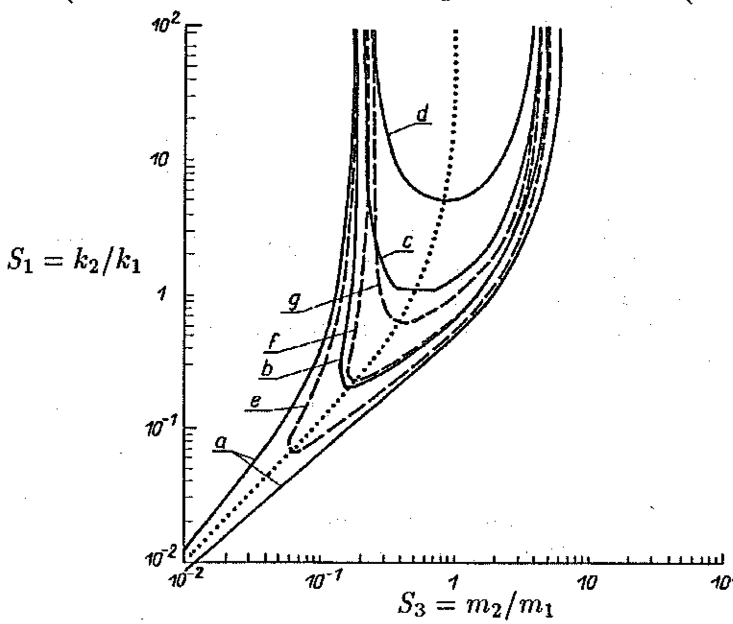

FIG. 11. Procedure I. Divergence regions for: $S_2 = m_1/k_1 = 0.0004(100/250000)$; a – system without damping, b – $WSP1 = WSP2 = 0.3$, c – $WSP1 = WSP2 = 0.6$, d – $WSP1 = WSP2 = 1.0$ (solid lines); e – $WSP1 = 0.1$, $WSP2 = 0.3$, f – $WSP1 = 0.1$, $WSP2 = 1.0$, g – $WSP1 = 0.1$, $WSP2 = 2.5$ (dashed lines).

Application of larger damping at the support of mass m_1 (not between the masses) is more advantageous. However, the difference is so slight that the location of larger damping can be said to have negligible effect on the procedure I. The best situation arises when damping is roughly uniform in the system. For example, to cause a similar effect as for $WSP1 = WSP2 = 0.3$, at one damping equal to 0.1 the other should amount to its critical value $WSP \simeq 1$.

