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Research Paper

Simultaneous Effects of Thermal Radiation and Chemical Reaction on Hydromagnetic Pulsatile Flow of a Casson Fluid in a Porous Space

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In the present study, the hydromagnetic pulsating flow of Casson fluid between two parallel plates in a porous space, thermal radiation and chemical reaction are investigated. The analytical solutions for flow variables are obtained by using a perturbation technique. The effects of pertinent parameters on velocity, temperature, concentration, Nusselt number and Sherwood number distributions are studied in detail.

Key words: Casson fluid, porous space, chemical reaction, Soret number, Hartmann number.

1. Introduction

The study of magnetohydrodynamic (MHD) flows of non-Newtonian fluid in a porous medium has attracted the attention of many researchers. This is due to a wide variety of applications for the MHD flows of non-Newtonian fluids in a porous medium which are encountered in irrigation problems, heat-storage beds, biological systems, processing of petroleum, and textile, paper and polymer composite industries. Numerous studies have been presented on various aspects of MHD flows of non-Newtonian fluid flows passing through a porous medium [1–4]. Rosca and Pop [5] investigated the flow and heat transfer of Powell-Eyring fluid over a shrinking surface in a parallel free stream. Wenchang *et al.* [6] introduced the fractional calculus approach in the constitutive relationship model of a generalized Maxwell fluid between two parallel plates.

Khan *et al.* [7] examined the unsteady squeezing flow of a Casson fluid between parallel plates. Attia and Sayed-Ahmed [8] analyzed the unsteady magnetohydrodynamic flow of a non-Newtonian Casson fluid bounded by two parallel non-conducting porous plates. Chiu-On Ng [9] developed analytical solutions for steady electroosmotic (EO) flow of a Casson fluid through a parallel-plate microchannel. Hayat *et al.* [10] investigated the stagnation point flow and heat transfer of a Casson fluid towards a stretching surface. The boundary layer flow of a Casson fluid over an exponentially permeable shrinking sheet has been examined by Nadeem *et al.* [11].

The pulsatile flow in a porous channel or pipe is important to understand the process of dialysis of blood in an artificial kidney [12–15]. An investigation about heat transfer to pulsatile flow in a porous channel was made by Radhakrishnamacharya and Maiti [15]. In their investigation, the walls were maintained at uniform temperatures and fluid was injected through one wall and removed at the opposite wall at the same rate. Siddiqui *et al.* [16] studied the pulsatile flow of blood in a stenosed artery by modeling blood as a Casson fluid. Shit and Roy [17] investigated the pulsatile flow and heat transfer of a magneto-micropolar fluid through a stenosed artery under the influence of body acceleration. Shawky [18] studied the pulsatile flow with heat transfer of dusty magnetohydrodynamic Ree-Eyring fluid through a channel.

The study of thermal radiation has received much attention of several researchers because of its many applications in environmental and scientific processes, physics and engineering. For example, the research on thermal radiation is used in aeronautics, fire research, heating and cooling of channels, nuclear power plants, gas turbines, and various propulsion devices for missiles, aircraft, space vehicles, and satellites [19–23]. Hossain *et al.* [24] studied the effect of radiation on free convection from a porous vertical plate. An analysis of thermal radiation on steady MHD asymmetric flow past a semi-infinite stationary plate was made by Raptis *et al.* [25]. MukhopadhyaY [26] examined the thermal radiation effect on unsteady flow of a Casson fluid and heat transfer over a stretching surface in presence of suction/blowing. The effects of slip and thermal radiation on the MHD free convection flow of a Casson fluid over a cylinder in a non-Darcy porous medium were investigated by Makanda *et al.* [27].

