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HYDROMAGNETIC FLOW OVER A STRETCHING SHEET OF A VISCOELASTIC FLUID WITH UNIFORM SUCTION AT THE WALL AND HEAT TRANSFER

E. F. ELSHEHAWEY, M. A. KAMEL and F. N. IBRAHIM (CAIRO)

A study is made for a hydromagnetic flow with heat transfer of an electrically-conducting, incompressible viscoelastic fluid past a porous plate. A solution for the velocity and temperature distributions in the flow with uniform suction at the wall is obtained. It is found that the velocities at any horizontal plane parallel to the plate in the fluid decrease as the constant magnetic field M, the variable suction parameter R and the viscoelastic parameter K increase individually keeping the other two parameters constants. It is observed that the temperature at a certain plane decreases with the increase of R, keeping K, M and the Prandtl number Pr constant. A similar effect is observed as Pr increases. However, the temperature gives a reverse relation by changing M or K. The boundary layer characteristics are estimated for different values of K, M and R.

1. Introduction

The viscous Newtonian flow on a wall stretched with a velocity proportional to x, which is the distance along the wall and where the free stream velocity is constant, has been studied by Danberg and Fansler [1]. An extention of such a work in which an electrically-conducting, incompressible fluid past a porous wall with a vanishing free stream velocity was investigated by Chakrabarti and Gupta [2].

There has been an increasing interest in the flow properties of viscoelastic fluids, especially in technological fields. The introduction of the fluid's elastic property will play an important role in modifying the flow fields. Oldroyd [3] and Walters [4] attempted to formulate rheological equations for viscoelastic fluids. The boundary layer equations of these fluids have been derived by Beord and Walters [5]. A numerical analysis was done by Soundalgekar and Ramana [6] of the dynamic boundary layer and the thermal boundary layer at a semi-infinitely large plate longitudinally streamlined by a viscoelastic fluid.

In our present work we consider the flow with heat transfer of an electrically-conducting, incompressible viscoelastic fluid (with electrical conductivity σ) past a porous plate coinciding with the plane $Y=0$, such that the flow is confined to $Y>0$. The wall is stretched keeping the origin fixed, and a uniform constant magnetic field B_0 is imposed along the Y-axis. The free stream velocity is taken to be zero and the motion of the fluid is caused solely by the stretching of the wall. A solution for the velocity and heat transfer characteristics in the flow with uniform suction at the wall is obtained.

This problem has applications to polymer technology, where one deals with stretching plastic sheets and metallurgy, and hydromagnetic techniques have recently been used. It may be pointed out that many metallurgical processes involve the cooling of continuous strips of filaments by drawing them through a quiescent fluid and that in the process of drawing, these strips are sometimes streched. An application of hydromagnetic to metallurgy is the purification of molten metals from nonmetallic inclusions by applying a magnetic field [7].

2. Formulation of the problem

The basic equations for the steady flow of an electrically-conducting, incompressible viscoelastic fluid are given by [5]:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu^*\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\varrho}u-K_0^*\left[u\frac{\partial^2 u}{\partial x\,\partial y}+v\frac{\partial^3 u}{\partial y^3}+\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}-\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\,\partial y}\right], \tag{2.1}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0. \tag{2.2}$$

Here the induced magnetic field is neglected, and it is assumed that the external electric field is zero and the electric field due to polarization of charges is negligible. The equation of heat (without dissipation) is given by

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=a\frac{\partial^2 T}{\partial y^2}. \tag{2.3}$$

In these equations u is the velocity in the x direction, v is the velocity normal to the plate, ν^* is the coefficient of kinematic viscosity, ϱ is the density of the fluid, K_0^* is the coefficient of the viscoelastic term, and a is the thermal diffusivity.

The boundary conditions of the problem are

$$\begin{aligned} u &= Cx, \quad v = -v_0, \quad T = T_w \quad \text{at} \quad y = 0, \\ u &= 0, \quad T \to T_\infty \quad \text{as} \quad y \to \infty, \end{aligned} \tag{2.4}$$

where $C > 0$, T_w is the constant temperature of the wall and T_∞ the temperature far away from it.

