ROZPRAWY INŻYNIERSKIE • ENGINEERING TRANSACTIONS • 26, 3, 457—470, 1978
Polska Akademia Nauk • Instytut Podstawowych Problemów Techniki

FINITE PLASTIC DEFORMATIONS OF ORTHOTROPIC CIRCULAR PLATES UNDER GAUSSIAN IMPULSE

S. ANANTHA RAMU and K. JANARDHANA IYENGAR (BANGALORE)

The permanent deflection of an orthotropic circular plate subjected to impulse loading is presented. The impulse is assumed to impart a transverse axisymmetric velocity having spatially a general Gaussian distribution. The simultaneous influence of the membrane forces and bending moments is considered in predicting the deformations. It is concluded that the simple bending theory overestimates the final plastic deformation and the time of deformation of the plate. The order of over estimation is not very much dependent upon the spatial distribution of the impulse.

NOTATIONS

r, θ	Radial circumferential coordinates,
b	radius of circular plate,
H	plate thickness,
σ	standard deviation,
a	$1/(\sqrt{2}\sigma)$,
c	Gaussian parameter $(a \cdot b)$,
M_r, M_θ	radial and circumferential bending moments,
N_r, N_θ	radial and circumferential membrane forces,
$M_{r0}, M_{\theta 0}$	yield moments in radial and cirmuferential directions,
$N_{r0}, N_{\theta 0}$	yield membrane forces in radial and cirmuferential directions,
$\dot{K}_r, \dot{K}$	radial and circumferential curvature rates,
$\dot{\varepsilon}_r, \dot{\varepsilon}_\theta$	radial and circumferential strain rates,
K	$M_{\theta 0}/M_{r0}$ or $N_{\theta 0}/N_{r0}$,
t	time,
t_1	time at which hinge circle comes to the centre without membrane forces,
$\bar{t}$	time at which hinge circle comes to the centre with membrane forces,
t_f	time of cessation of plate motion,
$W(r, t)$	plate deflection in transverse direction,
$\dot{W}(r, t)$	plate velocity,
V_0	$\dot{W}(0, 0)$,
u	displacement in radial direction,
x	r/b,
y	$\rho(t)/b$,
y_0	ρ_0/b,
J_0, J_1	Bessel functions of the first kind of zero and first order,
α_s	S roots of equation $J_0(\alpha_s)=0$,
S_s	α_s/b,

β $(V_0^2 b^2/M_{\theta 0} H)$,
μ mass per unit area of plate material,
$\delta(t)$ Dirac-δ function,
$\rho(t)$ radius of hinge circle at time t,
ρ_0 radius of hinge circle at $t=0$,
$(\dot{})$ $\frac{\partial}{\partial t}$,
$()'$ $\frac{\partial}{\partial r}$.

1. Introduction

It is well known that the simple bending theory is inadequate to describe the response of impulsively loaded rigid plastic structures. Significant improvement in theoretical predictions can be achieved if the influence of membrane forces is considered along with bending moments. Such an investigation was carried out by Symonds and Mentel [1] for the impulsively loaded, axially restrained rigid plastic beams. Florence [2] examined the behaviour of simply supported circular plates under uniformly distributed impulse and reported permanent deflections which are considerably smaller than those obtained by Wang [3] by the simple bending theory. Because of the difficulties in obtaining exact solutions when finite deflections interacting with membrane forces are retained, attempts were made to develop approximate bounding methods. Wierzbicki [4] extended Martin's [5] upper bound theorem on displacements in order to take into account the influence of membrane forces. This technique is of limited use because it is dependent on the availability of an exact static solution for the corresponding dynamic problem using Von Mises' yield condition. An extensive study of the inffuence of geometry changes on the behaviour of dynamically loaded, rigid plastic beams, plates and shells has been carried out by Jones [6] to [10]. A detailed literature review of the dynamic response of structures published by Jones [11] focuses attention primarily on the influence of finite disacements or geometry changes and material strain rate sensitivity.

All the reported theoretical investigations which retain the inffuence of finite deflections assume the material of the plate to be isotropic and the dynamical load to be uniformly distributed over the surface of the plate. However, many plates used in practice are of stiffened construction in order to achieve a high strength-to-weight ratio. Such plates can be considered to exhibit polarorthotropy. In addition, the applied dynamical load may have a general spatial distribution which may significantly influence the plastic response of the structure.

The objective of this paper is to present the plastic response of an impulsively loaded, simply supported, orthotropic circular plate considering the influence of membrane forces along with the bending moments. The impulse is assumed to impart instantaneously a transverse velocity which is axisymmetric with a general Gaussian distribution spatially. The solutions are obtained for a variety of distributions of impulse ranging from a uniform distribution over the entire plate to a concentrated impulse at the centre. The solution presented is an extension of an earlier work by th authors [12] in which the membrane forces were disregarded.

2. Basic equations

The problem of a thin, orthotropic circular plare under impulsive loading is considered. The impulse is assumed to impart an instantaneous velocity having a general Gaussian distribution (Fig. 1) given by

(2.1) $$\dot{W} = V_0 \, e^{-c^2 x^2} \delta(t).$$

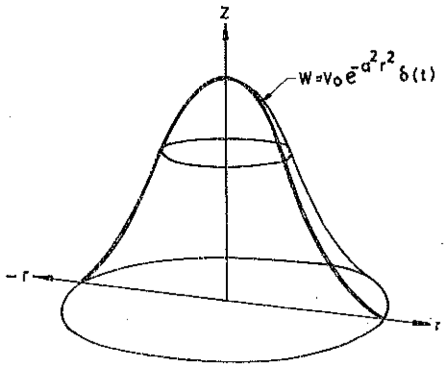

Fig. 1. Gaussian velocity distribution.

