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CHANGES OF THE YIELD CONDITION DUE TO ACCUMULATION OF DAMAGE OF METAL ALLOYS

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The results of the experimental investigations of fatigue damage accumulation and redistribution of residual stresses are reported in this paper. Local measurements of the inelastic response under constant stress amplitude were used to observe two phenomena for selected alloys. It was found that fatigue damage accumulation and redistribution of residual stress affect the yield condition for the investigated materials. Yield condition with damage parameter and the parameter representing residual stress state are proposed. The damage parameter is calculated, basing on the definition given in the author's previous paper. It was also found that the yield condition and damage parameters are different for dynamic (cyclic loading) and for static (for unloaded material) conditions. Physical interpretation for the observed experimental results is given in this paper. Fatigue damage accumulation is divided into three phases: cyclic stabilisation, local increase of crystal defects density, formation and propagation of the crack. Local methods of strain measurements, together with dynamic measurements of damage parameter, were found to be crucial for proper observation of fatigue damage accumulation.

Key words: fatigue damage accumulation, yield condition, residual stress.

1. Introduction

It is widely accepted that accumulation of the fatigue damage affects mechanical properties of elastic-plastic materials. Since local damage of crystal structure and formation of material discontinuities (cracks) underlie this process, progress of damage should be manifested by changes of the yield condition. These changes are usually described by hardening rules. Among the parameters of this rule, there should be at least one related to the progress of damage. If this is the case, using well-known techniques of yield locus determination we should be able to investigate accumulation of the fatigue damage due to service loads.

Many theoretical models have been proposed to describe the damage-introduced changes of the yield condition. Most of them assume that due to damage accumulation, the plastic anisotropy is introduced into material. Damage-introduced plastic anisotropy is a very complex phenomenon, general form of the equation describing yield condition of anisotropic material consists of 21 constants [14]. Determination of all the constants is impossible in experimental manner using uniaxial tests like those of tension, compression or shear. Sophisticated complex-stress testing techniques have to be used to determine the yield locus of anisotropic material. For this reason, simplified models of plastic anisotropy, describing results of experiments with satisfactory accuracy are still searched. However many propositions, usually based on nonlocal yield condition of Drucker–Prager type, can be found in literature [15, 16] and [17], most of them lack any experimental verification. In a few cases such verification can be also found [18, 19] and [20]. In this paper, simple form of the yield condition taking into account the damage-induced plastic anisotropy is proposed. Such simple, physically motivated yield condition should be useful for investigation of damage accumulation, quantifying damage and estimation fatigue life of engineering materials subject to complex stress states and complicated loading histories. Moreover, anisotropy parameters used in this model are easy to determine with the use of simple tests (tension, compression or shear tests).

Many researchers still intensively investigate accumulation of the fatigue damage due to service loads for elastic-plastic materials such as metal alloys. Basic concepts like the SN curve and Linear Damage Rule (LDR) were formulated long time ago ([1–3]), but the use of such a simple method for prediction of the fatigue life can lead to enormous over- or underestimation. There are many reasons for such a situation. The most important is the fact that traditional testing technique is not suitable for observation of the damage progress during the test. Usually, the number of cycles to failure at a given amplitude of test controlling parameter (stress, strain or others) is the only result of such a test. Having no data concerning the damage progress, one can only assume a damage model (the manner, in which accumulation of the fatigue damage progresses). In the engineering practice, fatigue life prediction is in most cases (more than 90%) based on the Palmgren-Miner concept of linear damage rule (LDR); in a very few cases more complex theories are used (double-linear [4], non-linear [5]). Experimental verification of the applied damage model is crucial for obtaining accurate and credible fatigue life prediction. Any observation of the damage progress requires a definition of the measurable damage parameter. Changes of this parameter during the fatigue test can be plotted as a function of the load cycle number or cycle ratio. Such an experimentally determined plot, usually described as the damage curve, uniquely determines the proper damage model to be applied.

In publications, many definitions of the damage parameters can be found. A good review of the state-of-the art was given in [6]. A brief summary of the most popular damage quantifying parameters is shown in Table 1. Those parameters are divided into three groups: mechanical, physical and metallurgical. Mechanical parameters are usually measured in the strength laboratory. Some of them, such as the elasticity modulus, strain or stress amplitude changes under constant stress or strain amplitude, inelastic strain or strain energy, can be measured during the fatigue tests. Many attempts to investigate the fatigue damage accumulation have been made, but consistent, sufficiently accurate and credible data were not collected. The reason for such a situation lies in the measurement technique and it will be discussed later. Other mechanical parameters: fatigue limit, tensile strength, ductility, hardness, must be measured with the use of the destructive test (e.g. static tension), so on-line observation of the damage progress during the fatigue test using such parameters is not possible.

Table 1. Damage parameters.

Mechanical	Physical	Metallurgical
Elastic modulus	Velocity or attenuation of ultrasonic waves	Number of dislocations
Stress amplitude	Magnetic properties	Diameter of the dislocation cell
Strain amplitude	Electric potential	Shear band spacing
Inelastic strain amplitude	Temperature	Surface density of shear bands
Strain energy	Acoustic Emission	Crack front length
Others: fatigue limit, tensile strength, ductility, hardness	Others: density, X-ray diffraction, positron annihilation	Crack area

However, changes of mechanical properties are undoubtedly related to the damage progress, for most of them the damage-induced changes are very small. Some of them are influenced by other phenomena such as strain-hardening or residual stress redistribution. An additional disadvantage of the above-mentioned mechanical damage parameters is that they cannot be used for inspections of real construction components as non-destructive inspection techniques (NDI). Such inspection techniques are usually based on measurements of physical properties. Measurement of: velocity or attenuation of ultrasonic wave, magnetic properties [21], electric conductivity, temperature, acoustic emission, density, X-ray diffraction or positron annihilation, is widely used for detection of material physical discontinuities. There have also been attempts to use such techniques for detection of damage accumulation in the phase preceding formation of physical discontinuities in the investigated material ([22] and [23]). If such an indirect damage detection technique could be considered to be credible, one has to prove that the changes of the physical property in question are related to the fatigue damage accumulation. This can be achieved only by performing calibration of this technique with the use of a set of the calibration specimens (specimens with a known amount of damage introduced in laboratory environment). For the preparation of such specimens it is necessary to define measurable and physically based damage parameter, allowing accurate measurements of the accumulated fatigue damage.