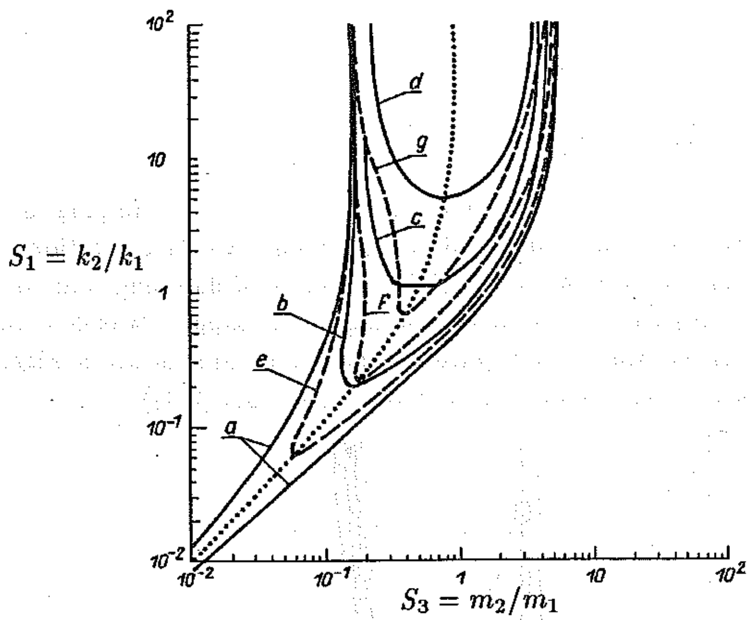

FIG. 12. Procedure I. Divergence regions for: $S_2 = m_1/k_1 = 0.0004(100/250000)$; a – system without damping, b – $WSP1 = WSP2 = 0.3$, c – $WSP1 = WSP2 = 0.6$, d – $WSP1 = WSP2 = 1.0$ (solid lines); e – $WSP1 = 0.3$, $WSP2 = 0.1$, f – $WSP1 = 1.0$, $WSP2 = 0.1$, g – $WSP1 = 2.5$, $WSP2 = 0.1$ (dashed lines).

The diagrams shown in Figs.10–12 are slightly different for other frequencies and coefficients S_2, but their character remains the same. The conclusions are also valid.

When a linear damping is assumed to exist in the system, the convergence condition for the procedure II takes the form

$$(3.5) \qquad \left| \frac{m_2^2 \times \omega^4}{[k_1 - (m_1 + m_2)\omega^2 + j\omega c_1] \times [k_2 - m_2\omega^2 + j\omega c_2]} \right| < 1 .$$

On putting (2.4) and (3.2) into the above inequality and rearranging, the final form of the condition (3.5) is

$$(3.6) \qquad \frac{(\omega^2 S_2 S_3)^2}{[(1 - S_2(1 + S_3)\omega^2]^2 + 2\omega^2 S_2 (WSP1)^2} \times \frac{1}{(S_1 - S_2 S_3 \omega^2)^2 + 2\omega^2 S_1 S_2 S_3 (WSP2)^2} - 1 < 0 .$$

Its analysis leads to similar conclusions as in the case of the procedure I, the only difference being that the region of divergence within the frequency

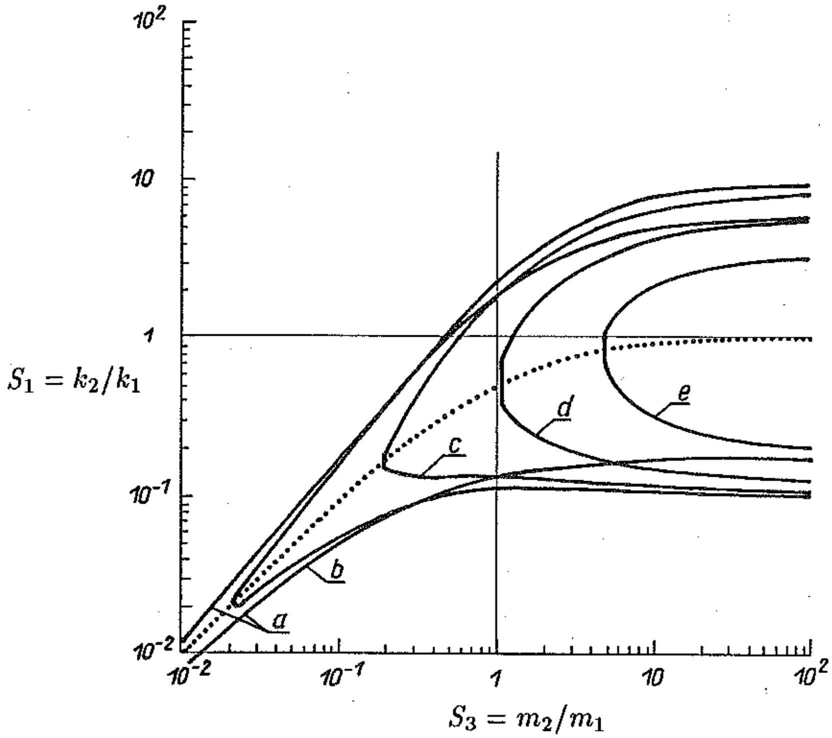

FIG. 13. Procedure II. Divergence regions for: $S_2 = m_1/k_1 = 0.0004(100/250000)$; a – system without damping, b – $WSP1 = WSP2 = 0.1$, c – $WSP1 = WSP2 = 0.3$, d – $WSP1 = WSP2 = 0.6$, e – $WSP1 = WSP2 = 1.0$.