The material of the plate is assumed to be rigid plastic and the principal directions of anisotropy coincide with the radial and tangential directions. Membrane forces and bending moments acting on an element of the plate are shown in Fig. 2. The basic equations governing the behaviour of the plate are

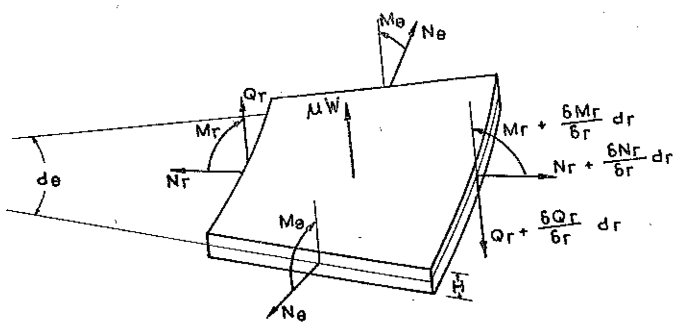

Fig. 2. Forces and moments.

(i) Equations of motion:

(2.2) $$(rM_r)' - M_\theta + rN_r W' = \int_0^r \mu r \ddot{W} \, dr, \qquad (rN_r)' - N_\theta = \mu r W' \ddot{W}.$$

(ii) Strain-displacement relations:

$$\dot{\varepsilon}_r = \dot{u}' + W' \dot{W}', \quad \dot{\varepsilon}_\theta = \dot{u}/r, \tag{2.3}$$

$$\dot{K}_r = (l+u') \dot{W}'' + \dot{u} W'' - \dot{u}'' W' - u'' \dot{W}, \quad \dot{K}_\theta = \dot{W}'/r. \tag{2.4}$$

(iii) Boundary conditions:

$$M_r(b, t) = 0, \quad \dot{W}(b, t) = 0. \tag{2.5}$$

3. Modes of deformations

The deformation of the plate takes place in two phases [12]. The first phase starts with the initiation of the plastic hinge at radius ρ_0 at time $t=0$. If the yield moment in the radial direction is greater than in the circumferential direction (i.e. $M_{r0} > M_{\theta 0}$), the first phase ends at a time $t=\bar{t}$, when travelling hinge comes to the centre. During the second phase the plate continues to deform as an inverted cone with a stationary hinge at $r=0$, until all the kinetic energy imparted to the plate is completely dissipated by plastic deformation. Alternatively, if the yield moment in the circumferential direction is greater than that in the radial direction (i.e. $M_{\theta 0} > M_{r0}$) at time $t=\bar{t}$, the travelling hinge does not shrink to the centre of the plate but has a radius ρ_1. This radius ρ_1 is dependent upon the parameter K which is the ratio of the circumferential yield moment to the radial yield moment and is given by the equation

$$2y_l^3 (K-l) - 3y_l^2 + K - l = 0.$$

During the second phase the plate continues to deform in the shape of an inverted truncated cone and comes to rest when the energy imparted to the plate by the impulse is completely dissipated by plastic deformation.

4. Yield condition

The equilibrium equations (2.2) must be solved using a four-dimensional yield surface relating N_r, N_θ, M_r and M_θ. Jones [6, 7] has shown that the simplified two-moment limited interaction surface proposed by Hodge [13] gives reasonably good results. Hence the same simplified yield condition and associated flow rules with a modification for orthotropy as given in Ref. [14] is used in this investigation. The yield condition and the associated flow rules are (Fig. 3):

Case 1.