Let us define another set of variables as follows:

$$\begin{aligned} u &= Cxf'(\eta), \quad v = -(\nu^* C)^{1/2} f(\eta), \\ \eta &= (C/\nu^*)^{1/2} y, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}. \end{aligned} \tag{2.5}$$

These variables satisfy the equation of continuity (2.2) Equations (2.1) and (2.3) under these transformations reduce to

$$f'^2 - ff'' = f''' - Mf' - K[2f'f''' - ff^{\mathrm{IV}} - f''^2], \tag{2.6}$$

$$\theta'' + \Pr f\theta' = 0, \tag{2.7}$$

where

$$M = \frac{\sigma B_0^2}{C\varrho}, \quad K = \frac{K_0^* C}{\nu^*}$$

and the Prandtl number $\Pr = \nu^*/a$. The boundary conditions for the function $f(\eta)$ and $\theta(\eta)$ are given by

$$\begin{aligned} f'(0) &= 1, \quad f(0) = v_0/(\nu^* c)^{1/2}, \quad \theta(0) = 1, \\ f'(\infty) &= 1, \quad f''(\infty) = 0, \quad \theta(\infty) = 0, \end{aligned} \tag{2.8}$$

where $v_0 > 0$ denotes the suction velocity at the wall, the fluid being at rest at infinity.

3. Solution of equations

To solve Eqs. (2.6) and (2.7) subject to the boundary conditions (2.8), we take

$$f(\eta) = A + Be^{-\alpha\eta}, \tag{3.1}$$

where A, B and α are constants with $\alpha > 0$. Substitution of Eq. (3.1) in Eqs. (2.6) and (2.7) and the use of the boundary conditions (2.8) gives

$$B = -1/\alpha, \quad A = R + 1/\alpha, \tag{3.2}$$

$$KR\alpha^3 - (1-K)\alpha^2 + R\alpha + M + 1 = 0, \tag{3.3}$$

where

(3.4) $$R = v_0/(v^* c)^{1/2}$$

and

(3.5) $$\theta(\eta) = \frac{\gamma(P, X_1 e^{-\alpha\eta})}{\gamma(P, X_1)},$$

where

$$P = \frac{\Pr A}{\alpha}, \qquad X_1 = \frac{\Pr B}{\alpha}$$

and $\gamma(P, X_1)$ is the incomplete gamma function given by

(3.6) $$\gamma(P, X_1) = \int_0^{X_1} e^{-t} t^{p-1}\, dt, \qquad Re\, P > 0.$$

Equation (3.3) has two positive and one negative roots. The negative root is, however, not admissible as α must be greater than zero. One of the positive roots closely agrees with results obtained by CHAKRABARTI and GUPTA [2] for a nonelastic Newtonian viscous liquid, for that we neglect the other one,

4. THE BOUNDARY LAYER THICKNESSES

4.1. *The boundary layer thickness* δ

This thickness is defined as the distance from the solid boundary at which the local value of the velocity reaches 0.01 of the stretched plate velocity, i.e.

(4.1) $$y = \delta \quad \text{when} \quad u = 0.01\,(Cx)$$

or

(4.2) $$\eta_\infty = \delta/(v^*/c)^{1/2} = -\frac{ln\,(0.01)}{\alpha}.$$

4.2. *The displacement thickness* δ_1

It is another type of boundary layer thickness which is useful under certain circumstances. This thickness is defined as the distance at which the undisturbed outer flow is displaced from the boundary by a stagnant layer which removes the same mass flow from the flow field as the actual boundary layer [8]. In mathematical terms this thickness is given by

$$\delta_1 = \int_0^{\eta_\infty} \left(1-\frac{u}{cx}\right) dy. \tag{4.3}$$

Using Eqs. (2.5) and (3.1) and the definition of δ from the relation (4.1) and (4.2), δ_1 in nondimensional form is given by

$$\delta_1/(\nu^*/C)^{1/2} = \eta_\infty - 0.99/\alpha. \tag{4.4}$$

4.3. The momentum thickness δ_2

The momentum thickness is defined as that thickness of layer which, at zero velocity, has the same momentum defect, relative to the outer flow, as the actual boundary layer [8]. Thus δ_2 is given by

$$\delta_2 = \int_0^{\eta_\infty} \frac{u}{cx}\left(1-\frac{u}{cx}\right) dy. \tag{4.5}$$

