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The Perzyna Viscoplastic Model in Dynamic Behaviour of Magnetorheological Fluid under High Strain Rates

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The extension of viscoplastic model of Perzyna for the field of magnetorheological materials is proposed. Perzyna's approach is adopted to identify the mechanisms of microscopic rearrangement of ferroelements producing visible increase of material stiffness, in particular increase of shear modulus. The project of laboratory test stand is presented. It is based on Split-Hopkinson pressure bar set-up equipped with container for magnetorheological fluid and coil to control it.

Key words: magnetorheological fluid, magnetorheological gel, Perzyna viscoplastic model, Split-Hopkinson pressure bar.

1. Introduction

The magnetorheological fluid (MR fluid) is a suspension of micro-sized ferroelements in a carrier viscous fluid. Ferroelements, mostly are made of Fe_3O_4 with diameter 12 μm or carbonyl iron particles of 4.5–5.2 μm diameter. Usually mineral oil is used as carrier fluid. To avoid aggregation of magnetic sensitive elements, ferroelements are surrounded by silicon coat.

According to the experimental data provided, e.g., in [13] the properties of magnetorheological fluid can be characterized by models describing nonlinear behaviour of fluid with magnetic active particles as regards the shear stress-shear strain rate relation. In the literature, the linear Bingham [21] model is commonly used. In many cases the linear relations between shear stress and shear strain rate is not adequate in describing mechanism of magnetorheological fluid flow. In literature there are analysed more accurate models. In articles [7, 10] the application of Bodner-Partom model is proposed. On the other hand the Herschel-Bulkely model is considered in [17]. Both models describe non-linear relation between shear stress and shear strain rate. However, the disadvantage of these models is that they are of empirical character and have no firm physical basis. Such models are valid for limited range of variables. Therefore, formulation of a new viscoplasticity model for magnetorheological materials that should be based on physical mechanisms responsible for rate dependency of yield stress remains an open question. The studies on application of adaptive materials, in particular magnetorheological fluids are of increasing interest, [15, 16]. Therefore, the recent importance of this paper is well justified.

The aim of the paper is the extension of viscoplastic model of Perzyna, typically used in metallic solids, for the area of magnetorheological materials. In this paper, Perzyna's approach will be adopted to identify the mechanisms of microscopic rearrangement of ferroelements producing visible increase of material stiffness, in particular increase of shear modulus. The proposed magneto- viscoplasticity model is to be specified with use of author's own experiments. The project of new laboratory test stand is presented. It is based on Split-Hopkinson pressure bar [22, 23] set-up equipped with container for magnetorheological fluid and coil to control it. The small strains approach is assumed in the description of elasto-viscoplastic behaviour of magnetorheological material subjected to quasistatic and dynamic loading conditions. It is also assumed that the studied processes are closed to isothermal conditions and therefore no temperature effects are considered. The proposed extension of viscoplastic model of Perzyna is a new one. Also the presented method of experimental investigations using the Split-Hopkinson pressure bar has not been proposed in the literature until now.

The Split- Hopkinson pressure bar experimental technique is used to evaluate the high- strain rates in the material. In many cases, this experimental technique is applied in experimental solid mechanics. What is important, this laboratory test stand could be used to provide high-rate squeeze flow [18–20]. The Split-Hopkinson pressure bar consist of a gas gun and three cylindrical bars, known as the striker bar (position 1 in the Fig. 1), incident bar (2), transmitter bar (4).

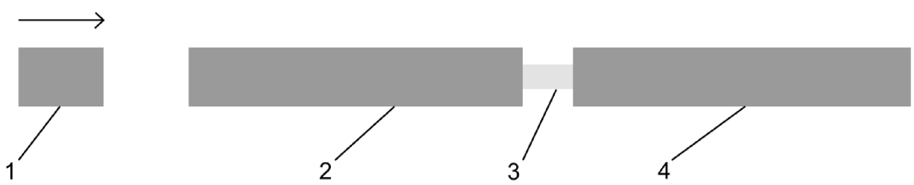

Fig. 1. Split-Hopkinson pressure bar experimental apparatus: 1) striker, 2) incident bar, 3) specimen, 4) transmitter bar.

