TENSILE STRENGTH OF BRITTLE MATERIALS IN HIGH STRAIN RATE PROCESSES

K. M. M I A N O W S K I (WARSZAWA)

Assuming that high strain rate tensile tests lead to brittle fracture (no slip mechanisms participate in the process), the phenomenon of strength growth in very fast processes is attributed to thermal vibrations of crystal lattice. Formulae are given enabling effective evaluation of strength as a function of duration of the loading process. Results of analysis are compared with experimental data published by Zieliński and concerning the tensile strength of concrete in high rate processes.

1. Introduction

This paper deals with the problem of dynamic strength of materials which are brittle "by birth", i.e. such in which the fracture processes under all possible time and temperature conditions do not exhibit any plastic phase; e.g. concrete and natural stone behave in this manner under tensile forces.

Experiments performed under very high strain rates (10—30 s^{-1}) prove that tensile strength of concrete is considerably higher than the static strength (by 100—200%)[1]. However, the known relations between the strength and process rate are mostly of empirical character, and the attempts have been made to formulate the problem on a theoretical basis.

Generalisations concerning the macro-level and given by BIRKIMER and LINDEMAN [2] are based on the assumption that the fracture energy has a constant value and is independent of the rate of the process. This assumptions, in view of the results obtained by this author [3] seems to be ungrounded.

MIHASI and IZUMI [4] used a model of the crack which lead to a statistical character of the relations describing the development of a critical

---

[1] Review of papers dealing with this problem may be found in the paper by A. J. ZIE-LIŃSKI [1].

crack. However, the physical sense of relationships between the rate of loading and critical crack growth remained rather vague.

According to author's opinion, the problem of dynamic strength of brittle materials should be analysed in two separate process rate ranges: moderately fast and very fast. These ranges are shown in Fig. 1 for the case of concrete.
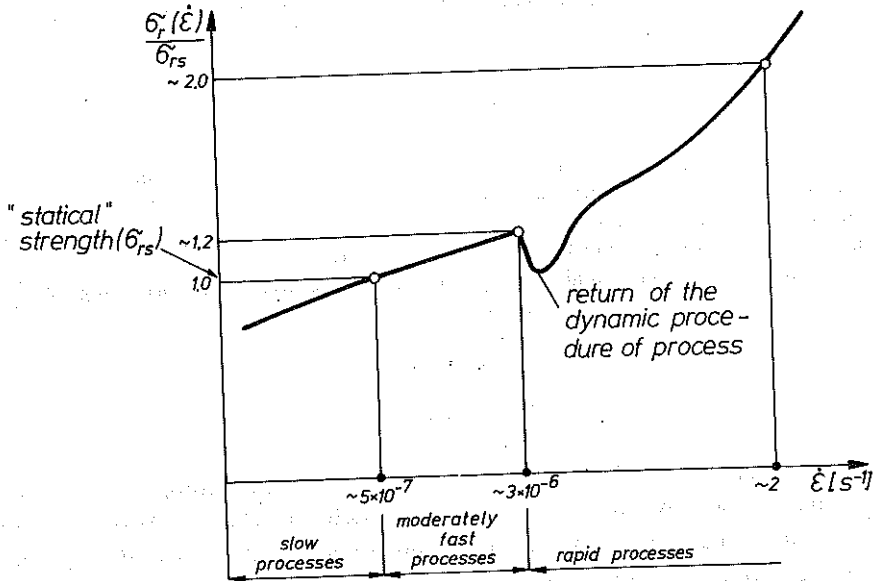

FIG. 1. Classification of fracture processes; example: tension test of concrete.

For the moderately high rate proess, analysis of strength should take into account the interaction occurring at the submacroscopic level, connected with the process dynamics and with the effects of its neutralisation. Dynamic effects appearing at the sub-macroscopic level are results of the stepwise, discontinuous character of fracture taking place in brittle materials and consisting in succesive damage increments. Such effects reduce the material strength.

The phenomena discussed above may be described in terms of the notions used in classical mechanics. In papers dealing with the problem of fracture dynamics [3, 6], use was made of the notions known from statistical strength theory, damage and stress waves theory. The author concluded that, under the conditions of brittle fracture, which do not allow for slip effects to occur, the increase in strength observed within the range moderately fast processes is a result of neutralisation of dynamic

effects. This neutralisation occurs by destructive interference of the stress waves produced by successive dynamic damage increments.

Passing to the problem of analysis of strength of brittle materials tested under very high strain and stress rate conditions, let us express the opinion that proper interpretation of such phenomena necessitates the application of a number of notions used in the physics of solids.

In this paper such a model of brittle material is proposed which takes into account the phenomenon of thermal oscillations of crystal lattice. This phenomenon was analysed by several authors in connection with the physical theory of metal flow based upon the thermally activated dislocational processes (to mention only a few papers [6, 7, 8]).

The amount of energy necessary to surmount the obstacles during motion of the dislocations over the slip planes is supplied by thermal fluctuations (thermal process) and by stresses acting in these planes (athermal process).