$$M_{r0} > M_{\theta 0}, \quad N_{r0} > N_{\theta 0},$$

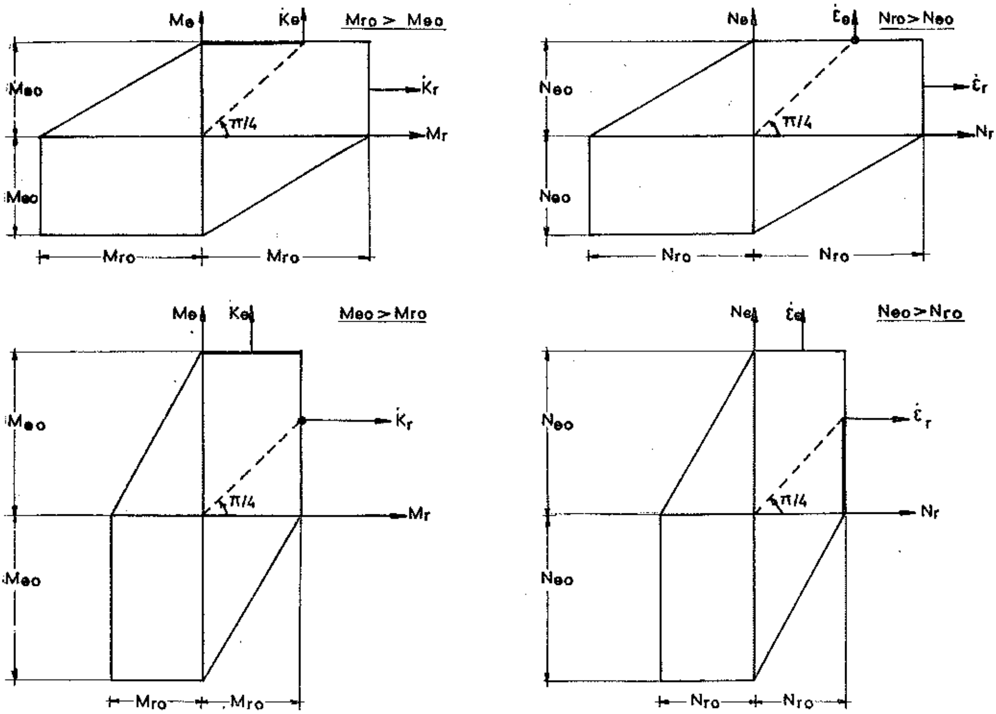

FIG. 3. Yield condition.

For $0 \leqslant r \leqslant \rho(t)$

$$
\begin{aligned}
&M_r = M_\theta = M_{\theta 0}, \\
&N_r = N_\theta = N_{\theta 0}, \\
&\dot{K}_r = 0, \quad \dot{K}_\theta \geqslant 0, \\
&\dot{\varepsilon}_r = 0, \quad \dot{\varepsilon}_\theta \geqslant 0.
\end{aligned}
\tag{4.1}
$$

For $\rho(t) \leqslant r \leqslant b$

$$
\begin{aligned}
&M_\theta = M_{\theta 0}, \quad 0 \leqslant M_r \leqslant M_{\theta 0}, \\
&N_r = N_{\theta 0}, \quad 0 \leqslant N_\theta \leqslant N_{\theta 0}, \\
&\dot{K}_r = 0, \quad \dot{K}_\theta \geqslant 0.
\end{aligned}
\tag{4.2}
$$

Case 2.

$$M_{\theta 0} \geqslant M_{r0}, \quad N_{\theta 0} \geqslant N_{r0},$$

For $\rho(t) \leqslant r \leqslant b$

$$
\begin{aligned}
&M_\theta = M_{\theta 0}, \quad 0 \leqslant M_r \leqslant M_{r0}, \\
&N_r = N_{r0}, \quad 0 \leqslant N_\theta \leqslant N_{r0}, \\
&\dot{K}_r = 0, \quad \dot{K}_\theta \geqslant 0, \\
&\dot{\varepsilon}_r > 0, \quad \dot{\varepsilon}_\theta = 0.
\end{aligned}
\tag{4.3}
$$

5. Solution

Case 1.

$$M_{r0} > M_{\theta 0}, \qquad N_{r0} > N_{\theta 0}.$$

Phase 1. $0 \leqslant t \leqslant \bar{t}$

At some time t after the application of the impulse the circumferential plastic hinge will have a radius $\rho(t)$. The motion of the plate is then described with the following velocity field:

$$\dot{W} = V_0 e^{-a^2 \rho^2} \quad \text{for} \quad 0 \leqslant r \leqslant \rho(t),$$

(5.1)

$$\dot{W} = V_0 e^{-a^2 \rho^2} \left[\frac{b-r}{b-\rho(t)} \right] \quad \text{for} \quad \rho(t) \leqslant r \leqslant b.$$

It is to be noted that $\dot{W}$ as given by Eq. (5.1) is continuous across the travelling hinge and satisfies all the flow conditions and the boundary conditions as well.

As a first approximation, if the membrane forces are neglected, the position of the plastic hinge at any time can be shown [12] to be given by the equation.

(5.2)
$$t = t_1 (B_1 + B_2)/B_3,$$

where t_1 is the time at which the plastic hinge comes to the centre and

$$B_1 = [1 - 6(1-y_0) c^{-2} + y_0 - 5y_0^2 + 3y_0^3] e^{-c^2 y_0^2} - 3/\pi\, c^{-3} \operatorname{erf}(cy_0),$$

(5.3)

$$B_2 = [-1 + 6(1-y) c^{-2} - y + 5y^2 - 3y^3] e^{-c^2 y^2} + 3/\pi\, c^{-3} \operatorname{erf}(cy_0),$$

$$B_3 = -1 + 6c^{-2} + B_1.$$

In Eqs. (5.3) y_0 is the position of the hinge at $t=0$. This is determined as a function of the Gaussian parameter c as in Ref. [12].

As the deflection in the first phase of deformation will not be too large, the time function for the case where the influence of membrane forces is taken into account can be written analogous to Eq. (5.2) as

(5.4)
$$t = \bar{t}(B_1 + B_2)/B_3,$$

where $\bar{t}$ is the time at which the plastic hinge comes to the centre of the plate and is determined later.