The combined study of heat and mass transfer problems with chemical reaction are of great practical importance in many processes and therefore have received a considerable amount of attention in recent years. Possible applications of this type of flow can be found in many industries such as electric power industry, chemical industry, food processing, etc. A reaction is said to be of first order if the rate of reaction is directly proportional to concentration itself. In many chemical engineering processes, a chemical reaction between a foreign mass and the fluid does occur. These processes happen in numerous industrial applications such as polymer production, manufacturing of ceramics or glassware, food processing [28–32]. Das *et al.* [33] analyzed the effects of a first order chemical reaction on the flow past an impulsively started infinite vertical plate with constant heat flux and mass transfer. Nable *et al.* [34] examined the effects of chemical reaction and heat radiation on the MHD flow of a viscoelastic fluid through a porous medium over a horizontal stretching flat plate. Shehzad *et al.* [35] studied the effects of mass transfer on the MHD flow of a Casson fluid with chemical reaction and suction. Recently, Srinivas *et al.* [36] have presented a note on thermal-diffusion and chemical reaction effects on the MHD pulsating flow in a porous channel with slip and convective boundary conditions.

To the best of authors' knowledge, no investigation has been made yet that would analyze the MHD pulsating flow of a Casson fluid in a porous space with thermal radiation and chemical reaction. Such consideration is of great value in engineering and science research. Keeping in view the wide range of applications both in engineering and science, an attempt is made in this paper to study the effects of thermal radiation and chemical reaction on the pulsating MHD flow of a Casson fluid between two walls with Joule heating. The structure of the paper is as follows: the formulation of problem is given in Sec. 2. Section 3 comprises the solution of the problem. The results and discussion are presented in Sec. 4. Section 5 contains the concluding remarks.

2. Formulation of the problem

Let us consider the pulsatile flow of electrically conducting Casson fluid between two parallel walls in Darcian porous medium, at a distance h in a porous medium apart, which is driven by the unsteady pressure gradient

$$-\frac{1}{\rho}\frac{\partial p^*}{\partial x^*} = A(1 + \varepsilon\, e^{i\omega t^*}), \tag{2.1}$$

where A is a known constant, ε ($\ll 1$) is a suitably chosen positive quantity, ω is the frequency, p^* is pressure and ρ is density of the fluid. As shown in Fig. 1 a cartesian coordinate system is taken in such a way that the x^*-axis is taken along the lower wall and the y^*-axis is normal to it. A magnetic field of uniform strength B_0 is applied perpendicular to the walls. The rheological equation of state for an isotropic and incompressible flow of a Casson fluid is [10, 11, 26]

$$\tau_{ij} = \begin{cases} 2(\mu_B + P_{y^*}/\sqrt{2\pi})e_{ij}, & \pi > \pi_c, \\ 2(\mu_B + P_{y^*}/\sqrt{2\pi_c})e_{ij}, & \pi < \pi_c, \end{cases} \tag{2.2}$$

where τ_{ij} is the (i, j)-th component of the stress tensor, μ_B is the plastic dynamic viscosity of the non-Newtonian fluid, P_{y^*} is the yield stress of the fluid,

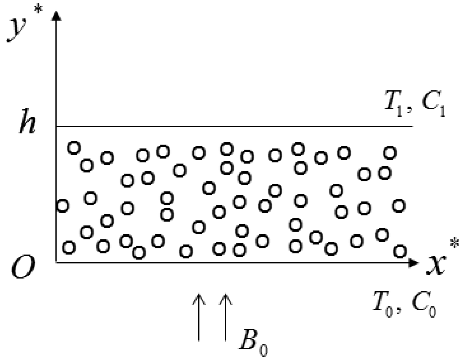

FIG. 1. Physical model of the studied problem.

$\pi = e_{ij}e_{ij}$; e_{ij} is the (i, j)-th component of the deformation rate and π_c is the critical value of this product based on the non-Newtonian model. Under these assumptions, the governing equations are

$$\frac{\partial u^*}{\partial t^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial x^*} + \nu\left(1 + \frac{1}{\beta}\right)\frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\sigma B_0^2}{\rho}u^* - \frac{\nu}{k}u^*, \tag{2.3}$$

$$\frac{\partial T^*}{\partial t^*} = \frac{\kappa}{\rho C_p}\frac{\partial^2 T^*}{\partial y^{*2}} + \frac{\mu}{\rho C_p}\left(1 + \frac{1}{\beta}\right)\left(\frac{\partial u^*}{\partial y^*}\right)^2 - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y^*} + \frac{\sigma B_0^2}{\rho C_p}u^{*2}, \tag{2.4}$$

$$\frac{\partial C^*}{\partial t^*} = D\frac{\partial^2 C^*}{\partial y^{*2}} + \frac{Dk_T}{T_m}\frac{\partial^2 T^*}{\partial y^{*2}} - k_1 C^*. \tag{2.5}$$