Elevation of the yield limit of ductile materials observed in high rate processes is explained by the fact that the shorter is the process duration, the smaller becomes the amount of thermal vibration energy engaged into dislocation displacement; hence, in order to produce these displacements, greater stress τ in the slip planes should be applied.

Taking into account the fact that dislocation processes occurring in brittle materials are too weak to produce slip effects in the entire test-piece, the assumption is made that in the model of brittle material such processes do not develop; thus the influence of thermal fluctuations of crystal lattice on strength is reduced to instantaneous drops of thermonuclear force potentials, that is to instantaneous macroscopic strength reductions.

In view of the probabilistic character of the relations governing the thermal fluctuation energy, tensile strength depends on τ_R — duration of the failure loading process: the faster the process is runing and the shorter is τ_R, the smaller will be the probability of occurring (within the period $\langle 0, \tau_R \rangle$) of large amplitude thermal fluctuations capable of considerably reducing the intermolecular (bonds activation energy) and the strength.

Hence, according to author's hypothesis, physical sense of brittle material strength increase observed in rapid processes consists in partial elimination of the internal thermal loading.

2. MODEL OF BRITTLE MATERIAL AT MICROSCOPIC LEVEL

Materials which are "brittle by birth", such as concrete and natural stone, exhibit considerable structural nonhomogeneity, what has twofold consequences: first, that the shear stress capable of producing slip are much

greater than the tensile strength. Owing to the fact that the absolute value of maximum shearing stresses (acting in planes inclined by 45° to the axis of specimen) equals the absolute value of principal tensile stresses, cleavage fracture (debonding) must occur.

Another consequence of material nonhomogeneity is a nonuniform development of the damage process which is concentrated in the critical (the weakest) material layer. When a crack develops in this layer, other portions of the test-piece are far from failure.

The above facts are taken into account in the model shown in Fig. 2. It consists of two grains particles and a contact zone lying between them, and transmitting the intermolecular forces (bonds).
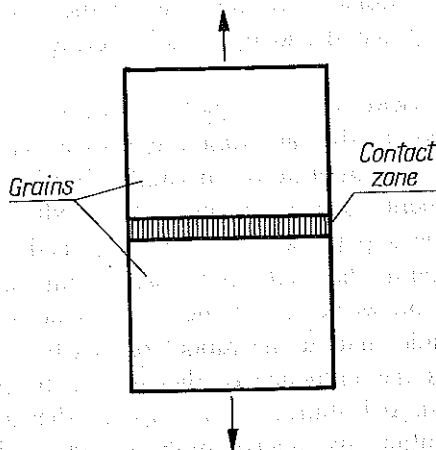

FIG. 2. Model of brittle material.

Failure of the model occurs by rupture of bonds in the contact zone. In view of the fact that strength of the grains is much higher than that of the contact zone, no slips or damage occur in the grains at the instant of model failure.

Strength of the bonds transmitted through the contact zone, R, is assumed to be non-uniform, and the failure process consists of successive rupture of individual bonds. According to the assumptions made in Sect. 1, strength of each bond depends on the rate of the process, what may be written in the form

(2.1) $$R = R(\dot{s}),$$

where R — strength of the bond, $\dot{s}$ — actual stress rate.

At a given rate of the process, bond strength distribution within the contact zone is given by a certain function $\varphi(R)$, determined in the interval $\langle R_{min}, R_{max} \rangle$.

Bond strength may also be characterised by the activation energy E_a, i.e. the amount of work to be done in order to disrupt the given bond under the given temperature conditions. Activation energy is given by the relation

(2.2) $$E_a = E_p - E.$$

Here: E_p — potential barrier of the bond, measured by the amount of work to be done in order to disrupt the given bond at absolute zero temperature, and E — time- and temperature-dependent thermal fluctuation energy.

Frequency of occurrence of bonds characterised by the given potential barrier E_{pi} is determined by means of a stationary probability density function $\varrho(E_p)$.

3. Process speed and strength of individual bonds

Tensile strength of a single bond is attained in time τ_r when V_p — potential of external forces P reaches the value of activation energy, that is when

(3.1) $$V_p = E_a(\tau_r).$$

It should be observed that τ_r is understood as the net time period during which the bond is exposed to the action of external forces of value R, under which disruption of the bond is probable (Fig. 3).

Value of the fluctuating activation energy is given by the formula

(3.2) $$E_a(\tau_r) = E_p - E_r(\tau_r).$$

Here: E_p — potential barrier (constant for the given bond), $E_r(\tau_r)$ — maximum energy of thermal vibrations whose attainment during period τ_r is probable.

Energy of thermal fluctuations is determined by Boltzmann function [9]

(3.3) $$p(E)|_{E_r} = Ae^{-\frac{E_r}{kT}}$$

with the notations: $p(E)|_{E_r}$ — occurrence probability of the given energy level E_r, A — characteristic constant of the material, k — Boltzmann constant, T — absolute temperature.

In formulating the condition (3.1) we simultaneously assume that the solo thermal vibrations (i.e. without external forces) are not capable of disrupting the bond in the period $\langle 0, \tau_r \rangle$. This postulate enables also the determination of τ_m, the longest possible time duration of the process necessary to disrupt the bond under the sole action of thermal vibrations.