The region of the plate between the radii ρ_0 and b lies in the plastic regime given by Eqs. (4.2) for all time, whereas the region between the radii ρ_0 and $\rho(t)$ lies in the plastic regime given by Eqs. (4.1) up to a time t_r, and for $t > t_r$ it lies in the plastic regime given by Eqs. (4.2). Hence, using Eqs. (5.1) the deflection of the plate is given by

$$W(r,t) = \int_0^t V_0 e^{-a^2 \rho^2} \left(\frac{b-r}{b-\rho} \right) dt \quad \text{for} \quad \rho_0 \leqslant r \leqslant b,$$

(5.5)

$$W(r,t) = V_0 e^{-a^2 \rho^2} t_r + \int_{t_r}^t V_0 e^{-a^2 \rho^2} \left(\frac{b-r}{b-\rho} \right) dt \quad \text{for} \quad \rho(t) \leqslant r \leqslant \rho_0(t).$$

Combining Eqs. (5.5) and $(2.2)_1$ and integrating the resulting equation of motion, we get the distribution of radial moment in the two regions of the plate. Using the continuity condition of radial moment at $r \neq \rho_0$, we get an equation of the form

(5.6)
$$\frac{V_0 \bar{t}}{H} = -\frac{B_3}{8B_4} + \frac{B_3}{8B_4}\left[\frac{4B_4 B_5}{3}\beta + 1\right]^{\frac{1}{2}},$$

where B_4 and B_5 are known functions of ρ_0 and c. The actual expressions are not given here as they are lengthy. For the particular case of a uniformly distributed impulse ($c \to 0$) Eq. (5.6) reduces to Eq. (25) of Ref. [6]. At $t = \bar{t}$, the deflection of the plate is obtained from Eqs. (5.4) and (5.5) as

(5.7)
$$W(r, \bar{t}) = \frac{V_0 \bar{t}}{B_3}(1 - x) B_6 \quad \text{for} \quad y_0 \leqslant x \leqslant 1,$$
$$W(r, \bar{t}) = \frac{V_0 \bar{t}}{B_3} B_7 \quad \text{for} \quad 0 \leqslant x \leqslant y_0,$$

where

$$B_6 = 1.5c^{-2} - 0.5 - (1.5y_0^2 + 1.5c^{-2} - 0.5 - y_0)\, e^{-2c^2 y_0^2},$$
$$B_7 = [-1 - x + 6(1 - x)\, c^{-2} + 5x^2 - 3x^3]\, e^{-c^2 x^2} + 3(\pi)^{\frac{1}{2}} c^{-3} \operatorname{erf}(cx) +$$
$$+ B_1 + (1 - x)\, [(1.5c^{-2} - 0.5 - (1.5x^2 + 1.5c^{-2} - 0.5 - x)\, e^{-2c^2 x^2}].$$

Phase 2 $\bar{t} \leqslant t \leqslant t_f$

After the time $t = \bar{t}$, it is reasonable to assume that deflections are large in comparison with plate thickness. Hence the rate of energy dissipated in stretching is more than that by bending. The plate could be considered to behave as a membrane and the second equation $(2.2)_2$ can be reduced to the standard Bessel equation.

(5.8)
$$W'' + W'/r = \mu \ddot{W}/N_{\theta 0}.$$

Following the conditions that deflection must be finite at $r = 0$ and zero at $r = b$, the solution of Eq. (5.8) is given by

(5.9)
$$W(r, t) = \sum_{S=1}^{\infty} (E_s \cos \gamma_s t + F_s \sin \gamma_s t)\, J_0(\alpha_s x),$$

where $\gamma_s = \left(\frac{N_{\theta 0}}{\mu}\right)^{\frac{1}{2}} \lambda_s$.

The constants E_s and F_s are evaluated using the condition that at $t = \bar{t}$ the deflection and the velocity from the first and second phases must be continuous. The plate comes to rest at time t_f when the velocity at every point of the plate vanishes. Using Eq. (5.9) we get

(5.10)
$$t_f = \gamma_s^{-1} \tan^{-1}(F_s E_s^{-1}).$$

Combining Eqs. (5.9) and (5.10) the final deformed shape of the plate is given by

$$\frac{W_f}{H}\sum_{S=1}^{\infty}[\Delta^2 B_{1S}^2+\beta B_{2S}^2]^{\frac{1}{2}},$$

where $\Delta=(V_0\,\bar{t})/B$.

B_{1S} and B_{2S} are calculated using the following equations:

$$B_8=(2J_1^{-2}(\alpha_s)\,B_6)/B_3\,,$$

$$I_1=\int_{y_0}^{1}(x-x^2)\,J_0(\alpha_s x)\,dx\,,$$

$$I_2=\int_0^{y_0}[(-x-x^2+5x^3-3x^4+6c^{-2}x+6c^{-2}x^2)\,e^{-c^2x^2}+ +(x-x^2)(1.5c^{-2}-0.5)-(1.5x^2+1.5c^{-2}-0.5-x)\,e^{-2c^2x^2}]+ +3(\pi)^{\frac{1}{2}}c^{-3}\operatorname{erf}(cx)+xB_{1S}]\,J_0(\alpha_s x)\,dx\,,$$

$$B_{1S}=B_8\,I_1+I_2\,,$$

$$B_{2S}=J_1^{-2}(\alpha_s)\,\alpha_s^{-1}\int_0^1(x-x^2)\,J_0(\alpha_s x)\,dx\,.$$

Case 2.

$$M_{\theta 0}\geqslant M_{r0}\,,\qquad N_{\theta 0}\geqslant N_{r0}\,.$$

Phase 1 $0\leqslant t\leqslant \bar{t}$.