Substituting Eqs. (2.5), (3.1) and (4.2) into Eq. (4.5), we obtain for δ_2 in nondimensional form the following expression:

$$\delta_2/(\nu^*/c)^{1/2} = \frac{0.9801}{2\alpha}. \tag{4.6}$$

5. Conclusions

In this paper we have studied the flow with heat transfer of an electrically-conducting, incompressible viscoelastic fluid past a stretched porous plate. The effect of a constant magnetic field B, the variable suction parameter R, the viscoelastic parameter K and the Prandtl number Pr on flow characteristics have been studied and are tabulated and illustrated graphically. The boundary layer thicknesses δ, δ_1 and δ_2 defined in the last section are calculated for different values of K, M and R.

Figures 1 and 2 show that the velocities u and v at any horizontal plane parallel to the X-axis in the fluid decrease as M (the magnetic field strength) increases, for fixed values of R and K. A similar effect is observed, for the velocity u, as the variable suction parameter R increases as shown in Fig. 3. Figures 4 and 5 show that u and v at a certain η in the fluid decrease with the increase of the viscoelastic parameter K.

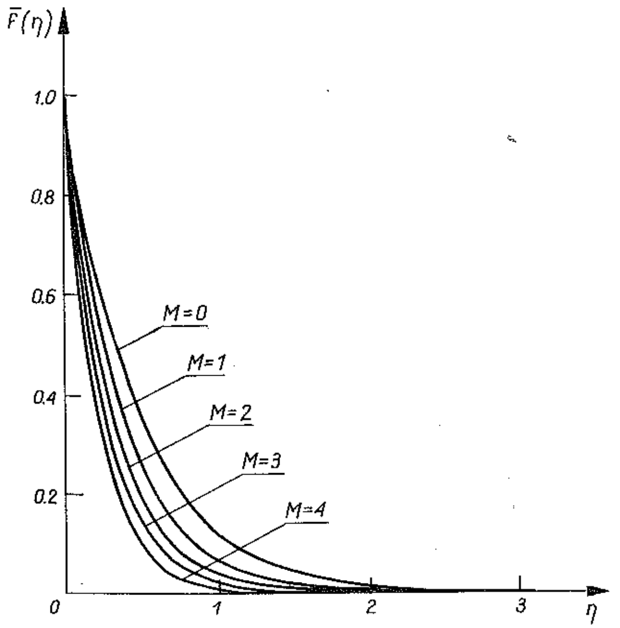

FIG. 1. Variation of $f'(\eta)$ for several values of M with $K = 0.1$ and $R = 1$.