The highest probable energy level E_r attained during the period τ_r, the assumed duration of the process, is reached only once (bond rupture).
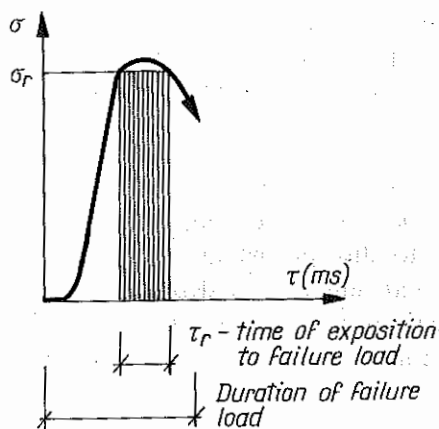
FIG. 3. Duration of fracture process in probabilistic sense.

Thus the relation between τ_r and the thermal vibration energy E_r assumes the form

(3.4) $$\tau_r \, vp \, (E)|_{E_r} = 1,$$

where v — thermal vibration frequency.

Assume now $\tau_r = \tau_m$; this time period enables realisation, within a single bond, of all possible thermal vibration levels, including the level E_m when the particle strikes the potential barrier. In the last case relation between the process duration and energy has the form

(3.5) $$\tau_r \, vp \, (E)|_{E_m} = 1.$$

Equating l.h.s. of Eqs. (3.4) and (3.5) we obtain

(3.6) $$\frac{p \, (E)|_{E_r}}{p \, (E)|_{E_m}} = \frac{\tau_m}{\tau_r}.$$

Transformation of l.h.s. of Eq. (3.6) yields

(3.7) $$\frac{\tau_r}{\tau_m} = e^{\frac{-E_m + E_r}{kT}},$$

whence

(3.8) $$E_r = E_m + kT \ln \frac{\tau_r}{\tau_m}.$$

Substitution of relation (3.8) into (3.2) yields the relation which makes it possible to calculate the minimum activation energy level which will possibly occur during the assumed time period

$$(3.9) \qquad E_a(\tau_r) = E_m - E_r(\tau_r) = -kT \ln \frac{\tau_r}{\tau_m}.$$

Strength of the bond expressed in terms of the loading period

$$(3.10) \qquad R = B\sqrt{-kT \ln \frac{\tau_r}{\tau_m}},$$

where B — a constant.

4. STRESS RATE AND STRENGTH OF A GROUP OF BONDS OF THE SAME POTENTIAL BARRIER

In the analysis of the problem stated above it was taken into account that once the exposition time τ_r expires, individual particles will exist at different thermal vibration energy levels, less or equal E_r.

Load carrying capacity of a group of bonds is exhausted when V_p, potential of external forces, attains the value $\bar{E}_a$ of the mean activation energy, that is when

$$(4.1) \qquad V_p = \bar{E}_a.$$

Value of the mean activation energy is found from the relation

$$(4.2) \qquad \bar{E}_a = E_m - \bar{E}.$$

Here $\bar{E}$ — mean value of thermal vibration energy in the interval $\langle 0, E_r \rangle$, E_m — potential barrier characteristic for the given group of bonds.

Mean value of thermal vibration energy in the interval $\langle 0, E_r \rangle$ is determined by the formula

$$(4.3) \qquad \bar{E} = \frac{\int\limits_0^{E_r} p(E)\,E\,dE}{\int\limits_0^{E_r} p(E)\,dE} = \frac{\int\limits_0^{E_r} e^{-\frac{E}{kT}}\,E\,dE}{\int\limits_0^{E_r} e^{-\frac{E}{kT}}\,dE},$$

that is

$$(4.4) \qquad \bar{E} = kT\,\frac{e^{-\frac{E_r}{kT}}\left(\dfrac{E_r}{kT} + 1\right) - 1}{e^{-\frac{E_r}{kT}} - 1},$$

Mean value of the activation energy for the group of bonds characterised by the same potential barrier equals

$$(4.5) \qquad \bar{E}_a = E_m - kT \frac{e^{-\frac{E_r}{kT}}\left(\frac{E_r}{kT}+1\right)-1}{e^{-\frac{E_r}{kT}}-1}.$$

The measure of potential barrier of a particle is assumed to be the energy kT attributed to thermal vibrations performed in each of the three directions; this energy is a characterised magnitude for the process analysed. Hence, the bond energy barrier may be determined by a dimensionless parameter

$$(4.6) \qquad \alpha = \frac{E_m}{kT}.$$

Assume $E_m = \alpha kT$ and express E_r by means of relation (3.8). Then, after certain transformations, relation (4.5) takes the form

$$(4.7) \qquad \frac{\bar{E}_a}{kT} = \alpha - \frac{e^{-\alpha-\ln\frac{\tau_r}{\tau_m}}\left(\alpha+\ln\frac{\tau_r}{\tau_m}+1\right)-1}{e^{-\alpha-\ln\frac{\tau_r}{\tau_m}}-1}.$$

Mean strength R_{ir} of bonds belonging to the given group i is calculated as before, namely

$$(4.8) \qquad R_{ir} = B\sqrt{\bar{E}_a}.$$

Here $\bar{E}_a$ is the mean value of activation energy to be derived from Eq. (4.5).

