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Probabilistic Method to Determine the Overall Rock Block Failure Based on Failure Mode

Hongze ZHAO[1,2], Zequan LI[1], Chi Seng KONG[3]

[1] *Faculty of Resources and Safety Engineering*
China University of Mining and Technology (Beijing)
Beijing 100083, China
e-mail: hzzhao78@163.com

[2] *State Key Laboratory of Coal Resources and Safe Mining*
China University of Mining and Technology (Beijing)
Beijing 100083, China

[3] *China Railway Construction (HK) Limited*
Hong Kong SAR

In rock slopes or underground excavations, due to the structure of the cutting surface, rock typically exhibits strong random properties, such as the structure of the surface geometry and mechanical parameters showed strong randomness, resulting in a high degree of rock mass uncertainty of the previous studies. In this paper, a new approach to estimating probability of failure of a rock block is proposed. Based on the previous studies investigating the potential failure of the block theory model, the block failure calculation took into account the geometrically formed probability, failure mode, and mechanical failure probability. As an engineering application of the presented model, the analysis of block failure, observed in a copper mine site in Australia, was performed. The Monte-Carlo simulation method was used for the evaluation where plane roughness, friction angle, and cohesion were considered as random variables. The results of this example show that the model can be used as a basis for evaluating the reliability of the block.

Key words: jointed rock mass, key block theory, reliability theory.

1. Introduction

Rock mass is a complex geological body; its formation has undergone a long geological age. In the whole geological history of the formation and existence of rock mass, rock mass has undergone different loading and unloading stress history which was accompanied by different stages of chemical and physical weathering, which produces discontinuous planes such as joint, fold, fault, and foliation, due to the impact of different geological processes and tectonic forces. Cut by these discontinuous planes, rock mass is divided into several structural bodies at different sizes. In addition, the random distribution characteristics of a geometric feature of a structural plane such as occurrence, interval, and trace length, as well as the mechanical strength such as bonding force and friction coefficient both have impact on the stability of the resulting rock block with the random distribution feature.

On the basis of the geometric topology theory, the key block theory [4, 10] judges the block cut by the structural and excavation planes by the way of the "limited theorem" and "movable theorem". These theories can determine the block formed by cutting of the structural plane with different occurrences, also the location and the form of characteristics of the instability mode of the block. Furthermore, it can perform a mechanical stability analysis, calculate the safety coefficient of block stability, and provide the probability of block instability. Therefore, it can be applied extensively.

In the analysis of the reliability of rock mass excavation stability, if the random distribution characteristics of the geometric features and mechanical strength of the structural plane are concurrently taken into consideration, together with the failure mode of the block, then the stability of the block can be presented more accurately, and the result from the probability of failure calculation is more realistically significant. For this issue, a statistical method is adopted for the analysis and calculation.

2. Model evaluating for probability of failure of overall rock block

As for the research on the probability of failure of a rock block, there is currently a rich literature; however, the definition for the probability of failure is ambiguous and diverse. Quek and Leung [9] held that the probability of failure is expressed as

$$P_f = \frac{N_F}{N_T}. \tag{2.1}$$

In the formula, N_F is the number of cycles of the block failure mentioned in Monte Carlo method, namely, the number of $F_s < 1$, where F_s is the factor of safety; N_T is the number of total cycles used to analyse the block stability. However, Feng and Lajtai [1] suppose that, N_T can have two meanings, that is, it can be the total number of cycles, or the total number of the geometrically unstable blocks. According to the above definitions, the probability of failure of a block will be varied. Lajtai and Carter [2] defined N_T as the total number of geometrically unstable blocks. Feng and Lajtai [1] adopted the above two definitions in their calculations, but did not draw a distinction between them.