interval $(\Omega_{31}, \Omega_{32})$ gets narrower in the "opposite direction". This situation is shown in Figs.13, 14 and 15. The curves were obtained for the frequency $\bar{\omega}_a$. Suitable formula for partial model in the procedure II follows from (2.17) and takes the form

$$\bar{\omega}_a = \frac{\bar{\omega}_{01} + \bar{\omega}_{02}}{2} = \frac{\sqrt{S_3} + \sqrt{S_1(1+S_3)}}{2\sqrt{S_2 S_3(1+S_3)}} \, . \tag{3.7}$$

Similarly as in the case of procedure I, increasing damping leads to shrinking regions of divergence. Proportional increase of damping in the supports of both masses is seen in Fig.13, whereas an effect of "point of application" of damping is visualized in Figs.14 and 15. General conclusion is that also in this case the convergence is better for proportional variations in damping. For nonproportional case tho region of divergence is found to become narrower for damping that is applied between the masses instead being concentrated at the support of the first mass.

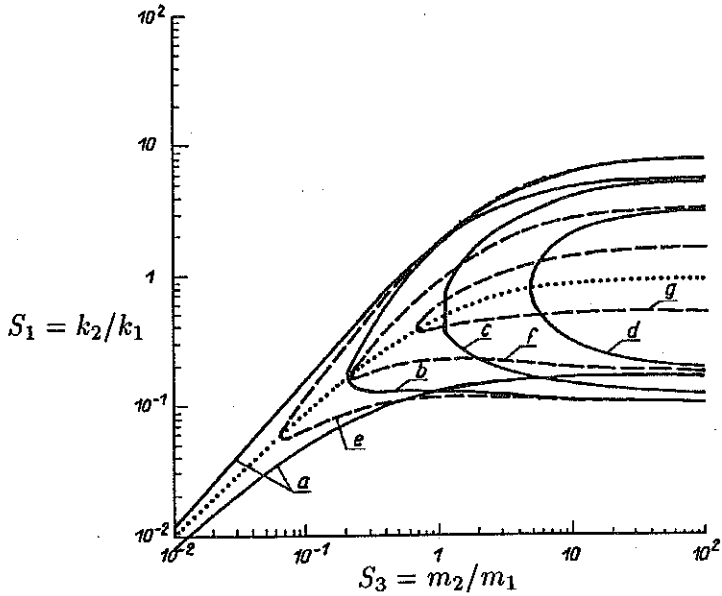

FIG. 14. Procedure II. Divergence regions for: $S_2 = m_1/k_1 = 0.0004(100/250000)$; a – system without damping, b – $WSP1 = WSP2 = 0.3$, c – $WSP1 = WSP2 = 0.6$, d – $WSP1 = WSP2 = 1.0$ (solid lines); e – $WSP1 = 0.1$, $WSP2 = 0.3$, f – $WSP1 = 0.1$, $WSP2 = 1.0$, g – $WSP1 = 0.1$, $WSP2 = 2.5$ (dashed lines).