The deformation of the plate in this phase is characterized by a plastically flowing annulus surrounding a central rigid portion of the plate. The plastic flow is governed by the yield and flow conditions given by Eqs. (4.3). Neglecting the influence of membrane forces, the differential equation for the movement of plastic hinge is derived as in Ref. [12], which is solved and modified to account for the membrane forces to get

$$t=(\bar{t}A_0)/D_0\,,$$

where

$$A_0=S_1\,A_1+A_2-S_1\,A_3-A_4\,,$$

$$D_0=S_1\,A_5+A_6-S_1\,A_3-A_4\,.$$

S_1, A_1 etc., are defined in Ref. [12].

Using essentially the same procedure as in Case 1, the equation for the time can be obtained in the form

$$\frac{V_0\,\bar{t}}{H}=-\frac{D_0}{8D_1}+\frac{D_0}{8D_1}\left[\frac{4D_2}{3}\beta+1\right]^{\frac{1}{2}}, \tag{5.11}$$

where β is the impulse value and D_1, D_2 are known functions of y_0, y_1, K and c. For an isotropic plate ($K\to 1$) subjected to a uniformly distributed impulse ($c\to 0$), Eq. (5.11) reduces to Eq. (25) of Ref. [6].

Phase 2 $\bar{t} \leqslant t \leqslant t_f$

The deformation of the plate during this phase is governed by the equation

$$W'' + W'/r = \frac{K\mu}{N_{\theta 0}} \ddot{W},$$

which can be solved to get the deflection of the plate at any time. The final deformed shape of the plate is given by

$$\frac{W_f}{H} = \sum_{S=1}^{\infty} [\Delta^2 D_{1S}^2 + D_{2S}^2]^{\frac{1}{2}} J_0(\alpha_s x) \quad \text{for} \quad y_1 \leqslant x \leqslant y_0,$$

$$\frac{W_f}{H} = \sum_{S=1}^{\infty} [\Delta^2 D_{1S}^2 + D_{2S}^2]^{\frac{1}{2}} J_0(\varkappa_s y_1) \quad \text{for} \quad 0 \leqslant x \leqslant y_1.$$

D_{1S}, D_{2S} are known functions of y_0, y_1, K, α_s and the Gaussian parameter c.

Numerical results and conclusions

From the preceding analysis it is clear that the plastic response of orthotropic circular plates under Gaussian impulsive loading depends on the magnitude of the impulse β, the Gaussian parameter c, the degree of orthrotropy K and the presence of the membrane forces. Figures 4 to 9 show the influence of the above parameters. Figure 4 shows the position of the plastic hinge as a function of time for various values of c. It is observed that the plastic hinge travels at a greater speed towards the centre of the plate when the influence of the memebrane forces is considered. Figure 5 shows the influence of the magnitude of impulse on central deflection for different values of c. Figures 6 to 9 show the deflection profile of the plate. The behaviour of the orthotropic plate with $K \leqslant 1$ is similar to that of an isotropic plate. With $c=0.1$ which approximates to a uniformly distributed impulse, the results compare very well with the results of Jones [6]. From the results obtained in Figs. 5 to 9 the following observations are made.

(i) The influence of the membrane forces is to reduce the deflections considerably. Consequently, the theory based on bending moments alone greatly overestimates the deflection.

(ii) The amount of overestimation is not very much sensitive to the spatial distribution of the impulse. It is almost the same for a uniformly distributed and the highly localized impulse near the centre of the plate. The difference in deflections and the time of movement of hinge obtained by the simple bending theory and the theory which retains the influence of membrane forces is considerably large for larger values of impulse.

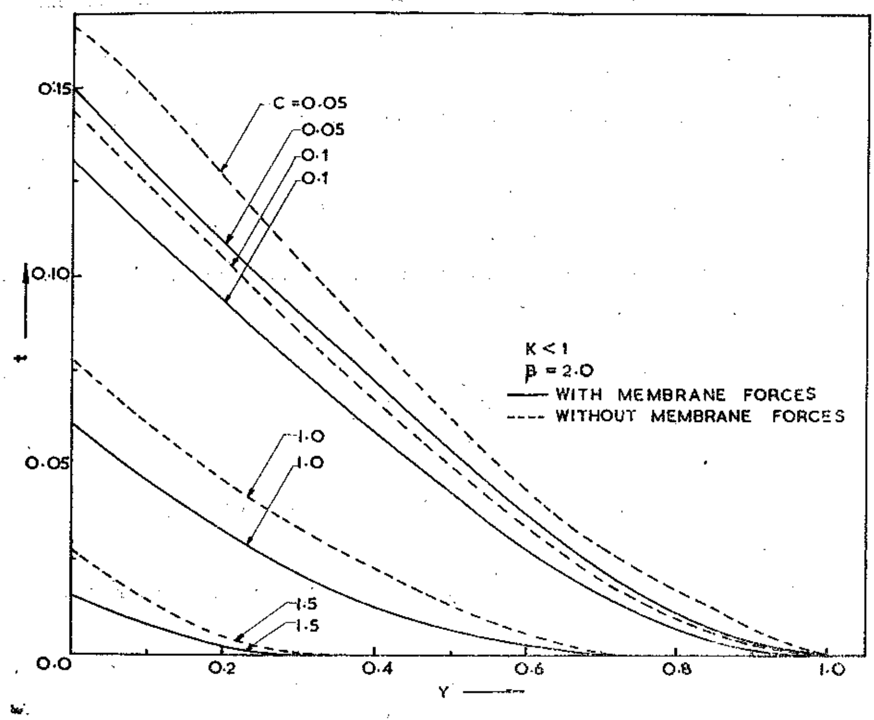

FIG. 4. Movement of hinges.