The gas gun is used to accelerate the striker, which impacts the incident bar end far from the specimen. The incident pulse is measured as it propagates along the bar axis by a pair of strain gauges positioned at the center. At the incident bar – specimen interface, the pulse is dispersed as a function of the specimen and the bar impedance (product of density ρ and sound velocity C_0). The resulting reflected and transmitted pulses are measured by strain gauges and due to this the strain rate can be determined.

The results in Fig. 2a [13] show the relation between shear stress and shear strain rate.

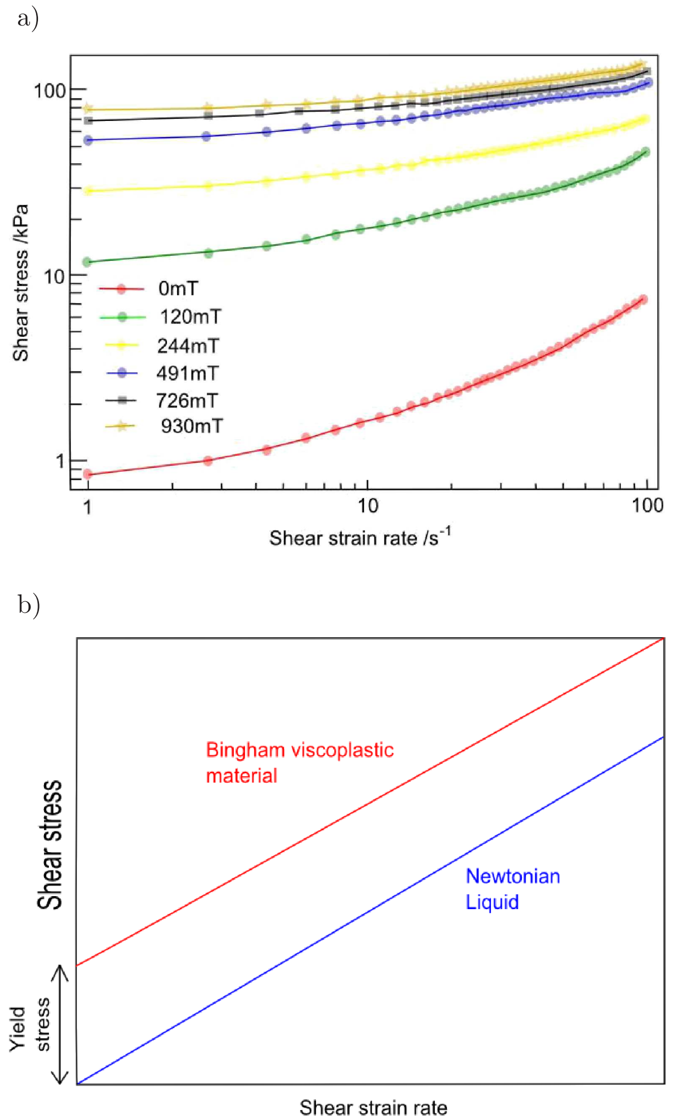

Fig. 2. Characteristics shear stress to shear strain rate [13]: a) the visualization of results of experiments of Xu *et al.* [13], b) the model of Bingham viscoplastic material.

In Fig. 2a there are presented the data of the experiment described in [13], where the magnetorheological gel with the content of 30% of ferroelements is studied. The test stand is based on rotational shear rheometer. The flow curves of magnetorheological gel differ remarkably from those which could be described by Bingham equation presented in Fig. 2b. The linespresenting experimental data tend to slowly curve therefore the linear Bingham model (showed in Fig. 2b) is not adequate to describe the mechanism of material behaviour.

Then, it can be observed that the results shown in Fig. 2a deviate from linear Bingham relations for higher shear strain rates. This observation reveals the necessity of the search for adequate viscoplastic model of magnetorheological materials.

The prepared laboratory test stand to investigate material behaviour under high strain rates is presented. The laboratory set- up and methodology of measurements based on Split-Hopkinson pressure bar with container for magnetorheological fluid and the controlling coil are discussed.

2. Laboratory test stand

Our area of interest is magneto-viscoplasticity describing magnetorheological fluids under high strain rates. The physical mechanism responsible for behaviour of such a kind of materials is presented in Fig. 3.