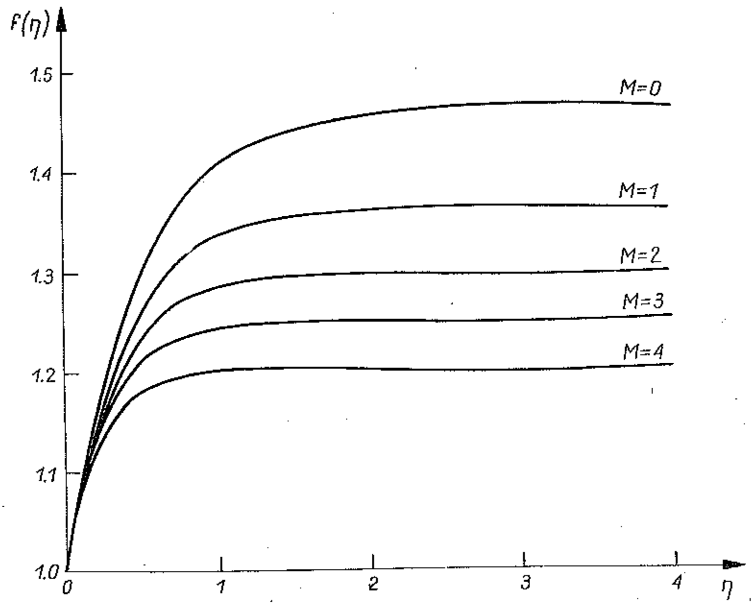

FIG. 2. Variation of $f(\eta)$ for several values of M with $K = 0.1$ and $R = 1$.

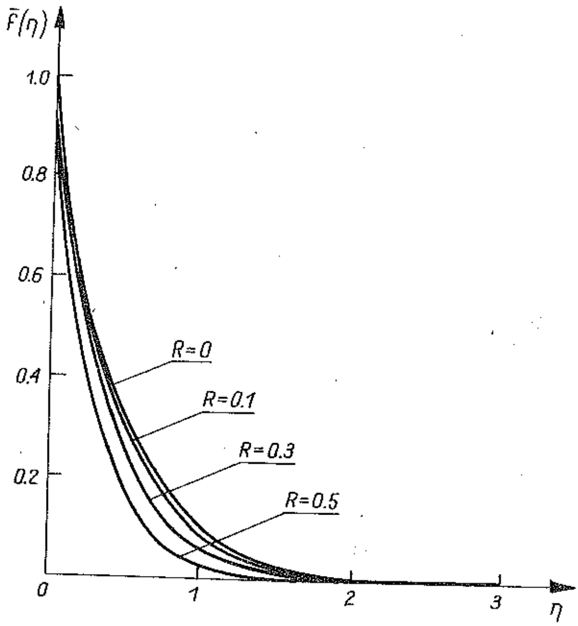

FIG. 3. Variation of $f'(\eta)$ for several values of R with $K = 0.2$ and $M = 3$.

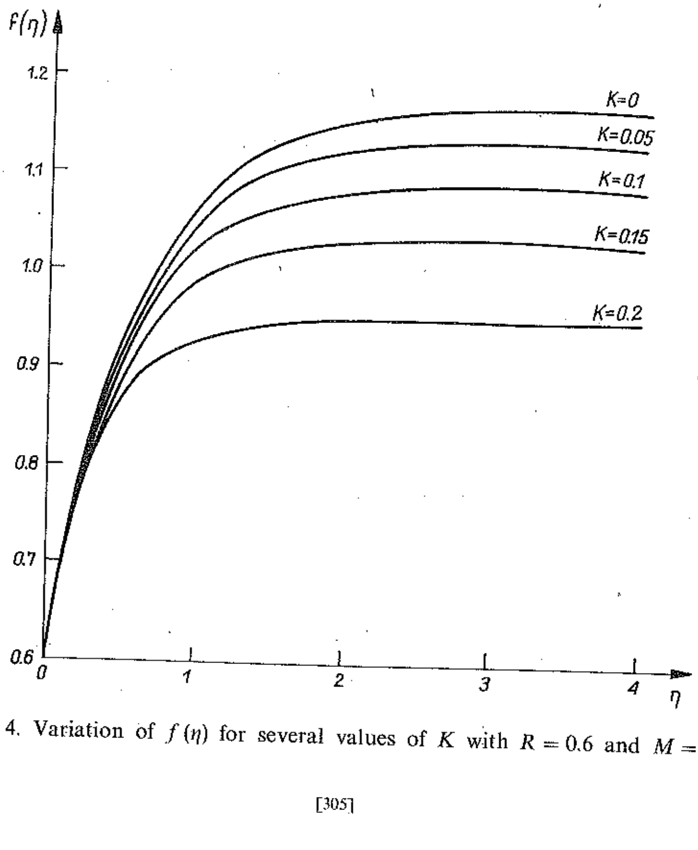

FIG. 4. Variation of $f(\eta)$ for several values of K with $R = 0.6$ and $M = 1$.