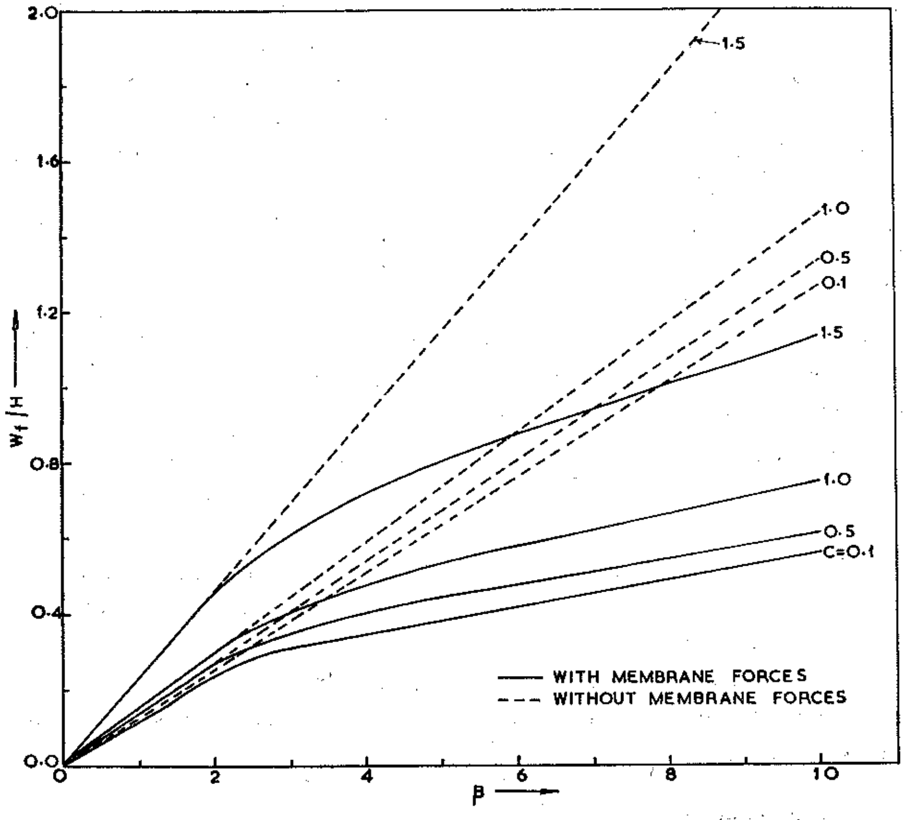

FIG. 5. Central deflection—impulse relationship.

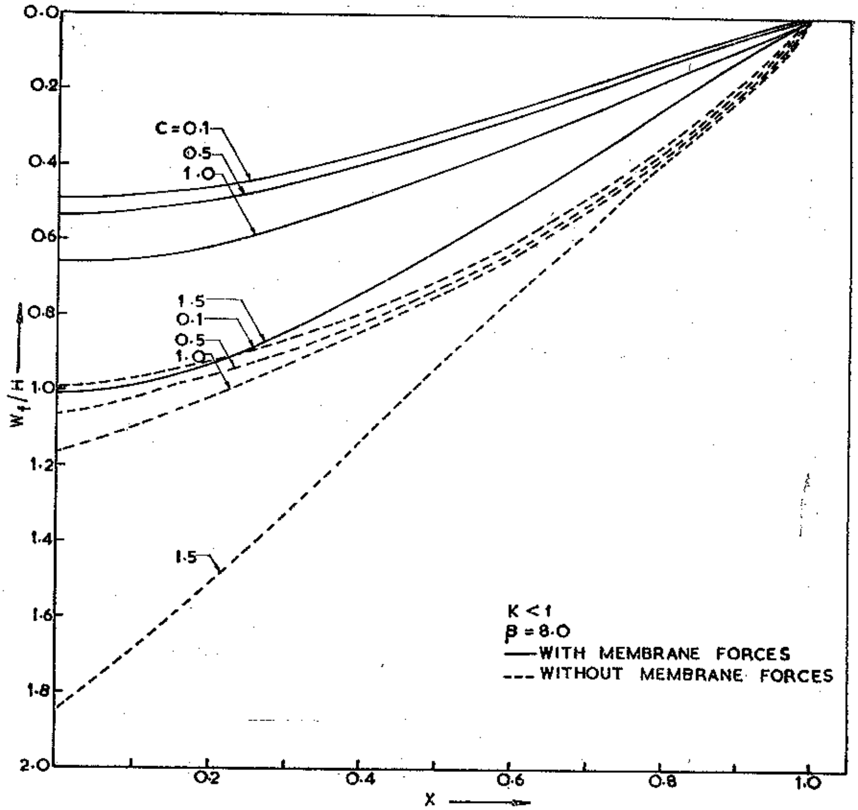

FIG. 6. Deflection profile.