Numerous researchers always suppose that, the total probability of failure of a rock block is the product of the geometrically unstable probability and mechanical unstable probability. The concept of the product stemmed from the definition given to the system probability by Marek and Savely [5]. The concept describes that the total probability of failure of a system is the product of the probability of failure of all individuals in the system. Using this approach, Glynn [3] also calculated the probability of failure, expressed as

$$P_f = \sum P[A]P[B]. \tag{2.2}$$

In the formula, $P[A]$ is the geometric probability of failure of the block, $P[B]$ is the mechanical probability of failure of the block, namely, the load exerted on the block outweighs the resistance. Park [6, 7] also used this method in analyzing the probability of stability of a slope. Meanwhile, Chinese scholars [12, 13] also employed the same definition in calculating the total probability of failure of the block.

However, in the calculation of the total probability of failure of the block in all the literature, the change in the block failure mode is not considered. After the block has been geometrically formed, because the dip angle is random in mechanical calculation, a change in the dip angle may affect the failure mode of the blocks, for example, with a change from a slide along a single plane to a slide along a double plane, the formula calculating the reliability also needs appropriate adjustment.

To give more exact results, the model calculating the probability of failure of the block is presented here once again. The total probability of failure of the block is the product of the geometrically formed probability, failure mode probability, and mechanical probability of failure of the block. It is noted that the total probability of failure of the block is a conditional probability, that is, only if the prerequisite condition is satisfied the latter can occur. This is expressed as

$$P_t = P_a \times P_m \times P_f. \tag{2.3}$$

In the formula, P_a is the formed block; P_m is failure mode/formed block; P_f is block mechanical failure/failure mode.

Described by the mathematical language, the following expression is obtained:

$$P_t = \sum_{m=1}^{3} P_a \times P_{m|a} \times P_{f|m}. \tag{2.4}$$

In the formula, P_t is the total probability of failure of the block, P_a is the geometrically formed probability, $P_{m|a}$ is the probability of failure in a certain mode after the block has been formed. As mentioned in the above block theory, there are three block failure modes, namely, direct drop, slide along single plane, and slide along double plane. $P_{f|m}$ is the mechanical probability of failure of the block after its failure mode has been finalised. The previous expression can be rewritten as

$$P_t = P_a \times P_{1|a} \times P_{f|1} + P_a \times P_{2|a} \times P_{f|2} + P_a \times P_{3|a} \times P_{f|3}. \tag{2.5}$$

In the formula, the subscripts 1, 2, and 3 are different failure modes of the block, namely, direct drop, slide along single plane, and slide along double plane.

3. Probabilistic method to determine the overall rock block failure based on statistical theory

In this paper, Monte Carlo is used mainly for calculation of the total probability of failure. The calculation process consists of three parts. The first two steps can be proceeded simultaneously. The first part is to calculate the probability of block formation in the geometric space. The second part is to calculate the probability of mass failure in a certain mode after the block has been formed. The third part is to calculate the mechanical probability of failure after the block has been formed and the failure mode has been finalised.

Detailed calculations are as follows:

1) The first step of simulation is to select the block. For this issue, FracSim3D software developed for three-dimensional fracture network simulation by Prof. Xu [11], Adelaide University, Australia is applied. Fisher distribution is adopted in the software for the orientation (including inclination and dip angle) of the structural plane. Owing to fewer parameters and better simulation results, the distribution is applied extensively [8]. The probability density function of Fisher distribution is

$$f(\theta) = \frac{K \sin\theta e^{K\cos\theta}}{e^K - e^{-K}}. \tag{3.1}$$

In the formula, θ is the true mean value of the dip angle of the structural plane, and $0 < \theta < \pi/2$. K is Fisher constant, reflecting the concentration of the data, refer to the literature [8]. To facilitate the calculation, Eq. (3.1) is simplified, and the following approximate cumulative probability distribution function is attained:

$$P(\theta) \approx 1 - e^{K(\cos\theta - 1)}. \tag{3.2}$$

In order to simulate Fisher distribution, parameter $R_U - U(0,\ 1)$ is introduced. The random number is generated by the linear congruent method. Moreover, parameter θ is replaced by R_F [6]. Hence, by solving the inverse function of Eq. (3.2), the following random number generation formula that complies with Fisher distribution is obtained:

$$R_F = \arccos(\frac{\ln(1 - R_U)}{K} + 1). \tag{3.3}$$

Through the sampling method and from the above formula, numerous random numbers that comply with Fisher distribution are generated. Then, by conversion, the expected data of inclination and dip angle of the structural plane can be attained.