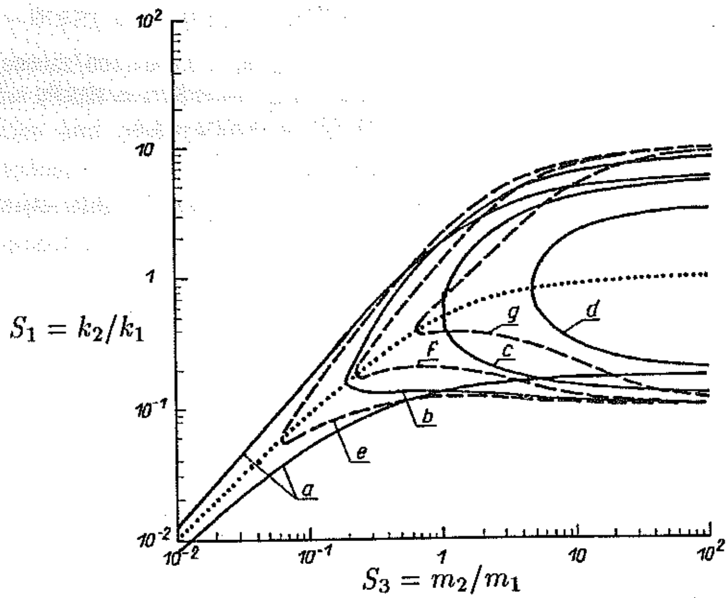

FIG. 15. Procedure II. Divergence regions for: $S_2 = m_1/k_1 = 0.0004(100/250000)$; a – system without damping, b – $WSP1 = WSP2 = 0.3$, c – $WSP1 = WSP2 = 0.6$, d – $WSP1 = WSP2 = 1.0$ (solid lines); e – $WSP1 = 0.3$, $WSP2 = 0.1$, f – $WSP1 = 1.0$, $WSP2 = 0.1$, g – $WSP1 = 2.5$, $WSP2 = 0.1$ (dashed lines).

4. Final remarks

The presented analysis of convergence of iterative procedures provides some guidelines for suitable decomposition of a complete model into partial ones and for creating effective iteration procedures. The conclusions drawn from the examples with two masses and their varying ratios S_1, S_3 remain valid for more complicated systems.

Damping is found to enhance convergence of the iterative procedures and to diminish the regions of divergence. The results can be considered satisfactory although even for relatively large damping neither procedures is convergent in the whole interval of variations in the coefficients S_1 and S_3. The divergence regions are also found to shrink rapidly with increasing damping. The regions in which both procedures yield divergent results are shown in Fig.16. For relatively small damping this region is very small; for $WSP1 = WSP2 > 0.55$ the divergence regions for both procedures have no common points. This means that for such damping the proposed analysis can be used in the systems with arbitrary ratios of the parameters S_1, S_3.

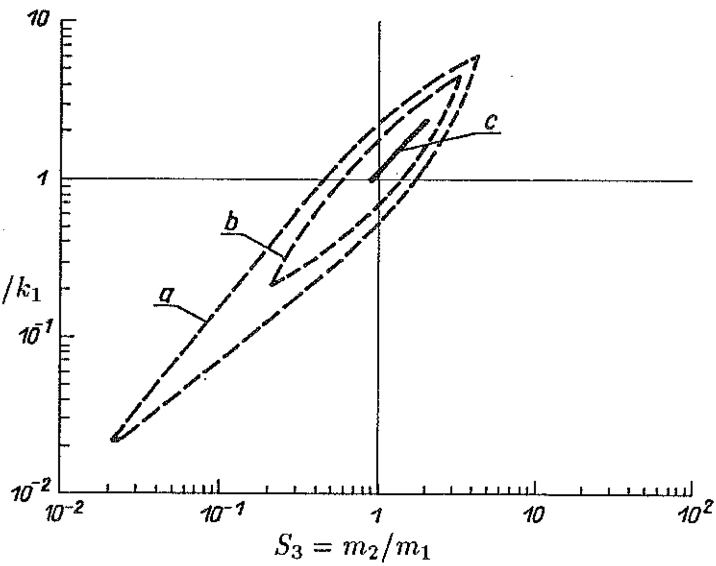

FIG. 16. Divergence regions for: $S_2 = m_1/k_1 = 0.0004(100/250000)$;
$a - WSP1 = WSP2 = 0.1$, $b - WSP1 = WSP2 = 0.3$, $c - WSP1 = WSP2 = 0.5$.

Figure 16 refers to the systems in which $S_1 \simeq S_3$, i.e. relevant regions are situated along the positive bisectrix of the coordinate system. Frequencies of free vibrations are almost equal and, in the presence of weak damping, beat may occur. No machinery with such properties should ever be constructed. Thus the proposed method of analysis of dynamic systems can be used for all machines which are designed correctly from the viewpoint of dynamics.

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