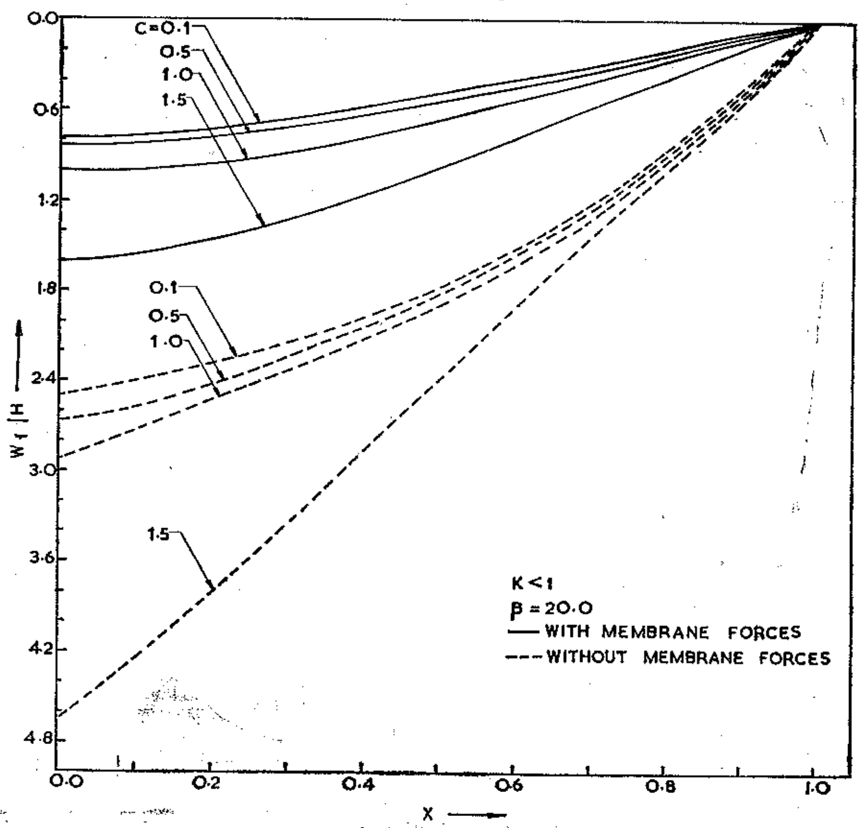

FIG. 7. Deflection profile.

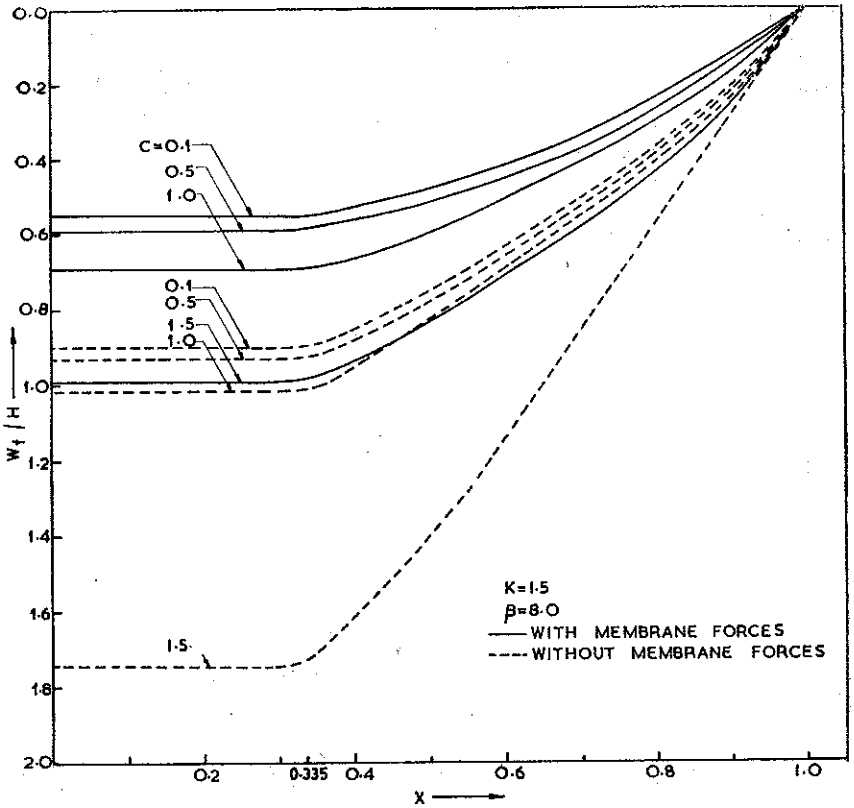

FIG. 8. Deflection profile.