The corresponding boundary conditions are

$$u^* = 0, \qquad T^* = T_0, \qquad C^* = C_0 \qquad \text{at} \quad y^* = 0, \tag{2.6}$$

$$u^* = 0, \qquad T^* = T_1, \qquad C^* = C_1 \qquad \text{at} \quad y^* = h, \tag{2.7}$$

where u^* is dimensional velocity in x^* direction, ν is the kinematic viscosity, $\beta = \frac{\mu_B\sqrt{2\pi_c}}{P_{y^*}}$ is the Casson fluid parameter, σ is electrical conductivity, μ is the dynamic viscosity, k is the permeability of porous medium, C_p is the specific heat at constant pressure, κ is the thermal conductivity, T^*, C^* are the temperature and concentration of the fluid, respectively, T_0, $T_1(> T_0)$ are the temperatures of the lower and upper walls, respectively, C_0, $C_1(> C_0)$ are the concentrations of the lower and upper walls respectively, D is the coefficient of mass diffusivity, k_1 is the first order chemical reaction rate, k_T is the thermal diffusion ratio, T_m is the mean temperature of the fluid, and q_r is the radiative heat flux. By using the Rosseland approximation for radiative heat flux, q_r is defined as [20, 21, 23]

$$q_r = -\left(\frac{4\sigma^*}{3\chi}\right)\frac{\partial T^{*4}}{\partial y^*}, \tag{2.8}$$

where σ^* is the Stefan-Boltzmann constant, and χ is the Rosseland mean absorption co-efficient. We assume that the temperature differences within the flow are sufficiently small such that T^{*4} may be expressed as a linear function of the temperature. This is accomplished by expanding T^{*4} in a Taylor series about T_0 and neglecting higher order terms, thus:

$$T^{*4} \cong 4T_0^3 T^* - 3T_0^4. \tag{2.9}$$

On substituting Eqs. (2.8) and (2.9) into Eq. (2.4), we obtain

$$\frac{\partial T^*}{\partial t^*} = \frac{\kappa}{\rho C_p}\frac{\partial^2 T^*}{\partial y^{*2}} + \frac{\mu}{\rho C_p}\left(1+\frac{1}{\beta}\right)\left(\frac{\partial u^*}{\partial y^*}\right)^2 + \frac{1}{\rho C_p}\frac{16\sigma^* T_0^3}{3\chi}\frac{\partial^2 T^*}{\partial y^{*2}} + \frac{\sigma B_0^2}{\rho C_p}u^{*2}. \tag{2.10}$$

By introducing non-dimensional parameters,

$$\begin{gathered} u = \frac{u^*\omega}{A}, \qquad t = t^*\omega, \qquad x = \frac{x^*}{h}, \qquad y = \frac{y^*}{h}, \\ \theta = \frac{T^* - T_0}{T_1 - T_0}, \qquad \phi = \frac{C^* - C_0}{C_1 - C_0}, \qquad p = \frac{p^*}{A\rho h}, \end{gathered} \tag{2.11}$$

Eqs. (2.3), (2.10) and (2.5) become

$$\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \left(1+\frac{1}{\beta}\right)\frac{1}{H^2}\left(\frac{\partial^2 u}{\partial y^2}\right) - \frac{1}{H^2}\left(\mathrm{M}^2 + \frac{1}{\mathrm{Da}}\right)u, \tag{2.12}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\mathrm{Pr}\,H^2}\left(1+\frac{4}{3}\mathrm{Rd}\right)\frac{\partial^2\theta}{\partial y^2} + \left(1+\frac{1}{\beta}\right)\frac{\mathrm{Ec}}{H^2}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\mathrm{M}^2\mathrm{Ec}}{H^2}u^2, \tag{2.13}$$