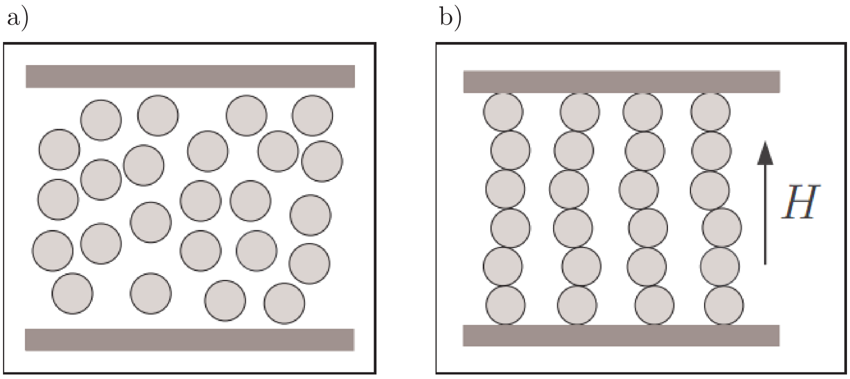

Fig. 3. a) Magnetorheological fluid at neutral state (without dynamic loads and magnetic field H), b) under the influence of magnetic field H: magnetic field strength [kA/m].

Magnetorheological fluid is colloidal suspension of ferromagnetic particles and viscous fluid that is very often mineral oil. Ferromagnetic elements are surrounded by the isolating coat to avoid sticking. Application of magnetic field causes aggregation of randomly distributed ferroelements producing solidification of the material volume.

The testing stand is made on basis of Split-Hopkinson pressure bar. The main idea is to use the present laboratory device and prepare it to test magnetorheological fluid in magnetic field at high strain rates.

Laboratory device to investigate dynamic properties of magnetorheological materials has to be modified. The idea of proposed modification, which is based on earlier studies with use of Hopkinson pressure bar [5, 18–20] is presented below.

In Fig. 4 the idea of new laboratory test stand is presented. The set-up is based on Split-Hopkinson pressure bar with novel grip (container) for MR fluid. The goal of experiment is to create incident wave inside the MR fluid under the influence of magnetic field. The container is equipped with the seals (to prevent leakage of fluid and keep constant pressure in the specimen), coils (to produce magnetic field) and water cooling system (to keep coils in constant temperature).