Since it is well known that accumulation of the damage results in structural changes, direct methods of damage measurements can be based on structural observations [24]. Among the propositions of metallurgical damage parameters the best known are: the number of dislocations, diameter of the dislocation cell, shear band spacing, surface density of shear bands, crack front length or summary crack area. Some of those parameters correspond to the phenomenon of initial phase of the fatigue damage process (dislocations, shear bands) and the remaining ones characterize physical discontinuities (cracks) propagating in the material. However, crack is the most obvious and measurable effect of the fatigue damage process, but it can be detected only in the final phase of the process and sometimes it is too late to avoid disaster. Damage parameters based on the crack size measurements are well known; probably the most popular is KACHANOW'S definition [7]. His proposition: the surface density of cracks, was well received by theoreticians and was later developed by MURAKAMI [8] into a second-rank tensor representing damage of the material. From the practical point of view, this proposition has two serious disadvantages: first of all, the use of this definition is limited only to the final phase of the process, so it is useless for early damage measurements and, what is even more important in engineering practice, it is not measurable before the final failure of the construction component takes place.

It is well known that the stress concentration zone forms around the crack tip under load. For the elastic-plastic material such stress concentration results in formation of a plastic zone, even if the bulk of undamaged material is still stressed below the yield limit. It means that for the load which should give us theoretically an elastic response of the material, due to the local yielding at the crack tip, this response starts to be non-linear. This phenomenon can also be macroscopically observed as the decrease of the yield limit. For a constant stress amplitude cyclic loading, nucleation and growth of the micro-cracks should in this case produce the increase of local inelastic strain amplitude (hysteresis loop width).

If the crystal structure defect such as dislocation is generated, it increases locally the Stacking Fault Energy and lowers the energy necessary to activate (move) the slip system. This effect can be also macroscopically observed as decrease of the yield stress. It is well known, that theoretical yield stress calculated for a perfect crystal on the basis of elastic constants and geometry of the crystal cell, is several times greater than the one observed in the case of real materials.

This is because real materials always possess some defects of their crystal structure (under cyclic load it leads to stress-strain hysteresis, even if the maximum stress is below the yield stress). Generation of new defects under cyclic load must then result in a decrease of macroscopically measured yield stress. If the yield stress for the investigated material decreases, increase of the inelastic response (amplitude) is observed under a constant stress amplitude.

As it was mentioned before, there are two kinds of structural changes revealed in the material due to the progress of the damage: defects of crystal structure and physical discontinuities. A physically based damage parameter should be sensitive to the increase of both the number of defects and propagation of physical discontinuities. Proposition of such a parameter was given in author's paper [9]. This proposition is based on the assumption, that both the mentioned kinds of damage-induced defects result in the local decrease of yield stress. For real materials, yielding is a continuous process, it may start locally much earlier than yielding of the material bulk takes place. Although different definitions of the yield stress are available (offset yield limit, upper or lower yield limit), none of them is to be regarded as the stress state separating purely elastic deformation and material yielding. This makes accurate measurements of yielding onset very difficult. A much better technique is based on the application of the constant amplitude cyclic stress, with simultaneous measurement of inelastic strain amplitude. Increase of such an inelastic response is related to changes of the yield stress and reflects redistribution of residual stress and progress of damage.

It is very important that the same result can be expected due to the generation of crystal defects and formation of physical discontinuities. This result – increase of inelastic response under constant load amplitude - was successfully detected and observed using experimental technique described in paper [10]. The definition of damage parameter, based on analysis of the obtained data is given below:

$$D = \frac{\Delta\varepsilon^i - \Delta\varepsilon^i_0}{\Delta\varepsilon^i_f - \Delta\varepsilon^i_0}, \tag{1.1}$$

where $\Delta\varepsilon^i$ denotes the value of inelastic strain range for the load cycle under consideration, $\Delta\varepsilon^i_0$ stands for the initial value of inelastic strain range at the considered stress amplitude and the final value $\Delta\varepsilon^i_f$ corresponds to the instant of material damage. It is crucial that all the parameters included in the Eq. (1.1) must be determined with the use of local methods of strain measurements. Application of the traditional methods requires uniform stress and strain distribution in the specimens gauge part. If this distribution is not homogeneous, such methods fail and production of consistent results regarding damage accumulation is impossible.

2. Fatigue damage of elastic-plastic material as a three-phase process

Physical phenomena underlying accumulation of the fatigue damage are nowadays well recognized. Structural observations carried out by many researchers allowed to conclude that in early stages, the fatigue damage results from slipping of crystal defects. As it was mentioned in the last paragraph, even if the global stress is below the yield limit, zones of stress concentration can be found in polycrystalline materials. In this zone, a local slip of crystal defects can be observed. As the result of that local slip, new crystal defects are generated and the Stacking Fault Energy increases locally. This process can finally lead to a situation in which maximum principal service stress is greater than the local decohesion stress. As a result, crack starter in the form of physical discontinuity is formed. This process was observed in paper [10], with the use of inelastic response measurements performed during the fatigue test. It must be recalled at this point, that the performed tests were stress-controlled with constant amplitude. Fully reversible stress cycle ($R = -1$) was applied to avoid any ratchetting behaviour. All tests were performed in ambient temperature, the load oscillation frequency was 20 Hz. Specimen designed according to ASTM requirements is shown in Fig. 1. Hourglass design was used to concentrate the stress in the narrowest cross-section. For this cross-section, the fatigue damage accumulation rate was assumed to be the highest. Transversal extensometer was used to measure the change of the specimen diameter. Local transversal strain ε_{22} was calculated for the narrowest cross-section, and using the well-known formula:

$$\varepsilon_{11} = -\frac{\varepsilon_{22}}{\nu}, \tag{2.1}$$

axial strain ε_{11} was calculated. The value of Poisson's ratio ν was assumed to be -0.33 for the elastic range and -0.5 for the plastic range. The recorded data