$$\frac{\partial \phi}{\partial t} = \frac{1}{\mathrm{Sc}\,H^2}\frac{\partial^2\phi}{\partial y^2} + \frac{\mathrm{Sr}}{H^2}\frac{\partial^2\theta}{\partial y^2} - \frac{\gamma}{H^2}\phi - \frac{K_1}{H^2}, \tag{2.14}$$

where $\mathrm{Da} = \frac{k}{h^2}$ is the Darcy number of the porous media, $\mathrm{Pr} = \frac{\mu C_p}{\kappa}$ is the Prandtl number, $\mathrm{Ec} = \frac{(\frac{A}{\omega})^2}{C_p(T_1 - T_0)}$ is the Eckert number, $\mathrm{Rd} = \frac{4\sigma^* T_0^3}{\kappa\chi}$ is the radiation parameter, $\mathrm{M} = \frac{B_0 h\sqrt{\sigma}}{\sqrt{\mu}}$ is the Hartmann number, $H = \frac{h\sqrt{\omega}}{\sqrt{\nu}}$ is frequency parameter, $\mathrm{Sr} = \frac{D K_T (T_1 - T_0)}{T_m \nu (C_1 - C_0)}$ is the Soret number, $\mathrm{Sc} = \frac{\nu}{D}$ is the Schmidt number, $\gamma = \frac{k_1 h^2}{\nu}$ is the chemical reaction parameter and $K_1 = \frac{k_1 C_0 h^2}{\nu(C_1 - C_0)}$.

The corresponding boundary conditions are

$$u = 0, \qquad \theta = 0, \qquad \phi = 0 \qquad \text{at} \quad y = 0, \tag{2.15}$$

$$u = 0, \qquad \theta = 1, \qquad \phi = 1 \qquad \text{at} \quad y = 1. \tag{2.16}$$

3. Solution of the problem

The velocity u, temperature θ and concentration ϕ can be assumed to have the form [15, 18]

$$u = u_0(y) + \varepsilon u_1(y)e^{it}, \tag{3.1}$$

$$\theta = \theta_0(y) + \varepsilon\theta_1(y)e^{it} + \varepsilon^2\theta_2(y)e^{2it}, \tag{3.2}$$

$$\phi = \phi_0(y) + \varepsilon\phi_1(y)e^{it} + \varepsilon^2\phi_2(y)e^{2it}. \tag{3.3}$$

Now, by substituting Eqs. (3.1)–(3.3) into Eqs. (2.12)–(2.14) and then equating the coefficients of various powers of ε, we obtain

$$\left(1+\frac{1}{\beta}\right)u_0'' - \left(\mathrm{M}^2 + \frac{1}{\mathrm{Da}}\right)u_0 + H^2 = 0, \tag{3.4}$$

$$\left(1+\frac{1}{\beta}\right)u_1'' - \left(\mathrm{M}^2 + \frac{1}{\mathrm{Da}} + iH^2\right)u_1 + H^2 = 0, \tag{3.5}$$

$$\left(1+\frac{4}{3}\mathrm{Rd}\right)\theta_0'' + \left(1+\frac{1}{\beta}\right)\mathrm{Ec\,Pr}\,u_0'^2 + \mathrm{M}^2\mathrm{Ec\,Pr}\,u_0^2 = 0, \tag{3.6}$$

$$\begin{aligned}\left(1+\frac{4}{3}\mathrm{Rd}\right)\theta_1'' - iH^2\mathrm{Pr}\,\theta_1 + 2\left(1+\frac{1}{\beta}\right)\mathrm{Ec\,Pr}\,u_0'u_1' & \\ + 2\mathrm{M}^2\mathrm{Ec\,Pr}\,u_0u_1 &= 0,\end{aligned} \tag{3.7}$$

$$\left(1+\frac{4}{3}\mathrm{Rd}\right)\theta_2'' - 2iH^2\mathrm{Pr}\,\theta_2 + \left(1+\frac{1}{\beta}\right)\mathrm{Ec\,Pr}\,u_1'^2 + \mathrm{M}^2\mathrm{Ec\,Pr}\,u_1^2 = 0, \tag{3.8}$$

$$\phi_0'' - \gamma\mathrm{Sc}\phi_0 - K_1\mathrm{Sc} + \mathrm{Sc\,Sr}\,\theta_0'' = 0, \tag{3.9}$$

$$\phi_1'' - (iH^2\mathrm{Sc} + \gamma\mathrm{Sc})\phi_1 + \mathrm{Sc\,Sr}\,\theta_1'' = 0, \tag{3.10}$$

$$\phi_2'' - (2iH^2\mathrm{Sc} + \gamma\mathrm{Sc})\phi_2 + \mathrm{Sc\,Sr}\,\theta_2'' = 0. \tag{3.11}$$

