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AN ENGINEERING MODEL FOR COMPACTION OF SAND UNDER CYCLIC LOADING

A. SAWICKI (GDAŃSK)

A model for compaction of saturated sand subjected to cyclic loading is proposed. The model has been formulated in terms of the cyclic stress and strain amplitudes. The first equation describing the model is the compaction law, the second one is the stress-strain relationship between cyclic amplitudes. A correlation of constitutive equations with cyclic loading data is presented. Pore pressure generation under cyclic loading in undrained conditions is evaluated to illustrate predictions of the model.

1. Introduction

The problem of sand compaction due to cyclic loading has been extensively investigated in the last two decades, and there already exists a vast literature on the subject, both experimental and theoretical. It seems that the paper of SEED and LEE [17] has inspired that research and, at the same time, has contributed to a better understanding of the behaviour of saturated granular materials subjected to cyclic loadings. The first stage of research involved the development of new experimental techniques and some attempts to describe qualitatively the behaviour of sand under cyclic loading, see: SILVER and SEED [20, 21]; SEED and PEACOCK [18], FINN *et al.* [4], YOUD [23]. The paper of MARTIN *et al.* [8] recapitulates paramount contributions in the field, up to the mid-seventies. The strongest of these contributions deal with the establishing of factors which influence compaction and so-called "shear modulus". MARTIN *et al.* [8] made also an important observation regarding the relationship between compaction of dry sand and pore pressure generation in a saturated granular material. According to that, the pore pressure increment in saturated sand caused by cyclic shearing is equivalent to the increment of compaction of dry sand divided by the skeleton modulus of compressibility.

Since the mid-seventies special efforts have been made to develop theoretical or empirical models describing the behaviour of sand subjected to cyclic shearing. It is not possible in this paper to make a survey of existing approaches to the problem. Extensive reviews are provided by ZIENKIEWICZ *et al.* [24], FINN [3], ISHIHARA and TOWHATA [6], and MARTIN and SEED [9].

Morland and Sawicki [11, 12] presented a constitutive theory for the compaction of saturated sand, which had the minimal ingredients necessary to reflect the observed cyclic loading phenomena. The constitutive relations of differential type were constructed heuristically from typical qualitative response. An influence of pore pressure on compaction was incorporated, and the generation of pore pressure under cyclic shearing was investigated. The theory was applied to shear wave propagation through a soil layer subjected to an earthquake — like motion, see: Sawicki and Morland [15, 16].

The model of Morland and Sawicki is general in the sense that it is three-dimensional and valid for an arbitrary stress (or strain) history. This generality, however, makes the engineering applications of the model rather difficult, except for some simple boundary value problems. The duration of cyclic loading can be described by the number of cycles N. In the case of an earthquake, N is a small number, usually $N < 20$. In the case of foundations loaded by machines, N is even of order 10^9. Solving a boundary value problem for each cycle of the loading history is practically possible when N is a rather small number, so the computer runs could not be quite long. For N measured in thousands of cycles one needs an approach appropriate for engineering purposes. Such an approach has to reflect the main features of soil behaviour on the one hand, and must be as simple as possible to enable us to solve various problems of practical importance on the other hand.

An attempt to construct such an engineering model for compaction of sand subjected to cyclic loading is presented in this paper.

2. Saturated sand as a two-phase mixture

Saturated sand is treated as a two-phase mixture, with the solid grains as one constituent and the water with dissolved air as the second one. For the practically important stress levels the behaviour of both phases is elastic, although in many approaches the fluid phase in treated as incompressible.

Let ${}^{E}\varrho^{S}$ and ${}^{E}\varrho^{f}$ denote densities of grains material and pore water respectively. A raised prefix E denotes quantity intrinsic to a constituent of a mixture, see Morland [10].

The respective partial densities are defined as follows:

$$\varrho^{s} = (1-\phi)^{E}\varrho^{s}, \quad \varrho^{f} = \phi^{E}\varrho^{f}, \tag{2.1}$$

where ϕ denotes porosity. The density of saturated sand is then

$$\varrho = \varrho^{s} + \varrho^{f}. \tag{2.2}$$

In a similar way, we shall define other "partial" and "intrinsic" quantities. The superscripts s and f will refer to the solid and the liquid phases, respectively. The total stress tensor acting on a saturated soil may be divided onto partial stress tensors in solid and fluid respectively.