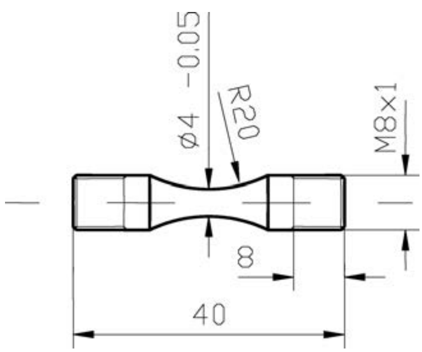

Fig. 1. Design of the specimen. Dimensions in mm.

(axial strain and stress) were used to plot the hysteresis loop shown in Fig. 2. Width of such a hysteresis loop $\Delta\varepsilon^i$ shown in this figure, called further the local inelastic response, was recorded for selected load cycles. Local inelastic response was plotted in a double logarithmic frame as a function of the load cycle number. As a result, the plot illustrating fatigue damage accumulation was obtained.

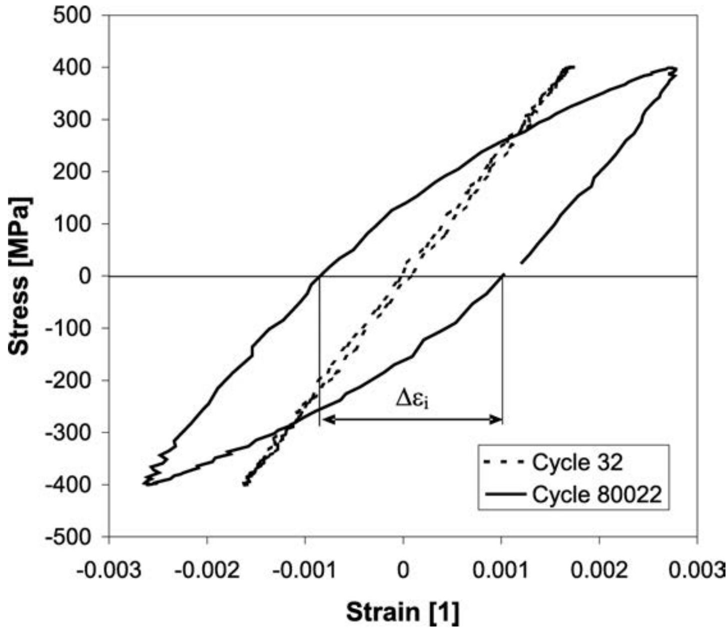

Fig. 2. Example of hysteresis loop recorded for selected cycles of load with indicated inelastic response for 80022 load cycle.

Examples of the recorded data are shown in Fig. 3 for the steel A 336 GR5 and in Fig. 4 for A 387 GR22. In both figures, one of the data sets represents a typical HCF test: for A 336 GR5 the stress amplitude 350 MPa is slightly above the endurance limit (342 MPa) and similarly, for A 387 GR22 the stress amplitude was set to 475 MPa (endurance limit 462 MPa). For such a small stress amplitude, the process of fatigue damage accumulation can be divided into three phases differing in the rate of inelastic response increase. During the first phase, no progress of damage can be observed, response of the material is quasi-elastic with constant width of the hysteresis loop – inelastic strain range for a cycle of load remains constant. At the end of the first phase, due to local increase of the Stacking Fault Energy new defects of crystal structure begin to be generated. This phenomenon, the local increase of crystal defects density, continues during the second phase of the fatigue damage process occupying about 80% of the fatigue life. As the local density of defects reaches the critical value,

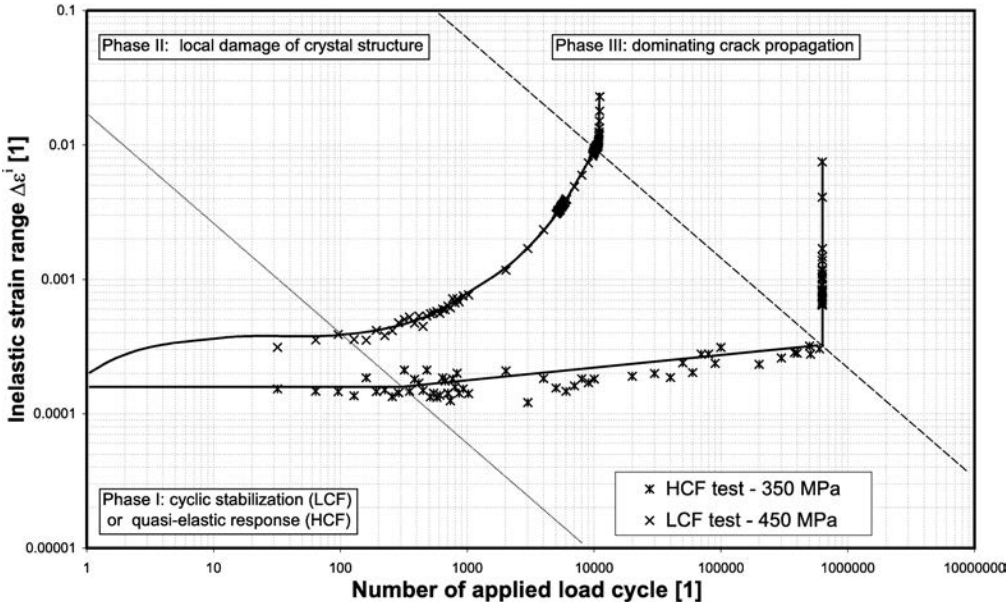

Fig. 3. Inelastic response as a function of the applied load cycle number for HCF and LCF test – A336 GR5 steel.