The corresponding boundary conditions are:

$$\begin{array}{llll} u_0(0)=0, & u_0(1)=0, & u_1(0)=0, & u_1(1)=0, \\ \theta_0(0)=0, & \theta_0(1)=1, & \theta_1(0)=0, & \theta_1(1)=0, \\ \theta_2(0)=0, & \theta_2(1)=0, & \phi_0(0)=0, & \phi_0(1)=1, \\ \phi_1(0)=0, & \phi_1(1)=0, & \phi_2(0)=0, & \phi_2(1)=0. \end{array} \tag{3.12}$$

By solving Eqs. (3.4)-(3.11) with the corresponding boundary conditions (3.12), one obtains

$$u_0 = A_1 \cos \sqrt{B_1} y + A_2 \sin \sqrt{B_1} y + A_3, \tag{3.13}$$

$$u_1 = A_4 \cos \sqrt{B_2} y + A_5 \sin \sqrt{B_2} y + A_6, \tag{3.14}$$

$$\theta_0 = A_7 + A_8 y + A_9 y^2 + A_{10} \cos \sqrt{B_1} y + A_{11} \sin \sqrt{B_1} y + A_{12} \cos 2\sqrt{B_1} y + A_{13} \sin 2\sqrt{B_1} y, \tag{3.15}$$

$$\begin{aligned}\theta_1 = {} & A_{14} \cos \sqrt{B_3} y + A_{15} \sin \sqrt{B_3} y + A_{16} \cos(\sqrt{B_1} - \sqrt{B_2}) y \\ & + A_{17} \cos(\sqrt{B_1} + \sqrt{B_2}) y + A_{18} \sin(\sqrt{B_1} + \sqrt{B_2}) y \\ & + A_{19} \sin(\sqrt{B_1} - \sqrt{B_2}) y + A_{20} \cos \sqrt{B_1} y + A_{21} \sin \sqrt{B_1} y \\ & + A_{22} \cos \sqrt{B_2} y + A_{23} \sin \sqrt{B_2} y + A_{24},\end{aligned} \tag{3.16}$$

$$\begin{aligned}\theta_2 = {} & A_{25} \cos \sqrt{B_4} y + A_{26} \sin \sqrt{B_4} y + A_{27} \cos 2\sqrt{B_2} y \\ & + A_{28} \sin 2\sqrt{B_2} y + A_{29} \sin \sqrt{B_2} y + A_{30} \cos \sqrt{B_2} y + A_{31},\end{aligned} \tag{3.17}$$

$$\begin{aligned}\phi_0 = {} & A_{32} \cos \sqrt{B_5} y + A_{33} \sin \sqrt{B_5} y + A_{34} \cos \sqrt{B_1} y \\ & + A_{35} \sin \sqrt{B_1} y + A_{36} \cos 2\sqrt{B_1} y + A_{37} \sin 2\sqrt{B_1} y + A_{38},\end{aligned} \tag{3.18}$$

$$\begin{aligned}\phi_1 = {} & A_{39} \cos \sqrt{B_6} y + A_{40} \sin \sqrt{B_6} y + A_{41} \cos \sqrt{B_3} y + A_{42} \sin \sqrt{B_3} y \\ & + A_{43} \cos(\sqrt{B_1} - \sqrt{B_2}) y + A_{44} \cos(\sqrt{B_1} + \sqrt{B_2}) y \\ & + A_{45} \sin(\sqrt{B_1} + \sqrt{B_2}) y + A_{46} \sin(\sqrt{B_1} - \sqrt{B_2}) y + A_{47} \cos \sqrt{B_1} y \\ & + A_{48} \sin \sqrt{B_1} y + A_{49} \cos \sqrt{B_2} y + A_{50} \sin \sqrt{B_2} y,\end{aligned} \tag{3.19}$$