2) In the second step, for a given limited tetrahedron, the changes in the inclination and dip angle of its three groups of the structural plane significantly affect its volume and the area of slide planes. We select certain sample data of inclination and dip angle obeying Fisher distribution, and substituted them into the calculation program of the block theory. Using the vector method, the values of volume and slide plane area responsive to the inclination and dip angle can be worked out. Also, from this calculation, the number of the samples among the total samples that form the block can be obtained, namely, the number of directly dropping blocks, blocks sliding along the single structural plane, and blocks sliding along the double structural plane. Therefore, the resulting block formation probability is

$$P_{fG} = \frac{N_G}{N_{TG}}. \tag{3.4}$$

In the formula, N_G is the number of cycles of block formation; N_{TG} is the total number of cycles, namely, the number of samples generating the inclination and dip angle. Only when the block is geometrically unstable, calculation for the mechanical probability of failure of the block should be made, which can be considered as calculation of the conditional probability.

3) In the third step, as mentioned previously, the inclination and dip angle obey Fisher distribution. The random number of the block volume and slide plane area in this distribution can be derived through calculation. These sample data are depicted in a frequency distribution histogram. When data-fitting is done, the approximate distribution obeyed by the block volume and slide plane area can be obtained, and then its probability density function and characteristic parameters such as mean value and variance can also be attained. In this process, fitting the test of distribution is required so that the coincidence of the established probability model with the given sample data as well as whether the sample data comply with the established model is verified and the most suitable probability density function is chosen.

4) The fourth step is to generate the random number of the distribution function of the volume and the area. Same as for the random number generating Fisher distribution, the linear congruent method is employed to generate the random number for other types of distribution. Namely, a uniform distribution obeying [0, 1] is generated, then through resolving the inverse function, the random number obeying the expected distribution is generated.

5) In the fifth step, after the limit state equation has been established, we substituted the mechanical parameters of the structural plane into it (internal friction angle φ and cohesion force c), solving the mechanical probability of failure and reliability indicator. Thus, the mechanical probability of failure of the block can be obtained, namely

$$P_{fM} = \frac{N_M}{N_{TM}}. \tag{3.5}$$

6) In the last step, the total probability of failure was calculated, expressed as

$$P_f = P_{fG} \times P_{fM} = \frac{N_G}{N_{TG}} \times \frac{N_M}{N_{TM}}. \tag{3.6}$$

4. Engineering example

Kanmantoo Copper Mine is located 100 km east to Adelaide, Australia. In the range of the mine area, uniaxial compressive strength is 12.4 MPa to 115.9 MPa, the size of which is related mainly to the direction of schistosity. Two major shear structures are identified in the pit currently excavated. One structure is situated on the south slope, with the dip angle of 80° and azimuth of 100°; the other is situated on the west slope, with the dip angle of 30° and azimuth of 40°. From the results of the structural plane group analysis, it is deemed that the mine area has three determined joint sets. The average volume weight of rock is 2.91 t/m^3. The geometric and mechanical properties of each set are shown in Table 1.

The statistical data in Table 1 into FracSim3D software are then substituted, and excavation is begun. A block located on the roadway roof is chosen for analysing. The block analysis program shows that the block tends to slide singly along the structural plane set 2.

From the random number generation method of Fisher distribution, 10,000 groups of data of the dip angle and inclination of three sets of the structural plane as listed in Table 1 are generated separately. From the data of these 10,000 groups, 1100 groups are drawn at random separately to serve as sample data. Then, these sample data are substituted in the finalised block analysis program. After 1100 cycles, the probability of a generated block (including a total block,

Table 1. Statistics of geometric and mechanical properties of the structural plane.