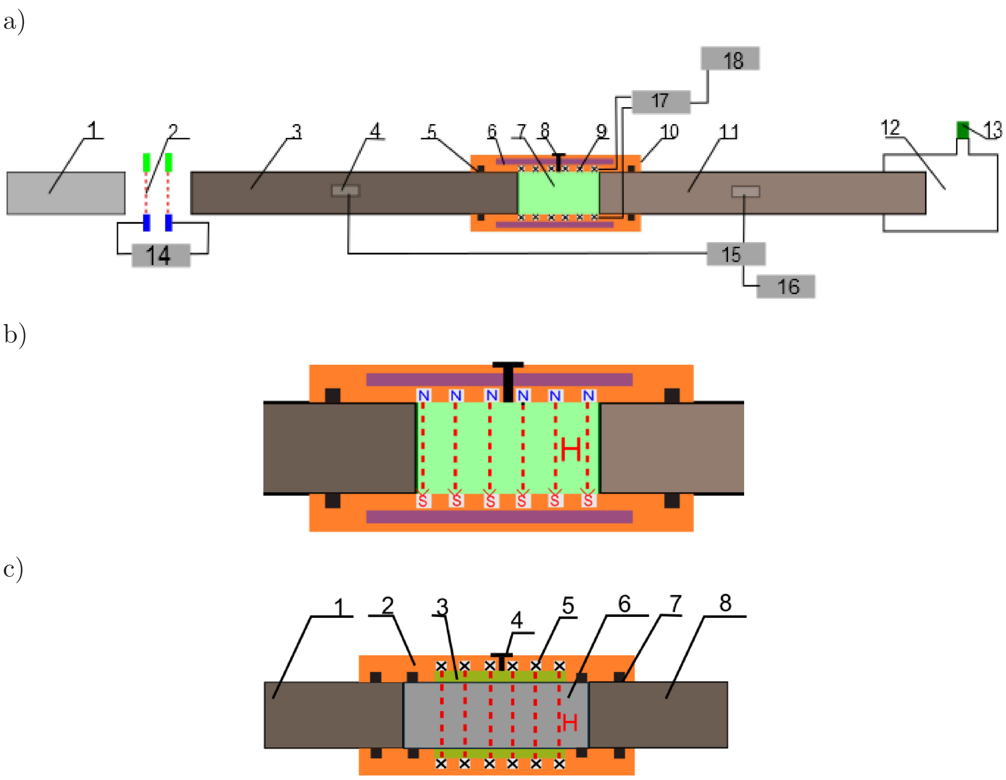

Fig. 4. a): 1) striker, 2) sensors to measure velocity of striker, 3) incident bar, 4) strain gauges, 5) seal, 6) water cooling for coils, 7) MR fluid, 8) infusion, 9) coils, 10) sleeve, 11) transmitter bar, 12) gas accumulator, 13) valve, 14) photo diode gates, 15) signal amplifier, 16) data acquisition system, 17) power supply for coils, 18) signal generator for coils controls; b) compression test device: sleeve with MR fluid and visualization of magnetic flux; c) shear test device: 1) incident bar, 2) sleeve, 3) MR fluid, 4) infusion, 5) coil, 6) rod, 7) seal, 8) transmitter bar.

At the end of transmitter bar the gas accumulator is fixed to prevent too large displacement of the bar. The idea of this kind of equipment is based on [8].

In Fig. 4c shear test device is presented. Inside the container the steel rod is placed which is responsible for shearing of the MR gel. The steel rod inside the magnetic field is supposed to increase the intensity of magnetic field, which should be estimated analytically. The classical shear test using the Hopkinson pressure bar is described in more detail, e.g., in [1].

The rest of the equipment is used in typical Hopkinson laboratory test stand. There are the diode matrixes responsible for measurement of the velocity of striker and data acquisition system based on strain gauges like strain sensor, signal amplifier and signal acquisition device – very often this is a digital oscilloscope. The control and power system responsible for generating magnetic field inside the container with MR fluid should be designed anew.

The novelty of the proposed laboratory test stand is also the applications of coils for the sleeves shown in Figs. 4b and 4c to induce the constant magnetic field. Also new shear test device will be designed. Due to this the dynamic axial compression and shear tests of solidified magnetorheological material will be possible.

2.1. Split-Hopkinson pressure bar-methodology of measurement

The theory of measurement of axial strain and determination of axial stress using the Split-Hopkinson pressure bar was presented by Klepaczko [1].

Striker accelerated by gas gun achieves velocity V_0 which is measured by matrix of diodes and photodiodes. In case of impact by striker against incident bar shock wave is triggered. It has the length $\varphi = 2L$, where L is the length of striker. The time period of this wave is $T = 2L/C_0$, where C_0 is the velocity of wave inside the bar.

During researches on Split-Hopkinson pressure bar we obtain:

i. $e_1(t)$: the axial strain in incident bar, triggered by incident wave, measured by strain gauges (number 1 in Fig. 5),

ii. $e_r(t)$: the axial strain in incident bar triggered by reflected wave, measured by strain gauges (number 1 in Fig. 5),

iii. $e_2(t)$: the axial strain in transmitter bar triggered by wave in transmitter bar, measured by strain gauges (number 2 in Fig. 5).

Value of mean strain rate in the specimen set-up is described by

$$\dot{\varepsilon}_{ns}(t) = \frac{1}{l_0}\left[\frac{du_a(t)}{dt} - \frac{du_b(t)}{dt}\right], \tag{2.1}$$