Strength R_{ir} may also be expressed by relation

$$(4.9) \qquad R_i = \bar{R}_{im}\,\psi\left(\alpha,\frac{\tau_r}{\tau_m}\right),$$

with the following notations: $\bar{R}_{im}$ — mean value of strength of group i in the case when the exposition time $\tau_r = \tau_m$, $\psi\left(\alpha,\frac{\tau_r}{\tau_m}\right)$ coefficient given by the formula

$$(4.10) \qquad \psi\left(\alpha,\frac{\tau_r}{\tau_m}\right) = \frac{\bar{R}_{ir}}{\bar{R}_{im}} = \sqrt{\frac{(e^{-\alpha}-1)\left[1-\alpha-e^{-\alpha-\ln\frac{\tau_r}{\tau_m}}\left(1+\ln\frac{\tau_r}{\tau_m}\right)\right]}{\left(e^{-\alpha-\ln\frac{\tau_r}{\tau_m}}-1\right)\left(1-\alpha-e^{-\alpha}\right)}}.$$

Values of function $\psi\left(\alpha,\frac{\tau_r}{\tau_m}\right)$ for several values of the coefficient α and various process rates are given in Fig. 4 and Tabl. 1. It follows that, with increasing rates of the process, strength of the group of bonds also
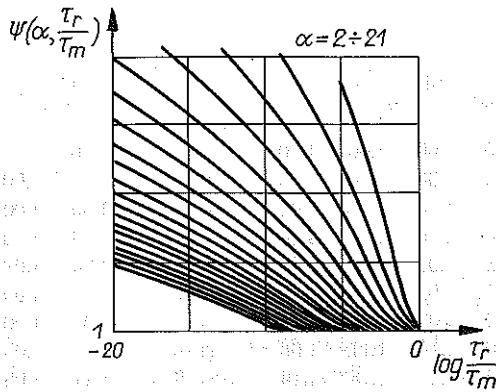
$$\psi\left(\alpha, \frac{\tau_r}{\tau_m}\right)$$

$\alpha = 2 \div 21$

$$\log \frac{\tau_r}{\tau_m}$$

Fig. 4. Values of function $\psi\left(\alpha, \dfrac{\tau_r}{\tau_m}\right)$.

Table 1.

$$E = a - \frac{EXP(-w) \times (w+1) - 1}{EXP(-w) - 1}$$

$$w = a + LN\, t$$

Values of
$$e = SQR[E(a, t)/(E(a, t=1)]$$

t	$a=2$	3	4	5	6	t	$a=7$	8	9	10	11
1	1	1	1	1	1	1	1	1	1	1	1
0.1	1.28	1.11	1.04	1.01	1.00	0.1	1.00	1.00	1.00	1.00	1.00
.01	1.70	1.36	1.18	1.09	1.04	.01	1.01	1.00	1.00	1.00	1.00
.001	2.12	1.66	1.40	1.24	1.14	.001	1.07	1.03	1.01	1.00	1.00
.0001	2.50	1.95	1.63	1.43	1.28	.0001	1.18	1.11	1.06	1.03	1.01
.00001	2.82	2.20	1.84	1.61	1.44	.00001	1.32	1.23	1.15	1.10	1.06
1E-6	3.12	2.43	2.04	1.78	1.59	1E-6	1.46	1.35	1.26	1.19	1.13
1E-7	3.39	2.64	2.21	1.93	1.73	1E-7	1.58	1.46	1.37	1.29	1.23
1E-8	3.64	2.84	2.38	2.07	1.86	1E-8	1.70	1.57	1.47	1.39	1.32
1E-9	3.87	3.02	2.53	2.21	1.98	1E-9	1.81	1.67	1.57	1.48	1.40
9.9999	4.09	3.19	2.67	2.33	2.09	9.9999	1.91	1.77	1.65	1.56	1.48
1E-11	4.30	3.35	2.81	2.45	2.20	1E-11	2.01	1.86	1.74	1.64	1.55
1E-12	4.50	3.51	2.94	2.56	2.30	1E-12	2.10	1.95	1.82	1.72	1.63
1E-13	4.69	3.66	3.06	2.67	2.40	1E-13	2.19	2.03	1.90	1.79	1.70
1E-14	4.87	3.80	3.18	2.78	2.49	1E-14	2.28	2.11	1.97	1.86	1.76
1E-15	5.05	3.94	3.30	2.88	2.58	1E-15	2.36	2.18	2.04	1.93	1.83
1E-16	5.22	4.07	3.41	2.98	2.67	1E-16	2.44	2.26	2.11	1.99	1.89
1E-17	5.38	4.20	3.52	3.07	2.75	1E-17	2.52	2.33	2.18	2.05	1.95
9.9999	5.55	4.33	3.62	3.16	2.83	9.9999	2.59	2.40	2.24	2.11	2.01
1E-19	5.70	4.45	3.72	3.25	2.91	1E-19	2.66	2.47	2.31	2.17	2.06
9.9999	5.85	4.56	3.82	3.34	2.99	9.9999	2.73	2.53	2.37	2.23	2.12