Structural plane set	Occurrence of structural plane			Internal friction angle			Cohesion force [kPa]		
	Fisher constant [K]	Average dip angle	Average inclination	Mean value	Standard deviation	Scheme	Mean value	Standard deviation	Scheme
1	13.12	49.3	93.3						
2	23.35	84.4	95.2	29.4	3.6	Normal	15	2	Normal
3	24.19	67.4	19.1						

directly dropping block, block sliding along single plane, and block sliding along double plane) has reached a balance, with fluctuations in a small range only. It is supposed that this method can effectively evaluate the block formation probability, as shown in Fig. 1. Therefore, statistics are made with respect to the formed block (the number of samples is 1000). 302 blocks have been generated,

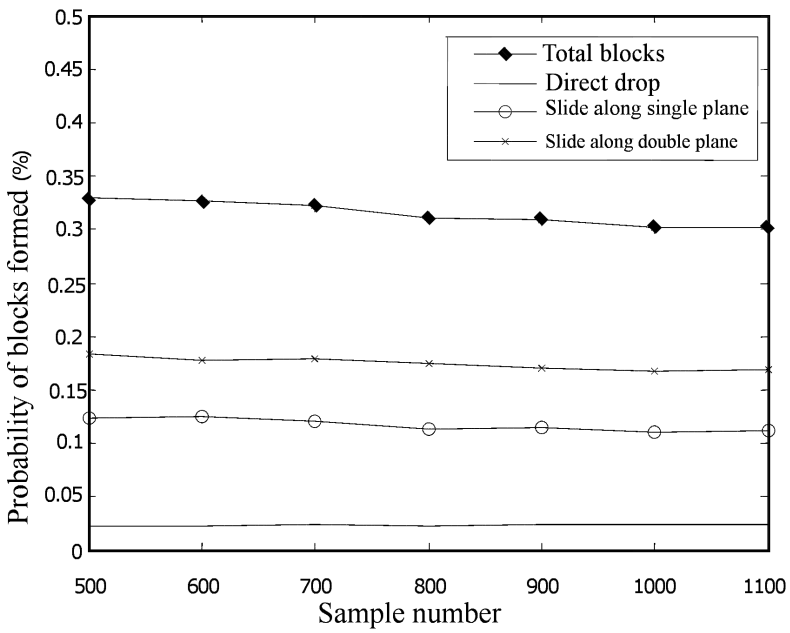

Fig. 1. Test of the sample number.

in which 110 blocks slide along the single plane. The statistical results are shown in Table 2.

Table 2. Statistics of block formation.

Block failure type	Number of blocks formed	Probability of blocks formed
Direct drop	24	0.024
Slide along single plane	110	0.11
Slide along double plane	168	0.168
Total blocks	302	0.302

Through the statistical analysis, the volume distribution scheme and slide plane area distribution scheme of the blocks sliding along the single plane have been chosen, namely, both the volume and slide plane area obey the normal distribution. As for mechanical parameters f_i and C_i, the normal distribution is taken as their distribution scheme. In addition, the reliability analysis model has been previously established for the blocks sliding along the single plane, namely, the relationship between the random variables $W(V)$, A_i, f_i, C_i and the limit state function. From the calculations it is found that the mechanical probability of failure of blocks is 0.35. It can be seen from Table 2 that the probability of blocks formed sliding along the single plane is $P_{fM} = 0.11$. Hence, according to Eq. (3.6), the total probability of failure of a block sliding along single plane is $P_f = 0.0385$.

5. Conclusion

The model evaluating the probability of failure of a block is redefined, and the formula calculating the total probability of failure of a block is presented. The total probability of failure is the product of the probability of a block formed geometrically, the probability of a formed block based on a given failure mode and the mechanical probability of failure of the block.

The dip angle, inclination, cohesion force, and friction coefficient of the structural plane are taken as random variables to solve the distribution of block volume, the slide plane area, and the random changing variables of the edge dip angle. A more precise and reliable value of the probability of failure of a block is obtained.

The case study results indicate that the probability of occurrence of a block sliding along a single plane is 11.12%, and the total probability of failure is 3.85%, which surpasses the risk level (1%) allowed for general rock engineering. The findings can be taken as the grounds for evaluating the reliability of blocks of such type.

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