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}^s + \boldsymbol{\sigma}^f. \tag{2.3}$$

It is very convenient to introduce the intrinsic stresses, which are defined as follows:

$$\boldsymbol{\sigma}^s = (1-\phi)^E\boldsymbol{\sigma}^s, \quad \boldsymbol{\sigma}^f = \phi^E\boldsymbol{\sigma}^f = -\phi^E p^f \mathbf{1}, \tag{2.4}$$

where ${}^E p^f$ denotes intrinsic pore pressure, i.e. the quantity which can be measured in a saturated soil (in geotechnical literature ${}^E p^f$ is often denoted by u, and means pore pressure). There is ${}^E p^f \geq 0$. Note that a minus sign in $\boldsymbol{\sigma}$ means compression, MORLAND and SAWICKI ([11, 12]). We shall be dealing with small porosity changes, so an initial porosity ϕ_0 will be used in Eqs. (2.1) and (2.4) as a reference one.

It has been assumed that the behaviour of both grains and pore fluid is purely elastic, so intrinsic stress tensors evoke the elastic strains in both phases. These changes are described by the intrinsic strain tensors ${}^E\boldsymbol{\varepsilon}^s$ and ${}^E\boldsymbol{\varepsilon}^f$ in grains and pore fluid respectively. The intrinsic volumetric strains are defined through densities (MORLAND [10]):

$${}^E\varepsilon^s = 1 - \frac{{}^E\varrho^s}{{}^E\varrho_0^s}, \quad {}^E\varepsilon^f = 1 - \frac{{}^E\varrho^f}{{}^E\varrho_0^f}, \tag{2.5}$$

where the subscript 0 denotes initial intrinsic density of a respective phase. Partial volumetric strains are defined as

$$\varepsilon^s = 1 - \frac{\varrho^s}{\varrho_0^s}, \quad \varepsilon^f = 1 - \frac{\varrho^f}{\varrho_0^f}. \tag{2.6}$$

For small porosity changes we have (MORLAND [10]):

$$\varepsilon^s = {}^E\varepsilon^s + \frac{\phi - \phi_0}{1-\phi_0}, \tag{2.7}$$

$$\varepsilon^f = {}^E\varepsilon^f - \frac{\phi - \phi_0}{\phi_0}. \tag{2.8}$$

From Eqs. (2.7) and (2.8) it follows that partial volumetric changes are possible in the case of both incompressible grains and pore fluid.

Intrinsic volumetric changes in the constituents are related to respective intrinsic pressures according to the following formulae:

$${}^E\varepsilon^s = -\varkappa_s\, {}^E p^s, \quad {}^E\varepsilon^f = -\varkappa_f\, {}^E p^f, \tag{2.9}$$

where $\varkappa_s$ and $\varkappa_f$ denote the moduli of compressibility for grains and pore water respectively, and ${}^{E}p^{s} = -\frac{1}{3} \operatorname{tr} {}^{E}\boldsymbol{\sigma}^{s}$. The constitutive relations between partial pressures p^{f}, $p^{s} = (1-\phi_0)\,{}^{E}p^{s}$ and partial dilatations ε^{f}, ε^{s} for the elastic behaviour of saturated sand are presented in MORLAND and SAWICKI [11]. In this paper we restrict our attention to a simplified situation, when the grains compressibility can be neglected in comparison with the pore fluid compressibility. The common approximation $\varkappa_s = 0$, adopted for example by VERRUIJT [22], is well founded for sands since $\varkappa_f \cong 30\varkappa_s$, see LAMBE and WHITMAN [7]. Elastic porosity variation is assumed to depend linearly on partial pressures

$$\frac{\phi-\phi_0}{\phi_0} = -ap^{s}+bp^{f} = \Delta, \tag{2.10}$$

where a and b are coefficients given by the following formulae (see MORLAND and SAWICKI [11]):

$$a = \frac{1-\phi_0}{\phi_0}\varkappa, \quad b = \frac{1-\phi_0}{\phi_0}\left(\frac{1-\phi_0}{\phi_0}\varkappa - \varkappa_f\right). \tag{2.11}$$

Here $\varkappa$ denotes the compressibility of the soil skeleton. For sands, $\varkappa$ is of the order 10^{-8} m^2/N.

The system of equations, including the momentum balance for both constituents, for the elastic behaviour of saturated sand is presented by MORLAND and SAWICKI ([11, 12]).

3. COMPACTION DUE TO CYCLIC SHEARING

It is well known that under cyclic shearing loose and medium dense sands compact (see SILVER and SEED [20], SEED and SILVER [19], CUELLAR *et al.* [2], YOUD [23]). It is so for both dry and free draining sands. The compaction is measured by a progressive irreversible decrease of volume induced by rearrangement of the granular structure.

Most of experiments have been performed in one-dimensional cyclic shearing conditions, under constant vertical load and constant cyclic shear strain amplitude. The main conclusions which follow from those experiments can be summarised as follows: 1) compaction depends on the amplitude of cyclic strain 2) the rate of compaction decreases as a number of cycles N increases, 3) compaction does not depend on frequency of cyclic loading, 4) compaction does not depend on the value of confining pressure, 5) compaction depends on the initial relative density D_r.