cd. tabl. 1

t	$a = 12$	13	14	15	16	t	$a = 17$	18	19	20	21
1	1	1	1	1	1	1	1	1	1	1	1
0.1	1.00	1.00	1.00	1.00	1.00	0.1	1.00	1.00	1	1	1
.01	1.00	1.00	1.00	1.00	1.00	.01	1.00	1.00	1.00	1.00	1
.001	1.00	1.00	1.00	1.00	1.00	.001	1.00	1.00	1.00	1.00	1.00
.0001	1.00	1.00	1.00	1.00	1.00	.0001	1.00	1.00	1.00	1.00	1.00
.00001	1.03	1.01	1.00	1.00	1.00	.00001	1.00	1.00	1.00	1.00	1.00
1E-6	1.09	1.05	1.03	1.01	1.00	1E-6	1.00	1.00	1.00	1.00	1.00
1E-7	1.17	1.12	1.08	1.05	1.03	1E-7	1.01	1.00	1.00	1.00	1.00
1E-8	1.25	1.20	1.15	1.11	1.08	1E-8	1.05	1.03	1.02	1.01	1.00
1E-9	1.33	1.28	1.23	1.18	1.14	1E-9	1.11	1.08	1.05	1.03	1.02
9.9999	1.41	1.35	1.30	1.25	1.21	9.9999	1.17	1.13	1.10	1.08	1.05
1E-11	1.48	1.42	1.36	1.31	1.27	1E-11	1.23	1.19	1.16	1.13	1.10
1E-12	1.55	1.48	1.43	1.37	1.33	1E-12	1.29	1.25	1.21	1.18	1.15
1E-13	1.62	1.55	1.49	1.43	1.38	1E-13	1.34	1.30	1.26	1.23	1.20
1E-14	1.68	1.61	1.55	1.49	1.44	1E-14	1.39	1.35	1.31	1.28	1.24
1E-15	1.74	1.67	1.60	1.54	1.49	1E-15	1.44	1.40	1.36	1.32	1.29
1E-16	1.80	1.72	1.66	1.60	1.54	1E-16	1.49	1.45	1.41	1.37	1.33
1E-17	1.86	1.78	1.71	1.65	1.59	1E-17	1.54	1.49	1.45	1.41	1.38
9.9999	1.91	1.83	1.76	1.69	1.64	9.9999	1.58	1.54	1.49	1.45	1.42
1E-19	1.97	1.88	1.81	1.74	1.68	1E-19	1.63	1.58	1.54	1.49	1.46
9.9999	2.02	1.93	1.86	1.79	1.73	9.9999	1.67	1.62	1.58	1.53	1.50

increases. Sensitivity of the strength to the process rate is the greater, the lower is the potential barrier (that is, the smaller is α). It is easily understood since reduction of the bond potential barrier (its lower strength) increases the contribution of thermal vibrations of crystal lattice to the work spent by external forces on the failure. Obviously, strength $\mathscr{R}$ of bonds observed in rapid processes cannot exceed the maximum strength resulting from the potential barrier

$$\text{(4.11)} \qquad \max \mathscr{R}_{max} = B \sqrt{E_p},$$

where E_p denotes the potential barrier for the group of bonds analysed. For processes running at moderate temperature (close to the room temperature) and for bonds characterised by a high potential barrier, the influence of thermal vibrations of crystal lattice on the strength is small and may be disregarded.

5. EFFECT PROCESS RATE ON THE STRENGTH OF A SYSTEM CONSISTING OF BONDS WITH DIFFERENT POTENTIAL BARRIERS

5.1. Distribution function of bond strength

The subject of the foregoing considerations was the strength of a single bond or of a group of bonds characterised by the same potential barrier.

However, according to the assumptions concerning the model of brittle material, bonds characterised by various potential barriers should also be taken into account. Distribution of potential belonging to the critical layer of the material is determined by means of the function $\varrho(E)$, and distribution of R-strength of the same bonds under static conditions, by means of function $\varphi(R)$, is determined in a certain interval $\langle R_{min}, R_{max}\rangle$.

During the very high rate processes, strengths of all bonds increase and reach the value of $\mathscr{R}$ instead of R. In particular the maximum and minimum values of bond strength increase and reach respectively values $\mathscr{R}_{max}$ and $\mathscr{R}_{min}$. New distribution of the bond strength is described by function $\Phi(\mathscr{R})$ defined in the interval $\langle \mathscr{R}_{min}, \mathscr{R}_{max}\rangle$. Function $\Phi(\mathscr{R})$ is obtained by transformation of the original function $\varphi(R)$.