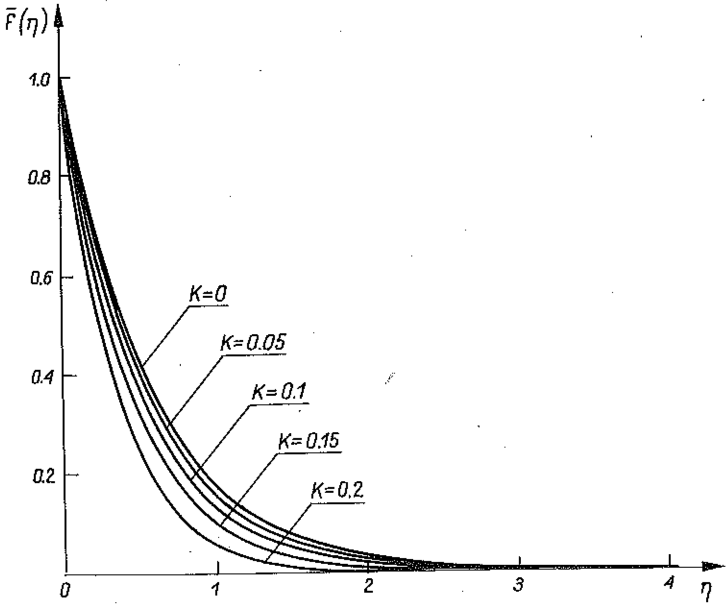

FIG. 5. Variation $f'(\eta)$ for several values of K with $R = 0.6$ and $M = 1$.

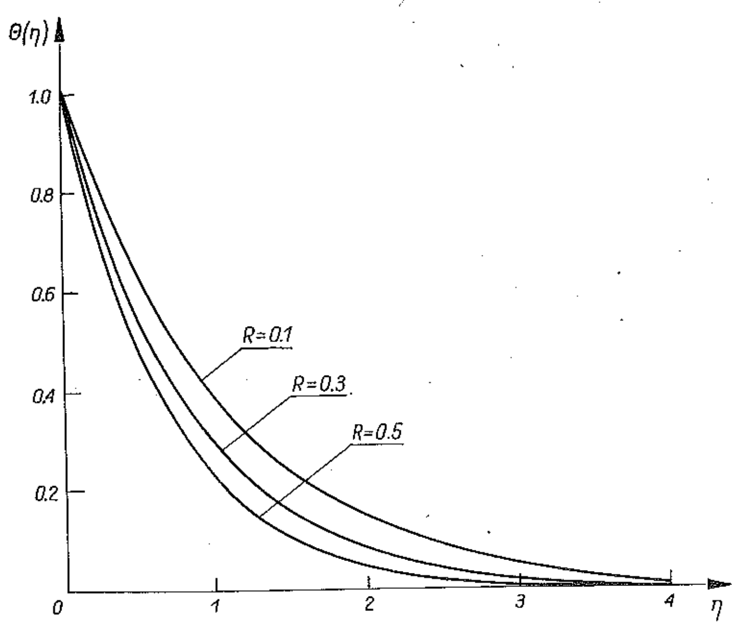

FIG. 6. Temperature profile for several values of R with $K = 0.2$, $M = 3$ and $\mathrm{Pr} = 2$.