These conclusions can be treated as the main features of the dry (or free draining) sand behaviour under cylic shearing (see SILVER and

SEED [20], YOUD [23], MARTIN *et al.* [8]). We shall neglect, in this paper, factors of secondary importance since we would like to construct a rather simple engineering model for compaction. The factor of secondary importance is, for example, a shape of cyclic loading which is assumed here not to influence the compaction. This means that a sinusoidal, trapezoidal or triangular wave causes the very same compaction if the magnitudes of cyclic shear strain amplitudes are equal. Compaction is also assumed to be frequency — independent, what is correct only in some range of applied frequencies, but we treat eventual frequency dependence as a factor of secondary importance and neglect it in the construction of our model.

After MORLAND and SAWICKI [11, 12], let us introduce the compaction Φ by the following formula:

$$\frac{\phi_0-\phi}{\phi_0} = -\Delta+\Phi, \tag{3.1}$$

where Δ is defined as the reversible porosity decrease (see Eq. (2.10)), and Φ defines the irreversible, positive porosity change due to cyclic shearing.

We shall assume the compaction law in the following form:

$$\dot{\Phi} = H(\Phi, J), \tag{3.2}$$

where the superposed dot denotes the rate of change with respect to some monotonically increasing parameter defined with respect to the cyclic loading sequence. This parameter was defined by MORLAND and SAWICKI [11, 12] as the accumulated deviatoric strain, after BAŽANT and KRIZEK [1] and ZIENKIEWICZ *et al.* [24]. We shall assume the cyclic loading to be harmonic in time, so it is convenient to introduce a number of cycles N as a loading parameter. Note that this variable has no meaning in a general constitutive law, and applies only to the case of simple harmonic cyclic loadings. The function H appearing in Eq. (3.2) has to be determined from experimental data, J denotes some invariant of the strain tensor.

Let us introduce the following decomposition of the strain tensor $\boldsymbol{\varepsilon}$:

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^c+\boldsymbol{\varepsilon}^N, \tag{3.3}$$

where $\boldsymbol{\varepsilon}^c$ and $\boldsymbol{\varepsilon}^N$ denote the cyclic and non-cyclic parts of $\boldsymbol{\varepsilon}$, respectively, and assume that cyclic strains are harmonic, i.e.

$$\boldsymbol{\varepsilon}^c = e^{i\omega t}\,\mathbf{E}, \tag{3.4}$$

where $\mathbf{E}$ denotes the tensor of cyclic strain amplitudes. The quantity J, appearing in Eq. (3.2), is defined as the second invariant of strain amplitudes deviator:

$$J = \frac{1}{2}\,\mathrm{tr}\,(\hat{\mathbf{E}})^2, \tag{3.5}$$

where $\hat{\mathbf{E}} = \mathbf{E} - \frac{1}{3}(\mathrm{tr}\,\mathbf{E})\,\mathbf{1}$. In the case of simple cyclic shearing there is:

$$J = \frac{1}{4}\gamma_0^2, \tag{3.6}$$

where γ^0 denotes the cyclic shear strain amplitude. Let us separate the variables in Eq. (3.2), so that

$$\frac{d\Phi}{dN} = JH_1(\Phi), \tag{3.7}$$

where $H_1(\Phi)$ is an unknown function. From Eq. (3.7) we have

$$\frac{d\Phi}{H_1(\Phi)} = JdN = dS(\Phi), \tag{3.8}$$

and, subsequently,

$$\frac{dS(\Phi)}{dN} = J. \tag{3.9}$$

When the cyclic shear strain history is prescribed, we can determine from Eq. (3.9) the function S, i.e.

$$S(\Phi) = \int_0^N J(N')\,dN', \tag{3.10}$$

where a number of cycles is treated as a continuous variable. For a simple cyclic shearing of dry sand at constant shear strain amplitude, there is

$$S(\Phi) = \frac{1}{4}\gamma_0^2 N = z, \tag{3.11}$$

where z denotes a new variable.

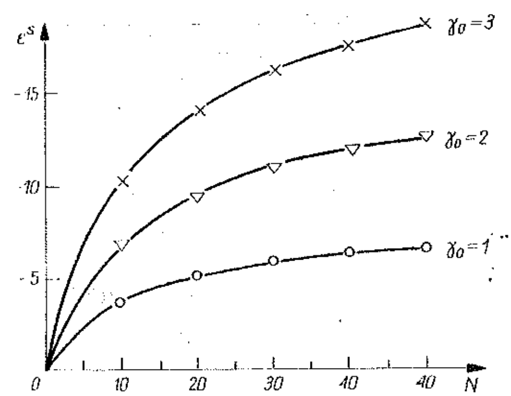

FIG. 1. Compaction curves at different strain amplitudes (MARTIN *et al.* [8]).