5.2. Example of transformation of $\varphi(R)$

To illustrate the procedure of defining $\Phi(\mathscr{R})$, assume initial bond strength distribution to be given by a possibly simple relation
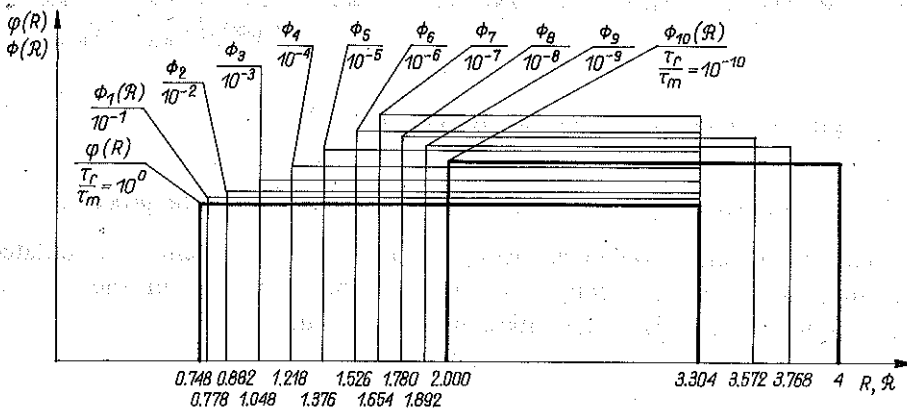
(5.1) $$\varphi(R) = \frac{1}{R_{max} - R_{min}}.$$

FIG. 5. Dependence of strength distribution function $\varphi(R) = \dfrac{1}{R_{max} - R_{min}}$ on the rate of process.

Diagram of this function has the form of rectangle Fig. 5. Values of the minimum and maximum values of strength $\mathscr{R}_{min}$ and $\mathscr{R}_{max}$ are determined from relation (4.9) having the form of two equations

(5.2) $$\frac{\mathscr{R}_{min}}{R_{min}} = \psi\left(\alpha_{min} \cdot \frac{\tau_r}{\tau_m}\right),$$

$$\frac{\mathscr{R}_{max}}{R_{max}} = \psi\left(\alpha_{max} \cdot \frac{\tau_r}{\tau_m}\right).$$

Assume now $\alpha_{min} = 4$, $\alpha_{max} = 16$ what means that max $\mathscr{R}_{max}$ (the greatest maximum strength) and max $\mathscr{R}_{min}$ the greatest minimum strength satisfy the relation

(5.3) $\max \mathscr{R}_{max} = 2\max \mathscr{R}_{min}.$

Results of calculations performed by means of Eqs. (5.2) for several high rate processes are given in Tabl. 2. It was assumed that the values of strength $\mathscr{R}_{min}$ and $\mathscr{R}_{max}$ obtained for a process of rate $\tau_r/\tau_m = 10^{-10}$ are close to the maximum values max $\mathscr{R}_{min}$ and max $\mathscr{R}_{max}$ (in the sense of relation (4.11)). Taking this into account, the fourth and fifth rows of Table 2 present the relationships between the quotients $\mathscr{R}_{min}/\max \mathscr{R}_{min}$, $\mathscr{R}_{max}/\max \mathscr{R}_{max}$ and the process rate. Values of the quotient $\mathscr{R}_{min}/\mathscr{R}_{max}$ are given in the fifth row of Tabl. 2; with increasing process speeds the value increases from 0.226 to 0.500.

Assume now (without proof) that diagrams of functions $\Phi(\mathscr{R})$ and $\varphi(R)$ are rectangular; the corresponding values are found from relations

(5.4) $\varphi(R)(R_{max} - R_{min}) = \Phi_1(\mathscr{R})({}_1\mathscr{R}_{max} - {}_1\mathscr{R}_{min}) = \Phi_2(\mathscr{R})({}_2\mathscr{R}_{max} - {}_2\mathscr{R}_{min}) =$
$$= ... = \Phi_i(\mathscr{R})({}_i\mathscr{R}_{max} - {}_i\mathscr{R}_{min}) = 1,$$

and graphs of functions $\Phi(\mathscr{R})$ are presented in Fig. 5.

5.3. Determination of strength of bond systems in high rate processes

Once the function $\Phi(\mathscr{R})$ is given, strength of the system is calculated according to the basic formulae of damage theory which, in the case of processes with neutralised dynamics, have the form

(5.5) $\sigma_{\mathscr{R}} = \mathscr{R}_r\left(1 - \int_{\mathscr{R}_{min}}^{\mathscr{R}_r} \Phi(\mathscr{R})\, d\mathscr{R}\right).$

Here $\sigma_{\mathscr{R}}$ — strength expressed in conventional stresses, $\mathscr{R}_r$ — strength of bonds (expressed in actual stresses) which, after fracture, makes the damage process develop spontaneously.

Value of stress $\mathscr{R}_r$ should be determined from the relation

(5.6) $\Phi(\mathscr{R})|_{\mathscr{R}_r} = \int_{\mathscr{R}_r}^{\mathscr{R}_{max}} \Phi(\mathscr{R})\, d\mathscr{R}.$

In view of similarly of all bond strength distributions (what takes place when the same material is analysed), all speed-independent processes indepen-

Table 2. Increase of strengths $\mathscr{R}_{min}$, $\mathscr{R}_{max}$ in high rate processes.