$$\begin{aligned}\phi_2 = {} & A_{51} \cos \sqrt{B_7} y + A_{52} \sin \sqrt{B_7} y + A_{53} \cos \sqrt{B_4} y + A_{54} \sin \sqrt{B_4} y \\ & + A_{55} \cos 2\sqrt{B_2} y + A_{56} \sin 2\sqrt{B_2} y + A_{57} \sin \sqrt{B_2} y + A_{58} \cos \sqrt{B_2} y,\end{aligned} \tag{3.20}$$

where A's and B's are constants given in the Appendix.

Next, the heat and mass transfer rates in terms of Nusselt number and Sherwood number at the walls respectively are defined as

$$\mathrm{Nu} = -\left(\frac{\partial \theta}{\partial y}\right)_{y=0,1} \qquad \text{and} \qquad \mathrm{Sh} = -\left(\frac{\partial \phi}{\partial y}\right)_{y=0,1}. \tag{3.21}$$

4. Results and discussion

In order to get the physical insight of the problem, velocity, temperature, concentration, Nusselt number and Sherwood number distributions have been discussed by assigning numerical values to various parameters obtained in mathematical formulation of the problem and the results are shown graphically. In this analysis u_s, θ_s, ϕ_s, u_t, θ_t, ϕ_t represent steady velocity, steady temperature, steady concentration, unsteady velocity, unsteady temperature, unsteady concentration respectively. Figure 2 shows the influence of Hartmann number (M), Darcy number (Da), Casson parameter/non-Newtonian parameter (β), and frequency parameter (H), on the velocity distribution. Figure 2a shows that for a given increase in Hartmann number, there is a decrease in velocity. This is due to the fact that the retarding forces (called Lorentz forces) generated by the applied magnetic field act as resistive drag forces opposite to the flow direction which results a decrease in velocity. Figure 2b depicts the variation of velocity for different values of Da. It is noticed that the velocity is an increasing func-