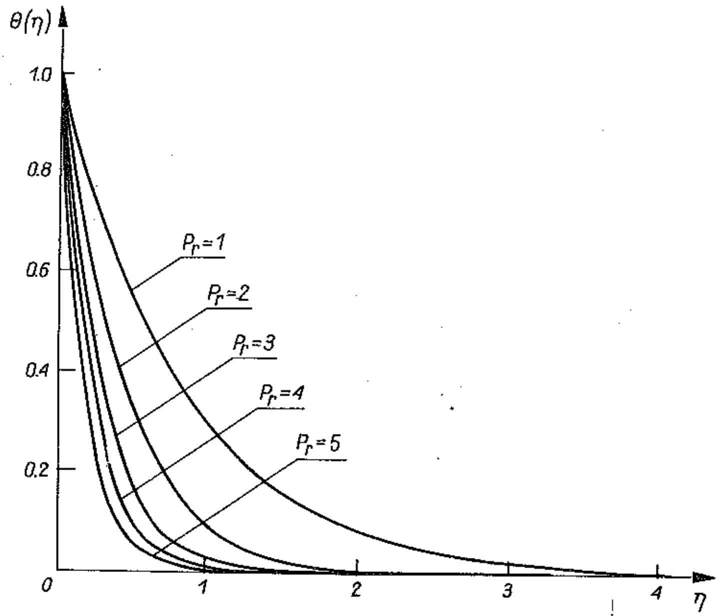

FIG. 7. Temperature profile for several values of Pr $K = 0.1$, $M = 3$ and $R = 1$.

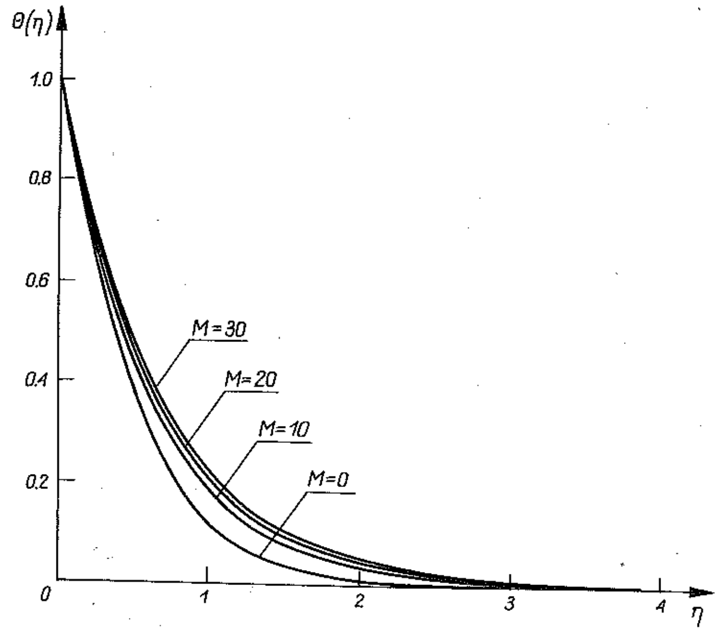

FIG. 8. Temperature profile for several values of M with $K = 0$, $R = 0.6$ $\mathrm{Pr} = 2$.

Table 1. The temperature distribution for a fluid with $\Pr = 2$ $R = 0.6$, $M = 0$ for several values of K.

η	$K = 0$ $\theta(\eta)$	$K = 0.1$ $\theta(\eta)$	$K = 0.2$ $\theta(\eta)$
0.4	0.4537	0.4603	0.4742
0.8	0.1837	0.1921	0.2087
1	0.1137	0.1213	0.1364
1.2	0.0694	0.0758	0.0886
1.6	0.0252	0.0290	0.0370

Table 2. The boundary layer thickness δ, the displacement thickness δ_1 and the momentum thickness δ_2 for a viscoelastic fluid for several values of M with $K = 0.1$ and $R = 1$.

M	δ	δ_1	δ_2
0	2.153042	1.690190	0.229112
1	1.666167	1.307982	0.177302
2	1.372392	1.077361	0.146040
3	1.151293	0.903793	0.122513
4	0.921034	0.723034	0.098010

Table 3. The boundary layer thickness δ, the displacement thickness δ_1 and the momentum thickness δ_2 for a viscoelastic fluid for several values of R with $K = 0.2$ and $M = 3$.

R	δ	δ_1	δ_2
0	2.059495	1.616753	0.219157
0.1	1.940735	1.523524	0.206520
0.3	1.669556	1.310642	0.177662
0.5	1.254053	0.984462	0.133448

Table 4. The boundary layer thickness δ, the displacement thickness δ_1 and the momentum thickness δ_2 for a viscoelastic fluid for several values of K with $R = 0.6$ and $M = 1$.