$\dfrac{\tau_r}{\tau_m}$	10^0	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}	10^{-7}	10^{-8}	10^{-9}	10^{-10}
$\dfrac{\mathscr{R}_{min}}{\mathscr{R}_{min}}$	1	1.04	1.18	1.40	1.63	1.84	2.04	2.21	2.38	2.53	2.67
$\dfrac{\mathscr{R}_{max}}{\mathscr{R}_{max}}$	1	1	1	1	1	1	1	1	1.08	1.14	1.21
$\dfrac{\mathscr{R}_{min}}{\max \mathscr{R}_{min}}$	0.374	0.389	0.441	0.524	0.609	0.688	0.763	0.827	0.890	0.946	1
$\mathscr{R}\,min$	0.748	0.478	0.882	1.048	1.218	1.376	1.526	1.654	1.780	1.892	2
$\dfrac{\mathscr{R}_{min}}{\mathscr{R}_{max}}$	0.826	0.826	0.826	0.826	0.826	0.826	0.826	0.826	0.893	0.942	1
$\mathscr{R}\,max$	3.304	3.304	3.304	3.304	3.304	3.304	3.304	3.304	3.572	3.768	4
$\dfrac{\mathbf{R}_{max}}{\max \mathscr{R}_{max}}$	0.226	0.235	0.267	0.317	0.369	0.416	0.462	0.500	0.500	0.500	0.500

dently on speed, will proceed in a similar manner, the same relations being preserved between the relative loading level (referred to the model load carrying capacity) and the damage extent, and the same proportions between the strength $\mathscr{R}_r$ and the interval limits $\mathscr{R}_{min}$, $\mathscr{R}_{max}$. Under such conditions the formula for strength $\sigma_{\mathscr{R}}$ in high rate processes (conventional stresses) may be written in the form

$$(5.7) \qquad \sigma_{\mathscr{R}} = R_r \, \psi \left(\alpha_r, \frac{\tau_r}{\tau_m} \right) \left(1 - \int\limits_{R_{min}}^{R_r} \varphi \, (R) \, dR \right),$$

where R_r — strength of bonds (in actual stresses) which are after fracture make the statical process develop spontaneously, and $\psi \, (\alpha_r, \tau_r/\tau_m)$ — a coefficient accounting for the rate effect, determined from Eq. (4.10) and referred to the bonds of strength R_r.

Since

$$(5.8) \qquad R_r \left(1 - \int\limits_{R_{min}}^{R_r} \varphi \, (R) \, dR \right) = \sigma_r,$$

with σ_r denoting the bond system strength (conventional stresses) attained in stationary processes, the final formula for the strength of brittle material during rapid processes has the form

$$(5.9) \qquad \sigma_{\mathscr{R}} = \psi \left(\alpha_r, \frac{\tau_r}{\tau_m} \right) \sigma_r.$$

Here σ_r — theoretical strength of the model or strength of brittle material determined in statical tests and $\psi \, (\alpha_r, \tau_r/\tau_m)$ — coefficient accounting for the process rate determined from Eq. (4.10).

6. CALCULATION OF TENSILE STRENGTH OF CONCRETE IN HIGH RATE PROCESSES. COMPARISON OF THEORETICAL RESULTS AND EXPERIMENTAL DATA

Calculations were performed on the basis of relation (5.9), choice of a proper value of α_r is of considerable importance; this coefficient characterises the bond potential barrier of strength R_r; fracture of the bonds makes the process develop spontaneously. Value of α_r was estimated by searching for such theoretical results which would be as close to the experimental data as possible; the procedure was based on the least squares method. It was finally assumed that $\alpha_r = 5$. The corresponding values of ψ are given in Table 3.

The results were compared with those given by A. J. ZIELIŃSKI [1]. Stress rates varied in a wide interval from 10^{-4} to 2.6 N/mm² ms.

Table 3. Coefficients accounting for the effect of process rate on the tensile strength of concrete.

τ_r/τ_m	10^0	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}	10^{-7}	10^{-8}
$\psi\left(\dfrac{\tau_r}{\tau_m}\right)$	1	1.019	1.090	1.221	1.430	1.613	1.780	1.930	2.078

(4.03351 B)