From Eq. (3.11) we have

$$\Phi = S^{-1}(z) = f(z), \tag{3.12}$$

where the function $f(z)$ has to be determined from experimental data. The typical compaction data are presented in Fig. 1, after MARTIN *et al.* [8]. There are three curves corresponding to various magnitudes of the cyclic shear strain amplitude γ_0. Here ε^s is an irreversible volume decrease of a dry sand specimen subjected to simple cyclic shearing. There is an obvious relationship between ε^s and the compaction:

$$\Phi = -\frac{1-\phi_0}{\phi_0}\varepsilon^s. \tag{3.13}$$

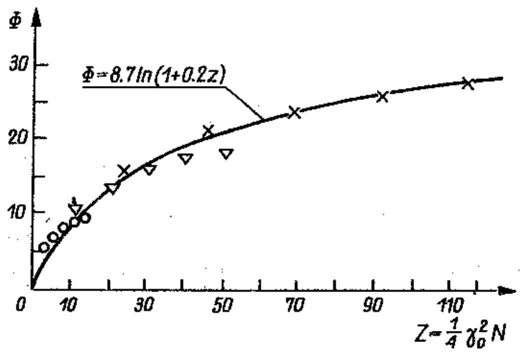

FIG. 2. Common compaction curve for the data of MARTIN *et al.* [8].

Figure 2 shows a new interpretation of the data presented in Fig. 2, using a new variable z introduced in Eq. (3.12). Those data can be approximated by the following function:

$$\Phi = C_1 \ln(1+C_2 z) = f(z), \tag{3.14}$$

where $C_1 = 8.7$ and $C_2 = 0.2$, if z has a unit 10^{-6}, and Φ has a unit 10^{-3}.

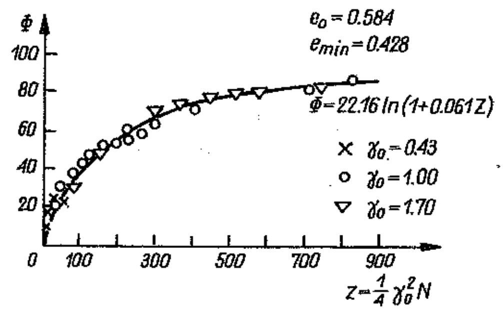

FIG. 3. Common compaction curve for the data reported by NEMAT–NASSER and SHOKOOH [14].

It follows from Eq. (3.15) that

$$z = \frac{1}{C_2}\left[\exp(\Phi/C_1) - 1\right] = S(\Phi). \tag{3.15}$$

From Eq. (3.9) we have

$$\frac{dS(\Phi)}{d\Phi} = \frac{1}{H_1(\Phi)}, \tag{3.16}$$

and finally

$$H_1(\Phi) = C_1 C_2 \exp(-\Phi/C_1) = D_1 \exp(-D_2 \Phi). \tag{3.17}$$

For our data there is $D_1 = 1.74$, $D_2 = 0.115$. Figure 3 presents the densification curve for another set of experimental data (NEMAT–NASSER and SHOKOOH, [14]). The examples presented in Figs. 2 and 3 suggest that in the case of cyclic shearing of dry sand, the compaction Φ is a function of a single variable z. The coefficients C_1 and C_2 appearing in Eq. (3.14) are, in this case, material parameters which in general depend on the type and initial structure of the soil. In order to establish respective relationships (C_1 and C_2 as functions of relative density, etc.), a lot of experimental data is needed, and this problem is beyond the scope of the present paper. The compaction law suitable for our purposes has the following form:

$$\frac{d\Phi}{dN} = D_1 J \exp(-D_2 \Phi), \tag{3.18}$$

where, for a given sand, D_1 and D_2 are numbers.

4. Stress-strain relations for cyclic amplitudes

Let us assume that a cyclic part of the stress tensor $\boldsymbol{\sigma}$ has the form

$$\boldsymbol{\sigma}^c = e^{i\omega t}\mathbf{T}, \tag{4.1}$$

where $\mathbf{T}$ denotes the tensor of cylic stresses amplitudes. We would like to construct the stress-strain relationship between the deviators of $\mathbf{T}$ and $\mathbf{E}$ (see Eq. (3.4)). For one-dimensional cyclic shearing this relationship is of the form

$$\tau_0 = G\gamma_0, \tag{4.2}$$

where τ_0 and γ_0 denote cyclic shear stress and strain amplitudes, respectively, and G is the so-called "shear modulus".

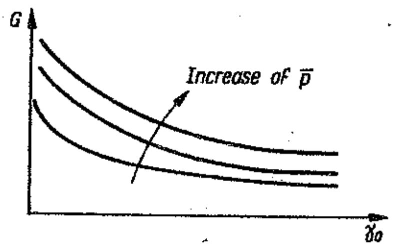

FIG. 4. Shear modulus as a function of strain amplitude and effective mean pressure.