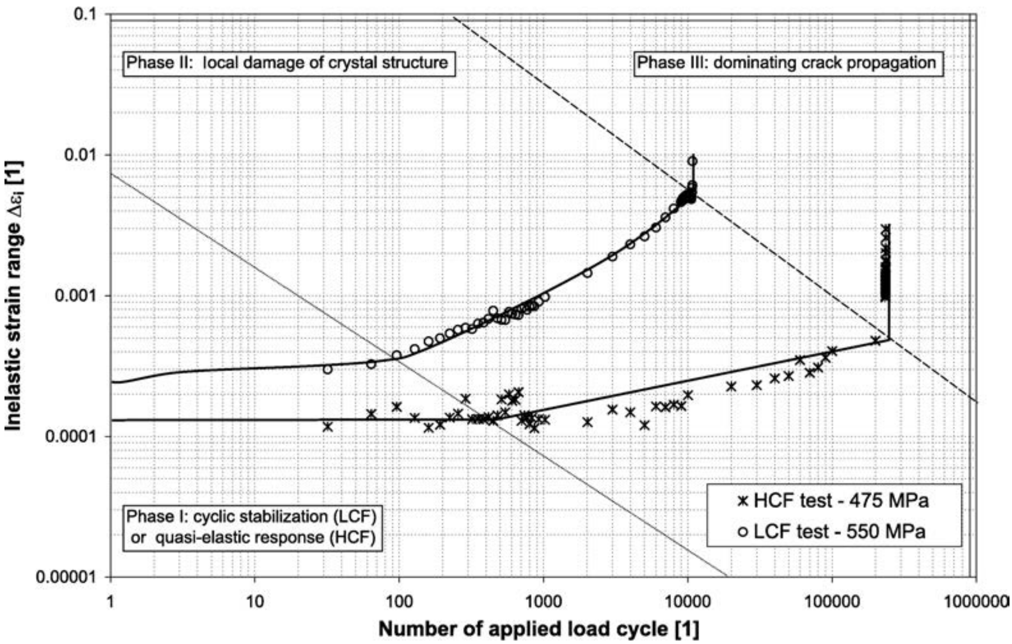

Fig. 4. Inelastic response as a function of the applied load cycle number for HCF and LCF test – A387 GR22 steel.

the Stacking Fault Energy can be locally so high that service stress may exceed the decohesion stress, discontinuity of the material is formed and the second phase of the process ends. At the beginning of the third phase, coalescence of few micro-discontinuities leads to formation of the dominant crack. This crack propagates during that phase until it reaches the critical size, when under the service load unstable propagation of the crack is triggered (critical value of the stress intensity factor K_c is exceeded) and final failure of construction component takes place.

For the HCF test, the defects generation and increase of defects density is strongly localised and the possibility of dislocation movement is limited. That situation is different in the case of a LCF test, when the bulk of material yields and the dislocation can move. This movement leads to redistribution of residual stress. Redistribution of residual stress can be macroscopically observed as a transient stabilization of the material response (saturation of the hysteresis loop). However, due to technical limitations, the initial hysteresis loops were not recorded in the case of tests shown in Figs. 3 and 4, special tests were performed to investigate redistribution of residual stress manifested by cyclic stabilisation. Results of those tests will be reported in the following Sec. 3.

3. Dynamic and static balance of defected crystal structure

The test program shown in Fig. 5, was in this case simple. Constant stress amplitude tests were performed in ambient temperature with the frequency of 1 Hz. The initially applied amplitude was selected below the endurance limit to obtain a quasi-elastic behaviour. After 50 load cycles with continuous recording of the stress and strain, loading was stopped for about 5 minutes, stress amplitude was increased by 25 MPa in case of A336 GR5 steel (50 MPa in case of A387 GR22) and cycling was restarted with cycle counter set to zero. All tests were performed in a sequence, higher amplitude following the lower one after 5 minutes pause. Strain measurement technique was described in the previous section. This procedure was repeated until the stress amplitude almost reached the yield limit. For each recorded stress-strain loop, the inelastic strain range was calculated. Results of the tests are shown in Figs. 6 and 7 for A336 GR5 and A387 GR22 steel respectively.

It can be seen in both figures that for low stress amplitudes (close to the endurance limit), the material response is stable. Width of the hysteresis loop (inelastic strain) remains almost constant. We can assume that there is no damage progress at this amplitude. For higher stress amplitudes, inelastic response starts to increase with the applied load cycles. Two phases of the process can be observed. During the first phase, rate of this process is higher and during the second phase, the decrease of inelastic response rate can be observed. This phe-

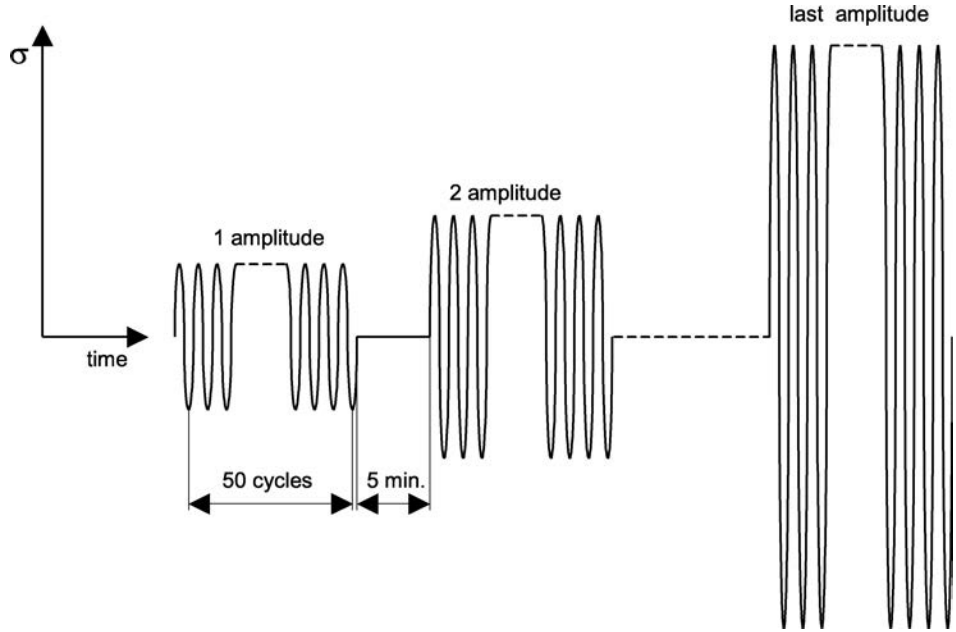

FIG. 5. Test program – cyclic stabilisation and redistribution of residual stress.