In comparing such results, it is of great importance to determine the limiting value of speed at which slips do not occur. In the macroscopic scale such a process will be manifested by absence of plastic deformations. Analysis of the results made by Evans and Marathe [10] lead the present author to the conclusion that, in the case of tension tests of concrete, this condition will be satisfied at the strain rates not less than $5 \cdot 10^{-7} \, s^{-1}$. The strain rate was finally assumed to be equal to $3 \cdot 10^{-6} \, s^{-1}$, what corresponded to the lowest rate value assumed by Zieliński.
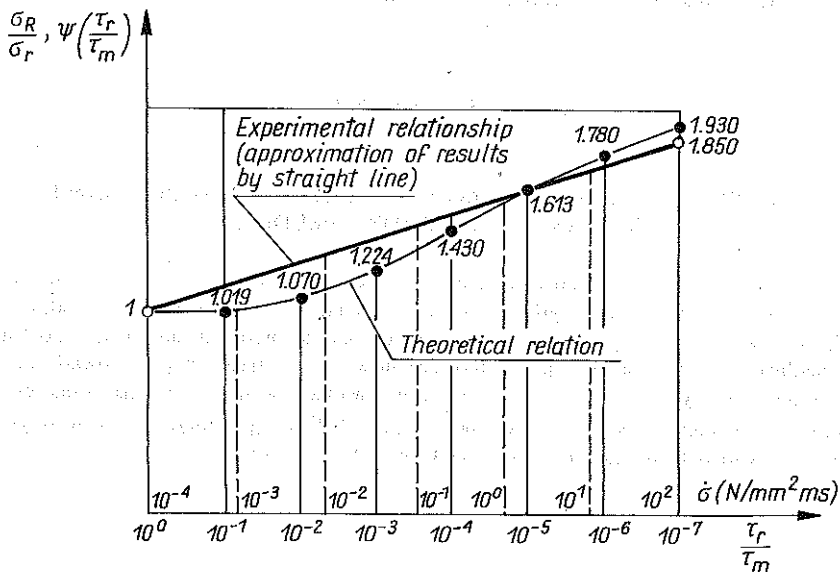

Fig. 6. Tensile strength of concrete in very high rate processes. Comparison of theoretical and experimental results.

In Fig. 6, diagram of the function $\psi(\tau_r/\tau_m)$ derived theoretically is compared with the experimental result. Both diagrams coincide fairly well.

The approach proposed in this paper seems to be applicable to the strength analysis of other brittle materials subject to rapid processes.

REFERENCES

1. A. J. ZIELIŃSKI, *Fracture of concrete and mortar under uniaxial impact tensile loading*, Delft University Press, 1982.
2. D. L. BIRKIMER, R. LINDEMANN, *Dynamic tensile strength of concrete materials*, J. ACJ, 47—49, 1971.
3. K. M. MIANOWSKI, *Dynamic aspects in fracture mechanisms*, Brittle Matrix Composites 1. Elsevier Applied Science Publishers, London–New York 1986 pp. 81—91.
4. H. MIHASHI, M. IZUMI, *A stochastic theory for concrete fracture*, Cement and Concrete Research, 7, 411—422, 1977.
5. K. M. MIANOWSKI, *Dynamic effect in fracture mechanics of brittle materials* [in Polish], Arch. Inż. Ląd., 1987.
6. P. PERZYNA, *Thermodynamics of inelastic materials* [in Polish], PWN, Warszawa 1978.
7. A. SEGHER, *The temperature dependence of the critical shear stress and of work-hardening of metal crystals*, Philosoph. Magazine, 45, 7, 771—773, 1954.
8. Z. BASIŃSKI, *Thermally acticated glide in the face-centered subic metals and its application to the theory of strain hardening*, Philosoph. Magazine, 4, 393—432, 1968.
9. C. H. WERT, R. M. THOMSON, *Physics of solids* [Polish translation], PWN, Warszawa 1974.
10. R. M. EVANS, M. G. MARATHE, *Microcracking and stress-strain curves for concrete in tension*, Materiaux et Construction, 1, 1, 61—64, 1968.

STRESZCZENIE

WYTRZYMAŁOŚĆ MATERIAŁÓW KRUCHYCH NA ROZCIĄGANIE W PROCESACH BARDZO PRĘDKICH

Zakładając, że zniszczenie pod działaniem sił rozciągających w procesach bardzo prędkich odbywa się w sposób kruchy, czyli bez udziału mechanizmów poślizgowych, podano interpretację zjawiska wzrostu wytrzymałości w procesach bardzo prędkich, nawiązując do zjawiska drgań cieplnych siatki krystalicznej. Podano zależności pozwalające na przeprowadzenie praktycznych obliczeń wytrzymałości w funkcji czasu trwania obciążenia. Wyniki obliczeń skonfrontowano z wynikami badań eksperymentalnych Zielińskiego dotyczących wytrzymałości betonu na rozciąganie (osiąganej w procesach bardzo prędkich).

Резюме

ПРОЧНОСТЬ ХРУПКИХ МАТЕРИАЛОВ НА РАСТЯЖЕНИЕ В ОЧЕНЬ БЫСТРЫХ ПРОЦЕССАХ

Предполагая, что разрушение под действием растягивающих сил в очень быстрых процессах совершается хрупким образом, т.е. без участия механизмов скольжения, приведена интерпретация явления роста прочности в очень быстрых процессах, навязывая ее к явлению термических колебаний кристаллической решетки. Приведены зависимости, позволяющие провести практические расчеты прочности в функции времени

продолжительности нагружения. Результаты расчетов сопоставлены результатам эксперментальных исследований Зелиньского, касающихся прочности бетона на растяжение достигаемой в очень быстрых процессах.

INSTITUTE OF CIVIL ENGINEERING TECHNOLOGY, WARSZAWA.