There are a few factors of primary importance which strongly influence the shear modulus G, see SILVER and SEED [20], HARDIN and DRNEVICH [5]: a mean effective pressure $\bar{p}$, a magnitude of cyclic shear strain amplitude γ_0, initial structure of soil. The typical qualitative dependence of G on $\bar{p}$ and γ_0 is shown in Fig. 4. As the first approximation we can assume that the shear modulus G depends only on a magnitude of mean effective pressure:

(4.3) $$G = G_0\,\bar{p}^m,$$

where $m = 0.5-0.7$ (see MARTIN *et al.* [8]). Usually $m = 0.5$. Here, G_0 is a material parameter. Equation (4.3) is a very good approximation of the shear modulus for small strains ($\gamma_0 < 10^{-4}$), see the experimental data of SILVER and SEED [20], HARDIN and DRNEVICH [5]. An advantage of using the formula (4.3) is its simple form. For bigger strains (say $\gamma_0 > 10^{-3}$) the influence of γ_0 on G may be significant. In this case we can assume, after HARDIN and DRNEVICH [5], the following form of shear modulus:

(4.4) $$G = \frac{\tau_{max}\,G_{max}}{\tau_{max} + \gamma_0\,G_{max}},$$

where G_{max} is a maximum value of shear modulus, τ_{max} denotes the maximum shear stress carried by a soil. The simplest form of expression for τ_{max} follows from the Coulomb–Mohr failure condition:

(4.5) $$\tau_{max} = \bar{p}\tan\psi,$$

where ψ denotes an angle of internal friction. The quantity G_{max} depends on the magnitude of mean effective pressure

(4.6) $$G_{max} = G^*\sqrt{\bar{p}},$$

where G^* is a coefficient which has to be determined experimentally. Substitution of Eqs. (4.4)—(4.6) into Eq. (4.2) gives the following relationship:

(4.7) $$\tau_0 = \frac{\bar{p}\tan\psi}{\frac{1}{G^*}\tan\psi\sqrt{\bar{p}} + \gamma_0}\gamma_0 = G\gamma_0.$$

From Eq. (4.7) we also have

$$\gamma_0 = \frac{\frac{1}{G^*}\tan\psi\sqrt{\bar{p}}}{\bar{p}\tan\psi - \tau_0}\tau_0 = Q\tau_0. \tag{4.8}$$

If the cyclic shear stress amplitude τ_0 tends to $\tau_{\max}$ then the denominator in Eq. (4.8) tends to zero and, subsequently, γ_0 rapidly increases. This situation corresponds to soil failure. Figure 5 shows the experimental data of SILVER and SEED [20] and the corresponding $G-\gamma_0$ curves calculated using Eq. (4.7). It is

$$G = \frac{0.65\bar{p}}{3.68\sqrt{\bar{p}}+\gamma_0}. \tag{4.9}$$

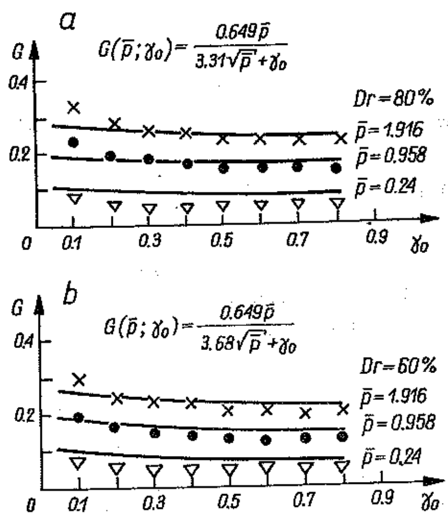

FIG. 5. Proposed shear modulus against experimental data of SILVER and SEED [20].

Here, G has unit $10^8\ N/m^2$, $\bar{p}$ has the stress unit $10^5\ N/m^2$, and γ_0 has the strain unit 10^{-3}. Equation (4.7) takes the following form in the three-dimensional case:

$$\hat{\mathbf{T}} = \frac{2\bar{p}\tan\psi}{\frac{1}{G^*}\sqrt{\bar{p}}\tan\psi + 2\sqrt{J}}\hat{\mathbf{E}} = 2G(\bar{p}, J)\,\hat{\mathbf{E}}, \tag{4.10}$$

where $\hat{\mathbf{T}} = \mathbf{T} - \frac{1}{3}\operatorname{tr}(\mathbf{T})\,\mathbf{1}$ is the second invariant of stress amplitudes deviator.