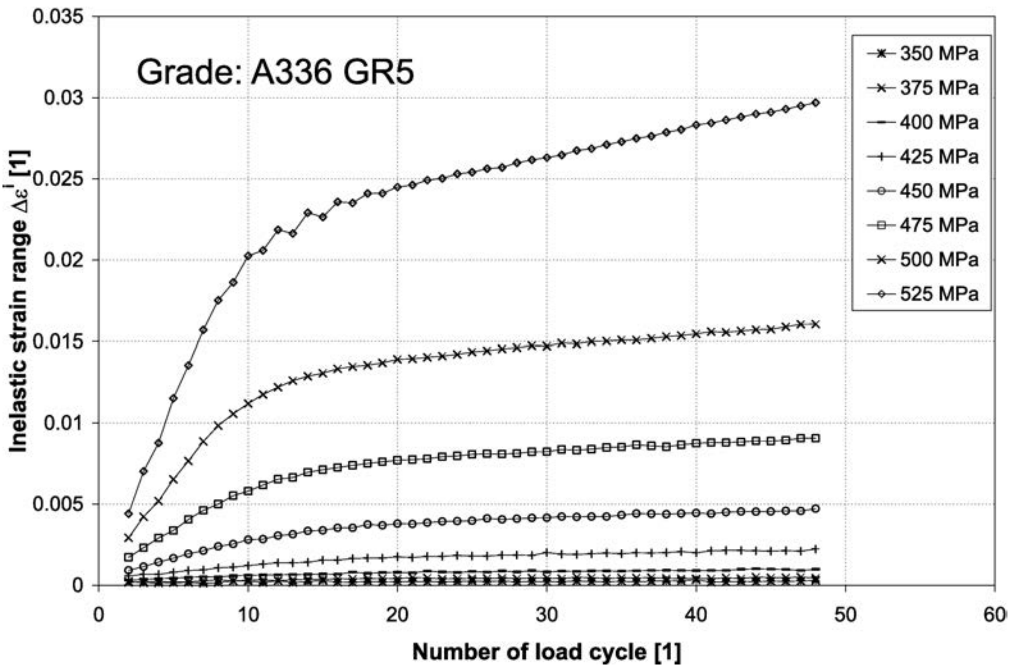

FIG. 6. Inelastic response as a function of the applied load cycle number for A336 GR5 steel.

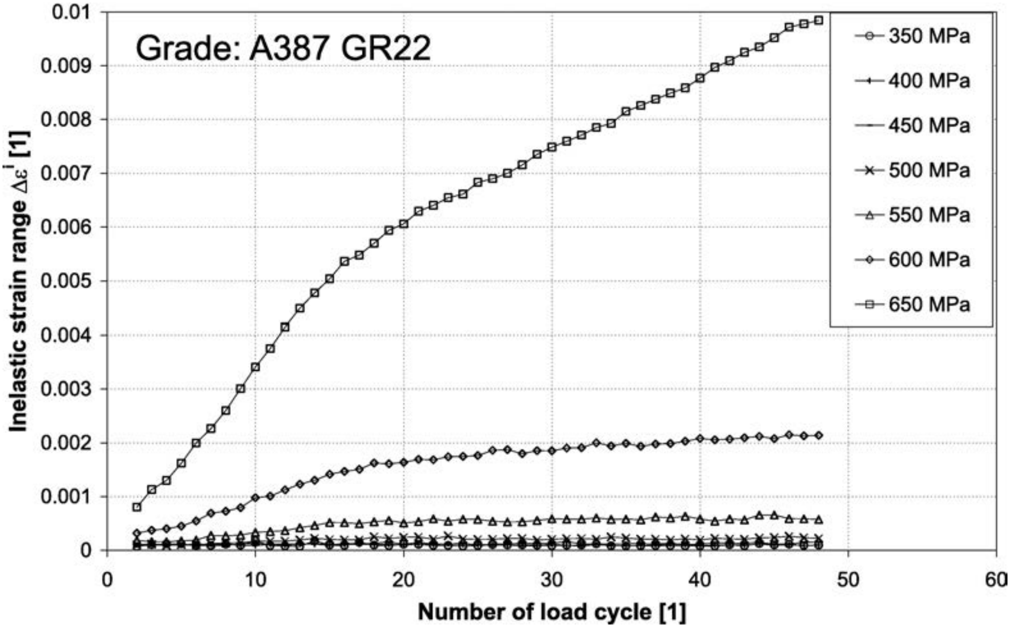

FIG. 7. Inelastic response as a function of the applied load cycle number for A387 GR22 steel.

nomenon can be explained as follows: during the first phase, the dislocations can move on a relatively long distance and this leads to redistribution of the residual stress. Movement of the dislocations leads to their mutual locking and finally, a certain quasi-stable state is achieved. Using the traditional testing method, this phenomenon would be observed as the stabilisation of the hysteresis loop. However, if the material response is observed locally, as in the case of the investigation presented, no saturation of the hysteresis loop (stabilisation of inelastic response) can be detected. This is because the fatigue damage accumulation is a localised phenomenon, and a proper (local) method of observation has to be used to obtain credible results. If there is no cyclic stabilisation, we can assume that the fatigue damage process in case of a LCF test starts immediately. During the first phase of the process, those two phenomena: redistribution of residual stress and accumulation of the fatigue damage overlap. This phase ends when the possibility of dislocation movement is strongly limited due to their mutual locking. Further increment of the inelastic response can be attributed to local generation of new defects appearing in agglomeration of dislocations – the rate of the process slows down and stabilizes.