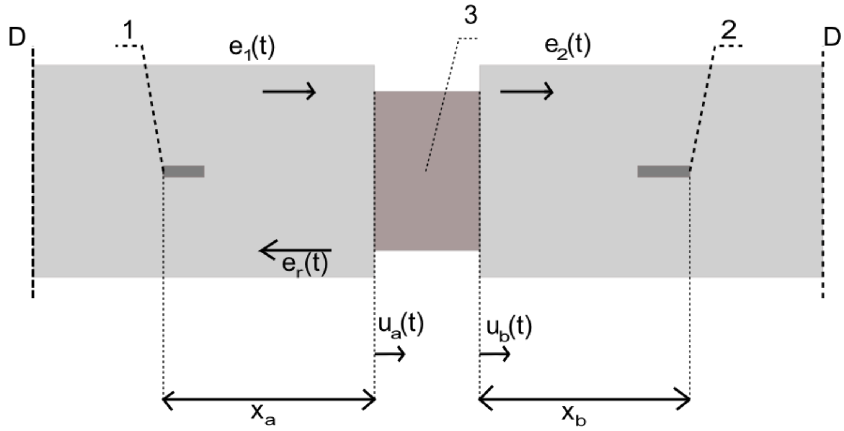

FIG. 5. Split-Hopkinson pressure bar: 1, 2) strain gauges, 3) specimen, D: bar diameter, $e_1(t)$: axial strain of incident wave in the bar (1), $e_2(t)$: axial strain of reflected wave in the bar (2), $e_r(t)$: axial strain of reflected wave, $u_a(t)$ incident displacement in bar (1), $u_b(t)$: transmitted displacement in bar (2).

where l_0 is the length of specimen, expressions for $u_a(t)$ is presented by:

$$u_a(t) = C_0 \int_0^t e_1(t)\, dt + (-C_0) \int_0^t e_r(t)\, dt \tag{2.2}$$

and for $u_b(t)$:

$$u_b(t) = C_0 \int_0^t e_2(t)\, dt. \tag{2.3}$$

Then, the equation of mean axial strain is obtained:

$$\varepsilon_{ns}(t) = \frac{C_0}{l_0} \int_0^t [e_1(t) - e_r(t) - e_2(t)]\, dt \tag{2.4}$$

and the strain rate:

$$\dot{\varepsilon}_{nS}(t) = \frac{C_0}{l_0} [e_1(t) - e_r(t) - e_2(t)], \tag{2.5}$$

as a result the relation for axial stress is obtained:

$$\sigma_{nS}(t) = \frac{\rho C_0^2}{2} \left(\frac{D}{D_{S0}}\right)^2 [e_1(t) - e_r(t) - e_2(t)]. \tag{2.6}$$

3. The Perzyna viscoplastic model

The original model of rate-sensitive plastic material was presented by Perzyna in [6]. Futher development and comprehensive discussion of viscoplasticity theory is given in [4].

3.1. One-dimensional formulation

The one-dimensional Perzyna viscoplastic model is described in the following way [4]:

$$\dot{\varepsilon} = \frac{\dot{\sigma}}{E} + \gamma\langle\Phi[\sigma - f(\varepsilon)]\rangle, \tag{3.1}$$

where ε – total nominal strain, E – Young modulus, γ – viscosity parameter, $\sigma = f(\varepsilon)$ is material characteristic for statical tension test.

The Φ factor describes the excess stress function:

$$\langle\Phi\rangle = \begin{cases} \Phi, & \text{when } \sigma > f(\varepsilon), \\ 0, & \text{when } \sigma \le f(\varepsilon). \end{cases} \tag{3.2}$$

3.2. General formulation

For Huber-Mises yield conditions the function of stress takes form:

$$f(\sigma_{ij}) = (J_2)^{1/2}, \tag{3.3}$$

where J_2 is second invariant of stress deviator s_{ij}.

The constitutive relations of rate-sensitive plastic (viscoplastic) material reads:

$$\dot{e}_{ij} = \frac{1}{2G}\dot{s}_{ij} + \gamma\left\langle\Phi\left(\frac{\sqrt{\mathrm{J}_2}}{\varkappa} - 1\right)\right\rangle \frac{s_{ij}}{\sqrt{\mathrm{J}_2}} \tag{3.4}$$

and

$$\dot{\varepsilon}_{ii} = \frac{1}{3K}\dot{\sigma}_{ii}, \tag{3.5}$$

where the symbols denote K is elastic compressibility modulus, $\dot{e}_{ij}$ is deviator of strain rate, $\dot{s}_{ij}$ is deviator of stress rate, s_{ij} is deviator of stress tensor, J_2 is second invariant of stress deviator, G is elastic shear modulus, γ is viscosity parameter, $\varkappa$ is quasi-static yield stress; dynamic yield condition can be derived from (3.4) in the form:

$$\sqrt{\mathrm{J}_2} = \varkappa(\mathrm{W_p})\left[1 + \Phi^{-1}\left(\frac{\sqrt{\mathrm{I}_2^{\mathrm{p}}}}{\gamma}\right)\right], \tag{3.6}$$

where $\mathrm{W_p}$ is the plastic work, $\mathrm{I}_2^{\mathrm{p}}$ the second invariant of strain rate tensor: $\mathrm{I}_2^{\mathrm{p}} = \frac{1}{2}\dot{\varepsilon}^p_{kl}\dot{\epsilon}^p_{kl}$.

4. The Perzyna viscoplastic model adapted for magnetic field effect

The Bingham model proposed in [2] reads

$$\tau = \tau_{0H}(H) + \mu\dot{\gamma},$$

where the symbols denote: τ – shear stress, τ_{0H} – yield stress dependent of magnetic field strength H, μ – visocsity factor, $\dot{\gamma}$ – shear strain rate, does not describe adequately the physical mechanisms in the range of high strain rates. The results displayed in Fig. 2a and the discussed relation of shear stress-shear strain rate by Xu *et al.* [13] and Quoc-Hung *et al.* [17] illustrated schematically in Fig. 6 show the necessity of new formulation of viscoplasticity model.

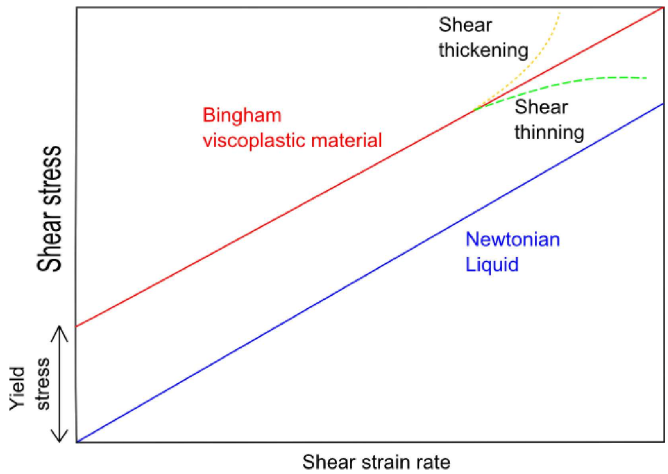

Fig. 6. Schematic view of the inadequacy of Bingham model.

The mechanism of microscopic rearrangement of ferroelements producing visible increase of material stiffness and nonlinear dependence of yield stress-strain rate can be better described by the Perzyna viscoplastic model (4.1) extended to account for the magnetic field strength H.

$$\dot{e}_{ij} = \frac{1}{2G(H)}\dot{s}_{ij} + \gamma(H)\left\langle \varPhi\left(\frac{\sqrt{\mathrm{J}_2}}{\varkappa(H)} - 2\right)\right\rangle \frac{s_{ij}}{\sqrt{\mathrm{J}_2}}, \tag{4.1}$$

where $2G(H)$ is elastic shear modulus, depending on magnetic field strength H, $\gamma(H)$ is the viscosity parameter of material depending on magnetic field strength H, $\varkappa(H)$ is quasi-static yield stress depending on magnetic field strength H.

To identify the proposed magneto-viscoplasticity model three kinds of experimental tests will be carried out: quasi-static compression test, dynamic axial compression and dynamic shear test using the modified Split-Hopkinson pressure bar.

5. Conclusions

The paper shows that new methodology of experimental investigation is required to indentify the proposed model accounting for magnetic field effect. Therefore, a new set-up of Split-Hopkinson pressure bar equipped with electromagnetic coils to investigate specimens of magnetorheological material is proposed. The presented state of the art and recent applications reveal that the investigation of behaviour of ferroelments under high strain rates is well motivated. The extension of Perzyna viscoplastic model which includes influence of magnetic field strength brings a new perspective into advanced mechanics of magnetorheological material. The Perzyna magneto-viscoplastic model accounting for mechanisms of microscopic rearrangements of ferroelements should better describe the material stiffness and nonlinear dependence of yield stress-strain rate than traditional Bingham model or empirical descriptions [7, 10, 17].

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