5. Pore pressure generation and liquefaction

Let us consider a simple cyclic shearing of a saturated sand sample at constant shear stress amplitude τ_0, and at constant confining pressure p. Consider the limiting case of undrained conditions such that internal diffusion is negligible and the stress and deformation is uniform through the saturated sand sample. This idealised situation corresponds to experimental condition described, for example, by Seed and Peacock [18], and Finn *et al.* [4]. When saturated sand undergoes cyclic shear in undrained conditions, the generation of pore water pressure occurs up to the soil liquefaction, what is demonstrated by the rapid increase of cyclic shear strain amplitude. The increasing pore pressure changes the distribution of total pressure between the soil skeleton and pore water. In other words, the mean effective pressure $\bar{p}$ decreases as pore pressure ${}^{E}p^{f}$ increases according to the formula

$$\bar{p} = p - {}^{E}p^{f}, \tag{5.1}$$

where $p = \text{const}$ is a total mean pressure. At the beginning of cyclic shearing there is ${}^{E}p^{f} = 0$. In undrained conditions the partial dilatations in solid and fluid phases are equal (no relative motion of both phases):

$$\epsilon^{s} = \epsilon^{f}. \tag{5.2}$$

Let us assume, for the sake of simplicity, that both grains and pore water are incompressible, i.e. ${}^{E}\epsilon^{s} = {}^{E}\epsilon^{f} = 0$. Substitution of Eqs. (2.7) and (2.8) into Eq. (5.2) leads to the following expression:

$$\phi - \phi_0 = 0, \tag{5.3}$$

what means that there is no volume change in a saturated sand if drainage of pore water is prevented. This is a common assumption accepted in soil mechanics. From Eqs. (3.1) and (5.3) it follows

$$\Delta = \Phi, \tag{5.4}$$

or

$$\frac{1-\phi_0}{\phi_0}\left(-\varkappa p + \varkappa\, {}^{E}p^{f}\right) = \Phi. \tag{5.5}$$

Differentiation of Eq. (5.5) with respect to N gives the following differential equation for pore pressure generated by cyclic shearing in undrained conditions:

$$\frac{d\,{}^{E}p^{f}}{dN} = \frac{1}{a}\,\frac{d\Phi}{dN}, \tag{5.6}$$

where a is given by Eq. (2.11). Integration of Eq. (5.6) assuming zero initial conditions (for $N = 0: {}^{E}p^{f} = \Phi = 0$), leads to the equation

$$ {}^{E}p^{f} = \frac{1}{a}\,\Phi. \tag{5.7} $$

Note that integration of Eq. (5.6) with the initial conditions $\Phi\,(N = 0) = 0$ and ${}^{E}p^{f}\ (N = 0) = p$ leads to Eq. (5.5). The experiments of Seed and Peacock [18] and Finn *et al.* [4] were performed at zero initial pore pressure, so in this case Eq. (5.7) is valid. Substitution of Eq. (3.18) into Eq. (5.6) gives, in the case of simple cyclic shearing,

$$ \frac{d\,{}^{E}p^{f}}{dN} = \frac{D_1\,\gamma_0^2}{4a}\exp\left(-D_2\,a\,{}^{E}p^{f}\right), \tag{5.8} $$

where γ_0 is given, for example, by Eq. (4.8). If we accept the simplest form of Eq. (4.2), i.e.

$$ \tau_0 = G_0\sqrt{\bar{p}}\,\gamma_0, \tag{5.9} $$

then we get the following ordinary differential equation for the pore pressure generation:

$$ \frac{d\,{}^{E}p^{f}}{dN} = \frac{D_1\,\tau_0^2}{4a\,G_0^2\,(p - {}^{E}p^{f})}\exp\left(-D_2\,a\,{}^{E}p^{f}\right). \tag{5.10} $$

Integration of Eq. (5.10) gives the pore pressure generated by a simple cyclic shearing of a saturated soil sample. Usually, in experimental conditions, the total confining pressure p and cyclic shear stress amplitude τ_0 are kept constant. The increasing pore pressure ${}^{E}p^{f}$ reduces the mean effective pressure $\bar{p}$ (Eq. (5.1)) down to zero. As $\bar{p}$ decreases, the shearing resistance of a saturated soil sample is progressively reduced, i.e. $G \to 0$. In that case the denominator of the right hand side of Eq. (5.10) tends to zero, so a numerical procedure of integration of Eq. (5.10) becomes unstable. That numerical instability corresponds to the so-called "final liquefaction", (see Zienkiewicz *et al.* [24]), which occurs when $p = {}^{E}p^{f}$, i.e. when a saturated soil loses entirely its shearing resistance and behaves macroscopically like a liquid. Some of the authors (Zienkiewicz *et al.* [24]) distinguish the so-called "initial liquefaction" which, for example occurs when the cyclic shear stress amplitude $\tau_0 = \tau_{max}$ (Eq. (4.5)). We can describe the onset of initial liquefaction as well if we substitute Eq. (4.8) instead of Eq. (5.9) into Eq. (5.8).