It has to be emphasized, that each time when loading starts after a 5 minutes pause, that kind of dynamic balance between the redistributed residual stress and the locally increased damage of the material has to be achieved. In Figs. 6 and 7 one can observe that at the end of each LCF test, inelastic strain amplitude reaches some value. After a short pause (approximately 5 minutes), this value decreases significantly – static balance between the residual stress and local damage of the material is achieved due to diffusion of the defects. If loading of the material is resumed with a higher stress amplitude, dynamic balance (cyclic saturation) is achieved again.

Transition between dynamic and static balance of local damage and residual stress was observed in one of the author's papers [11] for aluminium alloy, called PA6 according to Polish Standard. Simple test was repeated for a specimen cut out from one rod of the material. The test program is shown in Fig. 8. After stretching the specimen to 0.04 mm/mm of axial strain, direction of load was reversed by 180 degrees in the stress space (transition to compression). Tensile yield stress, measured for plastic offset 0.00001, 0.0001, 0.001, 0.01 mm/mm was 235.83, 236.85, 237.21 and 247.35 MPa, respectively. Subsequent compressive loading was performed after different intervals of time elapsed from the moment when 0.04 mm/mm of tensile strain was achieved: 0.25, 0.5, 2 and 380 hours. In Fig. 9 change of the yield limit under compressive load is plotted as a function of time. It can be seen that for a small offset, the yield limit changes with time. These changes can be attributed to diffusion of defects that results in redistribution of residual stress and leads to a new state of balance achieved with time. This balance is different than the state before prestraining, because during plastic flow of the material some of the defects moved to new positions. What is even more important, new defects were generated during this process. Movement and generation of new defects should affect the yield condition of the

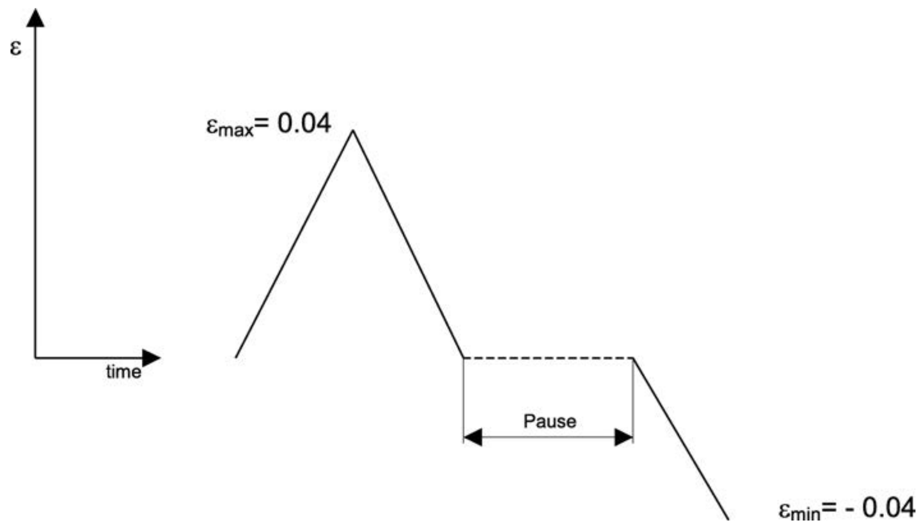

Fig. 8. Test program – material recovery.

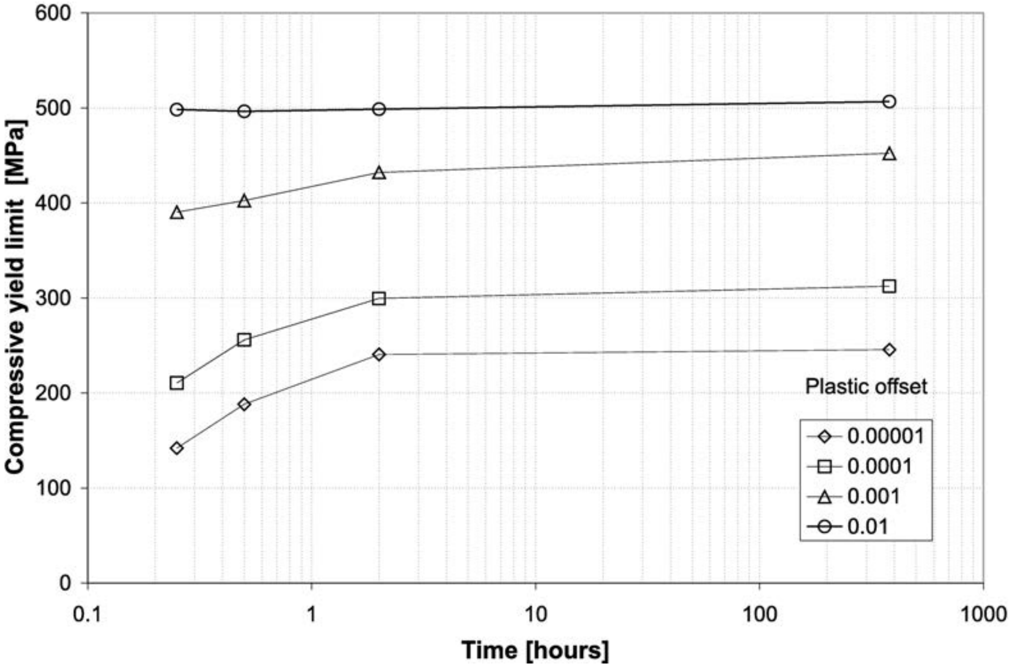

FIG. 9. Changes of the compressive yield limit as a function of the time elapsed after initial tensile prestrainig for PA6 aluminium alloy.

material in question. That change should be manifested by plastic anisotropy (directional dependence of the yield limit). In a general case, for complex stress states the yield condition is given by the following equation [12, 13]:

$$F\left(\sigma_{ik}-\alpha_{ik}\right)=k^{2}, \tag{3.1}$$

where two anisotropy parameters α_{ik} and k correspond to kinematic and isotropic strain hardening. First of them, α_{ik}, is usually identified as a tensor representing the residual stress state. We can assume that its changes should reflect redistribution of this residual stress caused by movement of the defects. The second anisotropy parameter k, represents increase (hardening) or decrease (softening) of the yield surface. Generation of new defects should be in this case manifested by the decrease of that parameter (yield limit should be smaller for all the loading paths in the stress space). Of course, it will also be affected by annihilation and diffusion of defects after unloading.