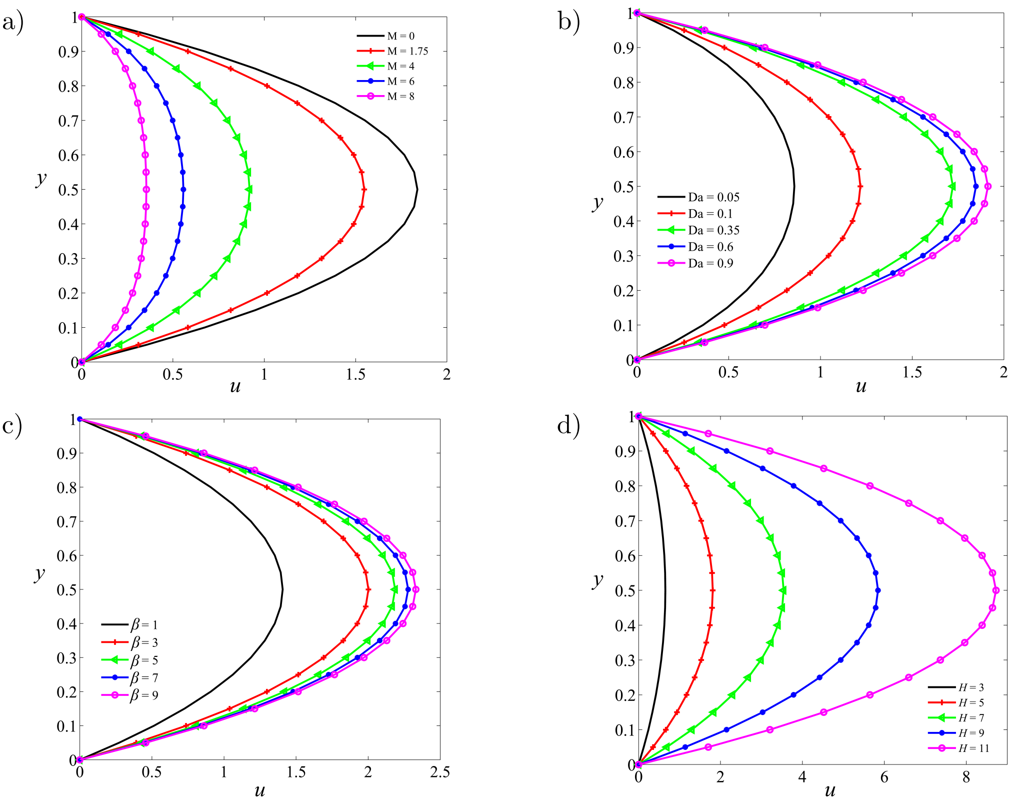

Fig. 2. Velocity distribution for $\varepsilon = 0.01$, $t = \pi/4$: a) effect of M when $\beta = 2$, Da $= 0.5$, $H = 5$, b) effect of Da when $\beta = 2$, $H = 5$, M $= 0.5$, c) effect of β when Da $= 0.5$, $H = 5$, M $= 0.5$, d) effect of H when $\beta = 2$, Da $= 0.5$, M $= 0.5$.

tion of Da. Because the linear porous drag force called the Darcian drag force is inversely proportional to Da (see the last term of Eq. (2.12), i.e., $-\frac{u}{H^2\mathrm{Da}}$) an increase in permeability of porous regions will increase Da which will act as the Darcian drag force. Hence, there is an increase in velocity with increase in Da. From Figs. 2c and 2d, it is clear that the velocity increases with an increase in Casson parameter and frequency parameter. Figure 3 demonstrate the variation of unsteady velocity for frequency parameter and various values of t. From Fig. 3a it is noticed that the unsteady velocity profiles exhibit oscillating character with an increasing frequency parameter. For small values of H the profiles are almost parabolic in nature. The maximum velocity is shifted to the boundary layers near the walls. From Fig. 3b, one can observe that the unsteady velocity profiles oscillate with increasing t.

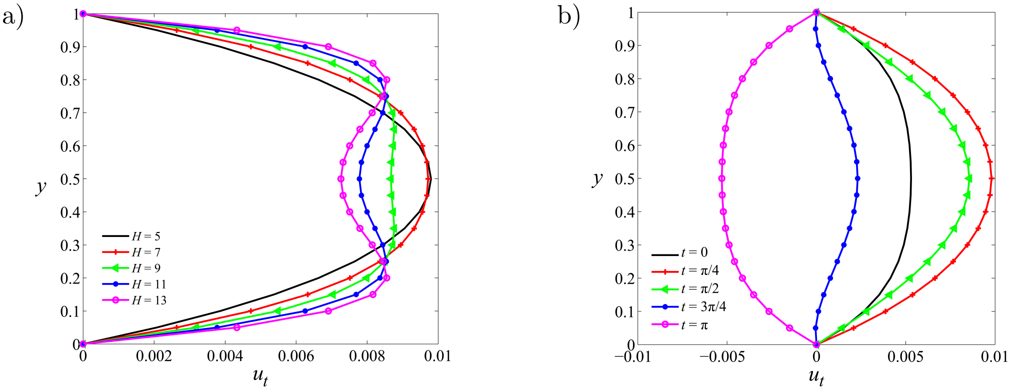

FIG. 3. Unsteady velocity distribution for $\varepsilon = 0.01$, $\beta = 2$, $\mathrm{M} = 0.5$, $\mathrm{Da} = 0.5$: a) effect of H, b) effect of t.