We have computed illustrations for the following data:

$$ \phi_0 = 0.4, \quad \varkappa = 2\,(\times 10^{-8}\ \mathrm{m^2/N}), \quad D_1 = 1.74, $$
$$ D_2 = 0.115, \quad G_0 = 0.72\,(\times 10^{8}\ \mathrm{N/m^2}). $$

which correspond to medium dense sand.

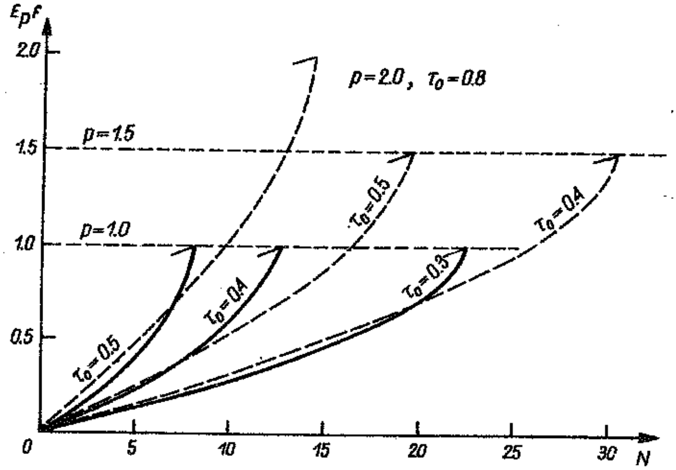

FIG. 6. Pore pressure generation prior to liquefaction.

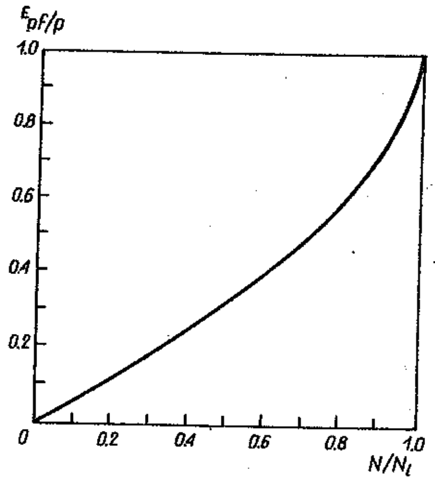

FIG. 7. Pore pressure generation curve in normalised coordinates.

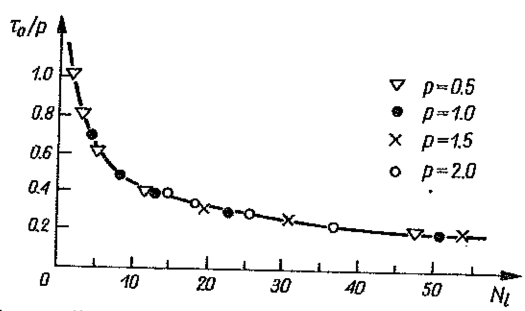

FIG. 8. Number of cycles to liquefaction in terms of shear stress-confining pressure ratio.

The pore pressure generation, for various cyclic shear stress amplitudes τ_0 (in unit 10^5 N/m^2) and various confining pressures p (in unit 10^5 N/m^2), is illustrated in Fig. 6. For example, for $\tau_0 = 0.4$ and $p = 1.5$, the final liquefaction occurs after $N_l = 30$ cycles. The ratio ${}^{E}p^{f}/p$ as a function of the normalised cycle count N/N_l, for the data shown in Fig. 6, is presented in Fig. 7 as a single curve. Figure 8 shows the results for a sequence of τ_0, p pairs as a relation between τ_0/p and the number of cycles N_l to liquefaction. There is a qualitative agreement with experimental data

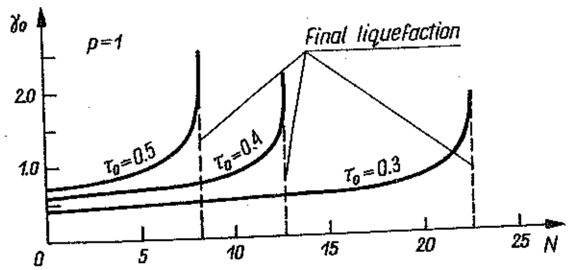

FIG. 9. Shear strain amplitude as a function of N for various shear stress amplitudes.