At this point we can assume (disregarding other effects such as element segregation, grain boundary diffusion of phase transformations) that these two mentioned anisotropy parameters are related to two phenomena: redistribution of residual stresses and accumulation of damage. For a simple case of uniaxial loading (tension – compression) we can easily determine the changes of parameters in question, with the use of tensile and compressive tests. In paper [11] the yield condition (3.1) for uniaxial stress states was simplified to the following form:

$$\sigma_{11} - \alpha_{11} = \pm\sqrt{3} \cdot k, \tag{3.2}$$

where k denotes the yield stress in shear. If Y_{11} denotes the tensile yield stress and Z_{11} denotes the compressive yield stress, we can determine the values of anisotropy coefficients in the following form:

$$\alpha_{11} = \frac{Y_{11} - Z_{11}}{2}, \qquad k = \frac{Y_{11} + Z_{11}}{2\sqrt{3}}. \tag{3.3}$$

Time changes of the anisotropy parameters after 0.04 mm/mm prestraining in tension and unloading are shown in Figs. 10 and 11.

In Fig. 10 the parameter α_{11} representing residual stress is shown as a function of time after prestraining. As it was mentioned, yield limit in tension and compression was determined for four values of the plastic offset. It can be seen that, immediately after deformation, value of residual stress is the highest for

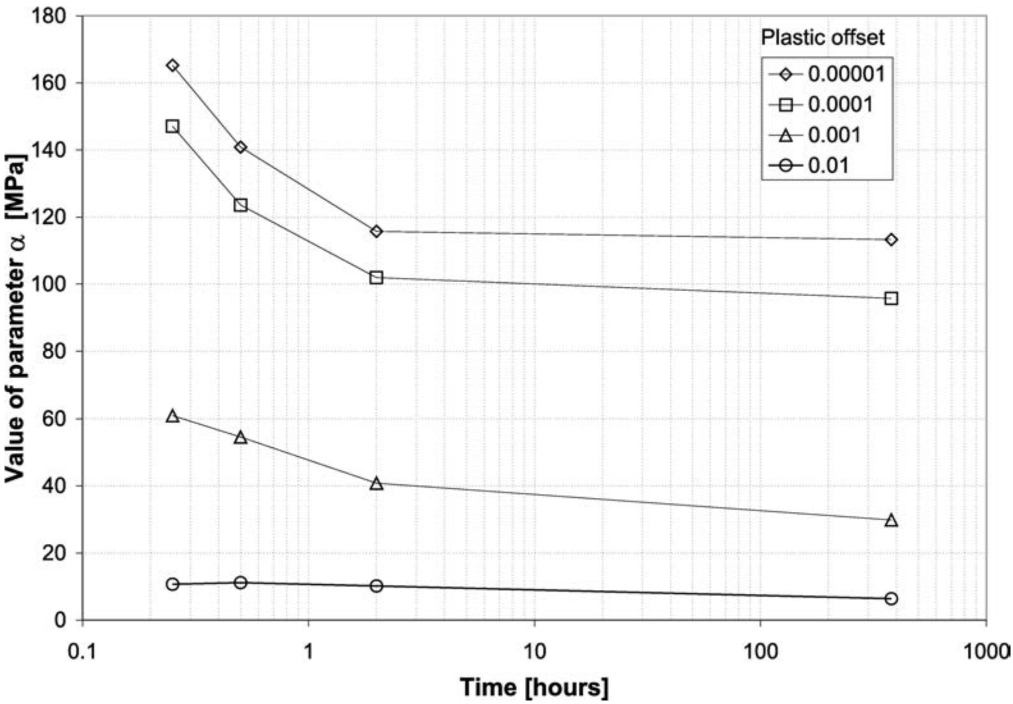

FIG. 10. Anisotropy parameter α representing residual stress as a function of the time elapsed after prestraining.

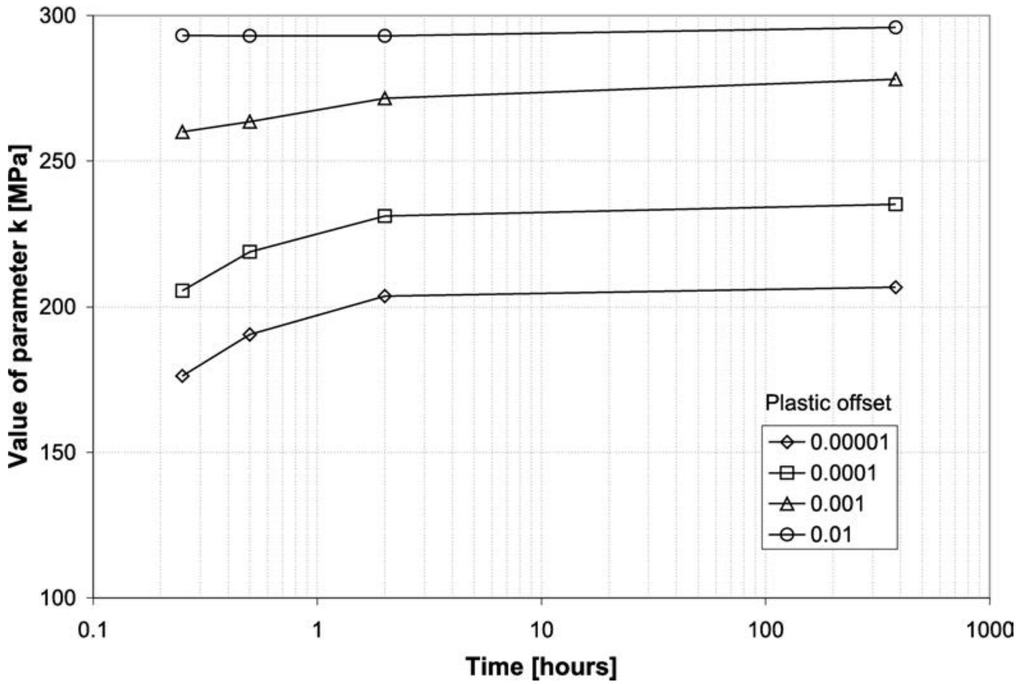

FIG. 11. Anisotropy parameter k representing damage accumulation as a function of the time elapsed after prestraining.