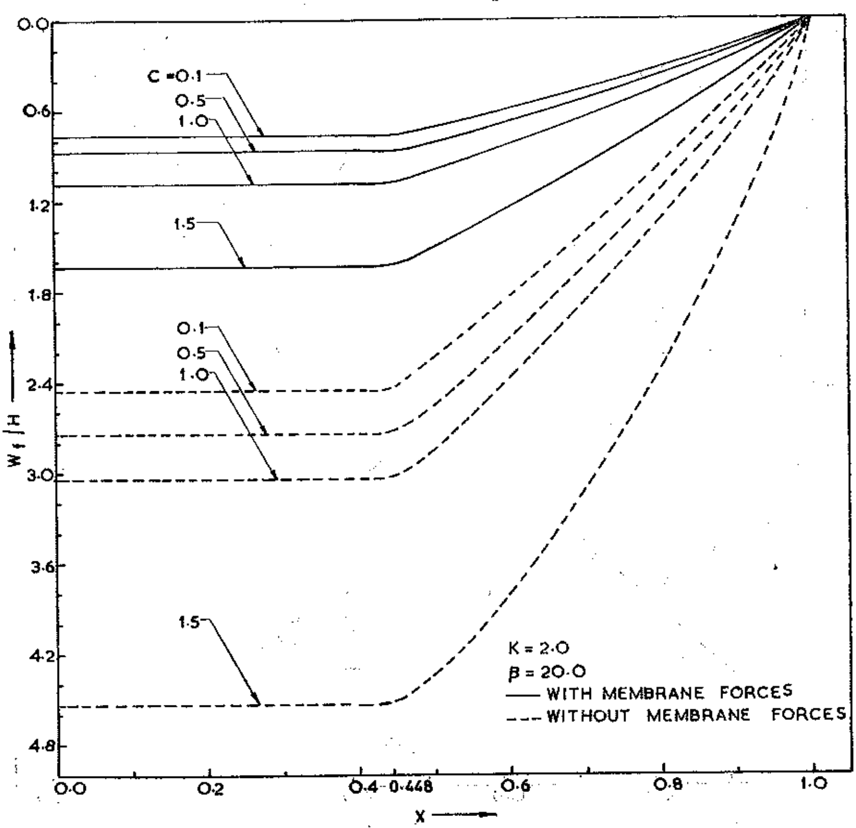

FIG. 9. Deflection profile.

Acknowledgement

The work reported herein was supported by the Aeronautics Research and Development Board, Ministry of Defence, Government of India.

References

1. P. S. Symonds, T. J. Mental, *Impulsive loading of plastic beams with axial constraints*, J. Mech. and Physics of Solids, **6**, 186, 1958.
2. A. L. Florence, *Annular plate under a transverse line impulse*, AIAA J. **3**, 1726, 1965.
3. A. J. Wang, *The permanent deflection of a plate under blast loading*, J. Appl. Mech. **22**, 375, 1955.
4. T. Wierzbicki, *Bounds on large dynamic deformations of structures*, J. Engn. Mech. Div., Proc. ASCE, 267, 1970.
5. J. B. Martin, *Impulsive loading theorems for rigid-plastic continua*, Proc. Amer. Soc. Civ. Engn., **90**, 27, 1964.
6. N. Jones, *Impulsive loading of a simply supported circular rigid plastic plate*, J. Appl. Mech. **35**, 59, 1968.
7. N. Jones, *Finite deflections of a simply supported rigid-plastic annular plate loaded dynamically*, Int. J. Solids, Struc., **4**, 593, 1968.
8. N. Jones, *Finite deflections of a rigid-viscoplastic strain hardening annular plate loaded impulsively*, J. Appl. Mech. **35**, 349, 1968.
9. N. Jones, *The influence of large deflections on the behaviour of rigid-plastic cylindrical shells loaded impulsively*, J. Appl. Mech., **37**, 416, 1970.
10. N. Jones, *A theoretical study of the dynamic plastic behaviour of beams and plates with finite deflections*. Int. J. Solids. Struc. 7, 1007, 1971.
11. N. Jones, *A literature review of the dynamic plastic response of structures. The shock and vibration digest*, **7**, 89, 1975.
12. S. Anantha Ramu, K. J. Iyengar, *Plastic response of orthotropic circular plates under blast loading*, Int. J. Solids. Struc. **12**, 125, 1976.
13. P. G. Hodge, *Yield condtions for rotationally symmetric shells under axisymmetric loading*, J. Appl. Mech., **27**, 323, 1960.
14. L. W. Hu, *Modified Tresca yield condition and associated flow rules for anisotropic materials. and application*, J. Franklin Inst., **265**, 187, 1958.

Streszczenie

SKOŃCZONE ODKSZTAŁCENIA PLASTYCZNE ORTOTROPOWYCH PŁYT KOŁOWYCH WYWOŁANE IMPULSEM TYPU GAUSSA

Przedstawiono trwałe ugięcia ortotropowej płyty kołowej poddanej obciążeniu impulsowemu. Przyjęto taki impuls, który wywoływał prędkość osiowo-symetryczną odpowiadającą w przestrzeni ogólnemu rozkładowi Gaussa. Do wyznaczenia odkształceń założono jednoczesny wpływ sił membranowych i momentów gnących. Wykazano, że teoria zgięciowa daje zbyt wysokie oszacowanie trwałego odkształcenia plastycznego i czasu odkształcenia płyty. Rząd przeceniania nie zależy w istotny sposób od przestrzennego rozkładu impulsu.

Резюме

КОНЕЧНЫЕ ПЛАСТИЧЕСКИЕ ДЕФОРМАЦИИ ОРТОТПОРНЫХ КРУГОВЫХ ПЛИТ ВЫЗВАННЫЕ ИМПУЛЬСОМ ТИПА ГАУССА

Представлены остаточные прогибы ортотропной круговой плиты подвергнутой импульсной нагрузке. Принят такой импульс, который вызывает осесимметричную скорость, отвечающую в пространстве общему распределению Гаусса. Для определения деформаций предложено одновременное влияние мембранных сил и изгибающих моментов. Показано, что изгубная теория дает слишком высокую оценку остаточной пластической деформации и времени деформации плиты. Порядок превышения не зависит существенным образом от пространственного распределения импульса.

DEPARTMENT OF CIVIL ENGINEERING
INDIAN INSTITUTE OF SCIENCE

Received March 21, 1977.