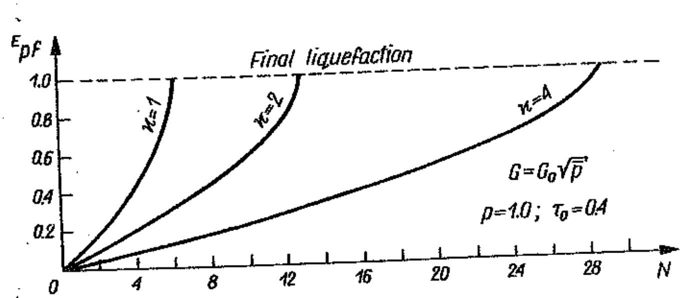

FIG. 10. Influence of skeleton compressibility on pore pressure generation.

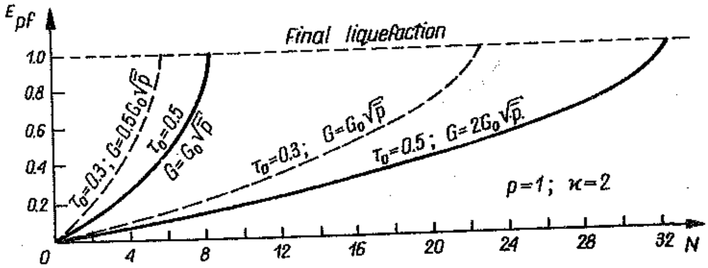

FIG. 11. Influence of shear modulus on pore pressure generation.

(MARTIN *et al.* [8]). The cyclic shear strain amplitudes γ_0 (in unit 10^{-3}), for $p = 1$ and various shear stress amplitudes, are shown in Fig. 9 as functions of N. The similar qualitative results are presented by NEMAT--NASSER and SHOKOOH [13]. It is visible that, at the onset of initial liquefaction, the strain amplitudes rapidly increase.

And finally, Fig. 10 shows the influence of the soil skeleton compressibility $\varkappa$ on pore pressure generation, and Fig. 11 illustrates the influence of G_0.

6. CONCLUSIONS

The aim of the present paper is to propose a simple model for compaction of saturated sand, which may be useful in various engineering applications. The model has been formulated in terms of the cyclic stress and strain amplitudes. The first equation describing the model is the compaction law (3.18), and the second one is the stress-strain relationship between cyclic amplitudes (4.10). For small strains Eq. (4.10) can be replaced by the following relation:

$$\hat{\mathbf{T}} = 2G_0 \sqrt{p}\, \hat{\mathbf{E}}. \tag{6.1}$$

Dry sand subjected to cyclic loadings is then characterised by three parameters, namely D_1, D_2 and G_0, which have to be determined experimentally. The other two parameters, i.e. initial porosity ϕ_0 and skeleton compressibility $\varkappa$ play an important role in studying pore pressure generation in saturated sand.

The examples of pore pressure generation presented in Figs 6—11 serve as the first test for the model proposed. The results obtained show good qualitative agreement with the available experimental data (see, for example, SEED and PEACOCK, [18], FINN *et al.* [4], MARTIN *et al.* [8]). There is also a qualitative agreement with the results obtained by using the model of MORLAND and SAWICKI [11, 12]. The substantial differences between the Morland–Sawicki approach and the model proposed herein are as follows:

i) The Morland–Sawicki model is formulated for full strain and stress tensors. The present model is formulated for the amplitudes of cyclic parts of stress and strain tensors.

ii) The number of cycles N has been chosen as a loading parameter in the present approach, against the accumulated deviatoric strain accepted by MORLAND and SAWICKI [11, 12].

iii) The shear response relationship (4.10) or (6.1) is of an algebraic form, against the hypoelastic type differential equation proposed by Morland and Sawicki.

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Streszczenie

INŻYNIERSKI MODEL ZAGĘSZCZANIA PIASKU POD OBCIĄŻENIEM CYKLICZNYM

Zaproponowano model zagęszczania piasku nasyconego wodą i poddanego obciążeniom cyklicznym. Parametrami modelu są wielkości amplitud naprężeń i odkształceń cyklicznych. Pierwszym równaniem modelu jest prawo zagęszczania, a drugim — związek między amplitudami cyklicznych naprężeń i odkształceń. Dla zilustrowania zastosowań modelu przeanalizowano problem powstawania ciśnienia w porach pod działaniem obciążeń cyklicznych.

Резюме

ИНЖЕНЕРНАЯ МОДЕЛЬ УПЛОТНЕНИЯ ПЕСКА ПОД ДЕЙСТВИЕМ ЦИКЛИЧЕСКОЙ НАГРУЗКИ

Предлагается модель уплотнения песка пропитанного влагой и подвергаемого циклическим нагрузкам. Параметрами являются амплитуды циклических напряжений и деформаций. Приложения модели проиллюстрированы на примере задачи о возникновении давления в порах под действием циклической нагрузки.

POLISH ACADEMY OF SCIENCES
INSTITUTE OF HYDROENGINEERING, GDAŃSK

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