K	δ	δ_1	δ_2
0	2.638035	2.070921	0.280721
0.05	2.464595	1.934767	0.262265
0.10	2.263205	1.776671	0.240835
0.15	2.012248	1.579663	0.214129
0.20	1.621385	1.272826	0.172536

It is seen from Fig. 6 that the temperature at certain η decreases with the increase of the suction parameter R, keeping K, M and Pr constant. A similar effect is observed as Pr increases (Fig. 7).

We observe from Fig. 8 that at a given η the temperature rises with the increase in M. This result agrees with that obtained by CHAKABARTI and GUPTA [2].

Table 1 gives the temperature distribution for a viscoelastic fluid with $\Pr = 2$, $M = 0$ and $R = 0.6$. Thus at a given η, the temperature rises as K increases. We notice that this method cannot be applied to viscoelastic fluids with a viscoelastic parameter K greater than 0.2.

Tables 2 and 3 and 4 show that the boundary layer thickness δ as well as the displacement thickness δ_1 and the momentum thickness δ_2 decrease with an increase in the parameters K, M and R individually, keeping the other two parameters constants.

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Streszczenie

HYDROMAGNETYCZNY PRZEPŁYW CIECZY LEPKOSPRĘŻYSTEJ WZDŁUŻ PŁYTY Z RÓWNOMIERNYM SSANIEM NA ŚCIANCE I WYMIANĄ CIEPŁA

Rozważono hydromagnetyczny przepływ wzdłuż porowatej płyty z wymianą ciepła w przypadku nieściśliwej cieczy lepkosprężystej przewodzącej elektryczność. Otrzymano rozwiązania dla rozkładu prędkości i temperatury w przepływie z jednorodnym ssaniem na ściance. Stwierdzono, że prędkości w płaszczyznach poziomych równoległych do płyty maleją, jeśli wzrasta tylko jeden z trzech parametrów: stałe pole magnetyczne M, zmienny parametr ssania R i parametr lepkosprężysty K (przy niezmiennej wartości pozostałych parametrów). Stwierdzono również, że temperatura w pewnej płaszczyźnie maleje ze wzrostem parametru R przy stałej wartości K, M i liczby Prandtla Pr. Podobne zjawisko zaobserwować można przy wzroście Pr. Jednak temperatura wywiera przeciwny skutek przez zmianę M lub K. Wartości parametrów warstwy przyściennej oceniono dla różnych wartości K, M i R.

Резюме

ГИДРОМАГНИТНОЕ ТЕЧЕНИЕ ВЯЗКОУПРУГОЙ ЖИДКОСТИ ВДОЛЬ ПЛАСТИНКИ С РАВНОМЕРНЫМ ВСАСЫВАНИЕМ НА СТЕНКЕ И ТЕПЛООБМЕНОМ

Рассмотрено гидромагнитное течение вдоль пористой пластинки с теплообменом в случае несжимаемой вязкоупругой электропроводящей жидкости. Получены решения для распределения скорости и температуры в течении с однородным всасыванием на стенке. Констатировано, что скорости, в горизонтальных плоскостях параллельных. пластинке, убывают, если возрастает только один из трех параметров: постоянное магнитное поле M, переменный параметр всасывания R и вязкоупругий параметр K (при неизменяющихся значениях остальных параметров). Констатировано тоже, что температура в некоторой плоскости убывает с ростом параметра R, при постоянных значениях K, M и числа Прандтля Pr. Аналогичное явление можно наблюдать с ростом Pr. Однако температура вызвает противонаправленный эффект при изменении M или K. Значения параметров пограничного слоя оценены для разных значений K, M и R.

FACULTY OF EDUCATION, AIN SHAMS UNIVERSITY ROXY. CAIRO
AND
FACULTY OF SCIENCE AIN SHAMS UNIVERSITY. ABASEIA. CAIRO, EGYPT.

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