all the offset definitions. It decreases with time due to recovery of the material. That recovery is performed by diffusion of defects in the field of the residual stress. After approximately 2 hours, static balance of material is achieved.

The second one of the above-mentioned parameters k representing accumulation of the damage also undergoes similar changes shown in Fig. 11. In this case, increase of the value can be observed indicating transition from dynamic to static balance of the material. It has to be mentioned that, in contrast with cyclic loading, where significant amount of damage was introduced to the material, in the case of static prestraining, mainly slips of the crystal defects took place. This means that in case of static balance, the value of k after prestraining was close to the one before loading. Time changes of parameter k shown in Fig. 11 can be attributed mainly to transition from the dynamic to static balance of crystal structure. This effect is consistent with the results obtained for A336 GR5 steel. For dynamic balance, the observed inelastic strain was much greater than the static one. It means that yield limit for static conditions is greater than that for dynamic conditions. To observe the progress of damage, we shall compare static or dynamic anisotropy parameters. However, we should be careful to avoid mixing the static and dynamic parameters since in this case the inconsistent picture of the damage accumulation would be obtained.

4. Distribution of residual stress and local character of damage accumulation

As it was suggested in the previous section two anisotropy parameters: α and k can be identified with the residual stress state and with the accumulated damage. The problem is that both phenomena are not uniformly distributed in the investigated bulk of the material. This makes the measurements of the accumulated damage extremely difficult. Traditional material testing technique assumes that strain measurement is performed in the bulk of material with uniform strain distribution – gauge part of the specimen. Using an extensometer it is possible to measure the displacement between two points of the specimen's gauge part and dividing it by the extensometer gauge length (measurement base), one can obtain the value of strain for the stressed material. This works well if the gauge part of the specimen is uniformly deformed. Measurements of elastic constants like Young's modulus or Poisson's ratio are typical examples of such measurements. Moreover, initial yielding (before deformation localisation onset) exhibits tendency to homogenise the strain field due to mutual interaction of dislocations. Therefore, the yield limit can be properly determined using the traditional material testing technique. Since stress is usually determined as the load divided by the area of gauge part cross-section, we can consider all the above-mentioned parameters as credible. The homogeneity of stress and strain distribution cannot be assumed in case of damage accumulation. This process reveals a tendency to localisation: if it starts at some spot of the material, it develops there. High Stocking Fault Energy for a such spot of material facilitates generation of new defects. It is very difficult to estimate the size of damaged area but strain distribution is obviously not uniform in the gauge part of the specimen. Additional complication is that residual stress distribution is related to damage distribution. Balance between the two phenomena is different for static and dynamic conditions. If we want to obtain a consistent picture of damage accumulation, all the measurements of anisotropy parameters should be performed in a static or dynamic manner.

Assuming that the two mentioned damage parameters were measured for static conditions, the progress of damage would be hardly observable. For this reason in paper [9] and [10] the value of damage parameter was measured for dynamic conditions. In this case we can postulate the following form of the yield condition:

$$F\left(\sigma_{ik}-\alpha_{ik}\right)=\left[\left(1-\beta D\right)k_0\right]^2, \tag{4.1}$$

where D stands for the damage parameter and β is a coefficient representing reduction of the yield limit. This coefficient can be defined as follows:

$$\beta=\frac{k_f}{k_0}, \tag{4.2}$$

where k_f stands for the yield stress in shear at material failure (formation of crack) and k_0 stands for the yield stress in shear corresponding to the virgin (undamaged) material assuming isotropy. It has to be stressed once more that there are two conditions necessary to obtain credible measurements results: all the parameters have to be measured locally and mixing of static and dynamic measurements must be avoided.

5. Conclusions

The following conclusions can be drawn from this study:

- Changes of plastic anisotropy are related to accumulation of damage and residual stress redistribution.
- Accumulation of damage and residual stress are local phenomena. Assuming uniform distribution of stress and strain in a Representative Volume Element of the material in order to use traditional material testing techniques, can result in inconsistent picture of the investigated phenomenon.
- State of plastic anisotropy is different for static and dynamic conditions. Transition from dynamic to static balance is achieved after load removal. Transition from static to dynamic balance is usually observed for cyclic loading as stabilisation of the hysteresis loop after the initial load cycles. In case of local measurements of material inelastic response, decrease of inelastic strain changes rate was observed instead of cyclic stabilisation.
- To observe accumulation of damage and distribution of residual stress, local methods of inelastic strain measurements are necessary. Such methods nowadays exist and can be applied in material testing. Using traditional material testing techniques (assuming uniform stress and strain distribution for the gauge part of the specimen), results in obtaining inconsistent picture of damage accumulation.
- To obtain consistent picture of damage accumulation, measurement of anisotropy parameters should be performed in static or dynamic manner. Mixing of the two kinds of measurements can lead to many misunderstandings.
- To describe the damage progress, yield condition in the form given in this paper can be used. Such condition represents local state of the material. Measurements performed on the Representative Volume Element (RVE) can be regarded as the averaged result. Result of such measurements depend on the measurement technique (measurement base and position of extensometer, stress distribution etc.).

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