Figure 4 shows the influence of Casson parameter and radiation parameter on the temperature distribution. Figure 4a shows that for a given increase in Casson parameter, there is an increase in temperature. Figure 4b depicts the variation of temperature distribution for different values of radiation parameter. It is noticed that the temperature is a decreasing function of Rd. The influence of Eckert number (Ec), Hartmann number (M) and radiation parameter on steady and unsteady temperature distributions are shown in Figs. 5–7. Figure 5 shows the effect of Ec on steady and unsteady temperature distributions. It is observed that the steady temperature increases with an increasing Ec. This increase in temperature may be due to heat created by viscous distribution (see Fig. 5a). From Fig. 5b it is observed that the unsteady temperature exhibits oscillating character and oscillations increase with an increasing Ec and the maximum is shifted to the boundary layers near the walls. From Fig. 6 it is clear that the steady and unsteady temperatures decrease near the walls with an increasing M while they increase near the center. Furthermore, the unsteady

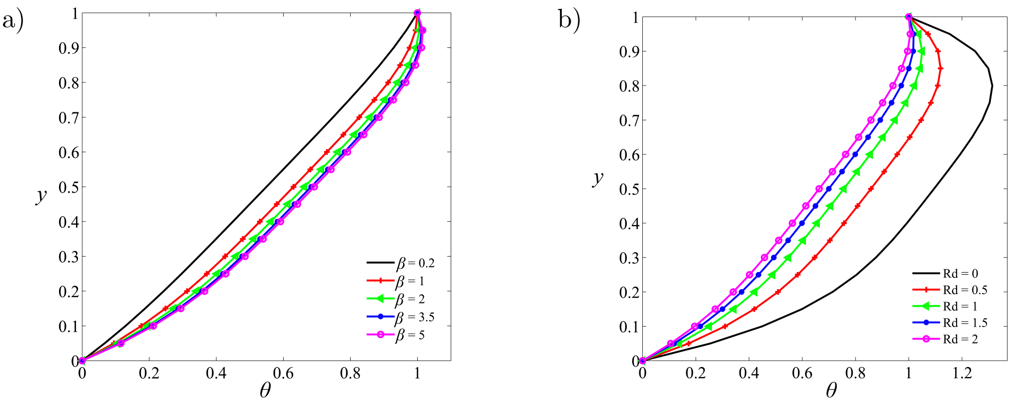

FIG. 4. a) Effect of β on temperature distribution when $\varepsilon = 0.01$, $\mathrm{M} = 0.5$, $t = \pi/4$, $H = 5$, $\mathrm{Ec} = 0.5$, $\mathrm{Pr} = 0.71$, $\mathrm{Rd} = 2$, $\mathrm{Da} = 0.5$, b) effect of Rd on temperature distribution when $\varepsilon = 0.01$, $\beta = 2$, $t = \pi/4$, $H = 5$, $\mathrm{M} = 0.5$, $\mathrm{Pr} = 0.71$, $\mathrm{Ec} = 0.5$, $\mathrm{Da} = 0.5$.

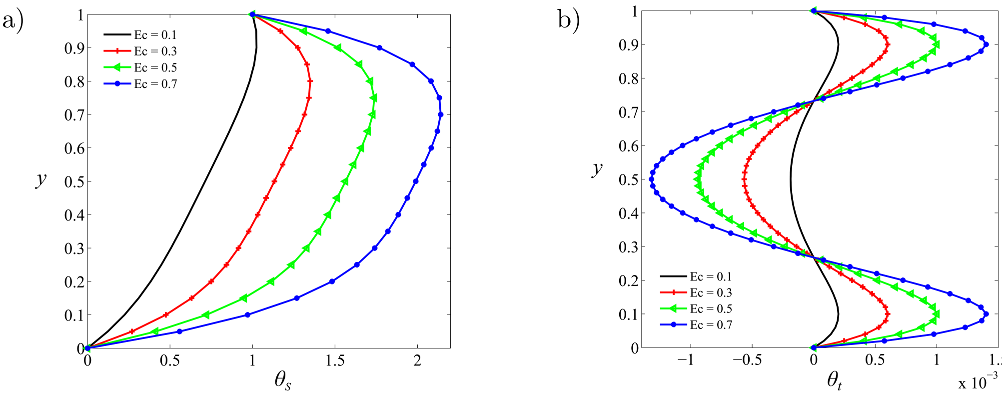

FIG. 5. Effect of Ec on temperature distribution when $\varepsilon = 0.01$, $\beta = 2$, $t = \pi/4$, $H = 8$, $\mathrm{M} = 0.5$, $\mathrm{Pr} = 0.71$, $\mathrm{Rd} = 2$, $\mathrm{Da} = 0.5$.

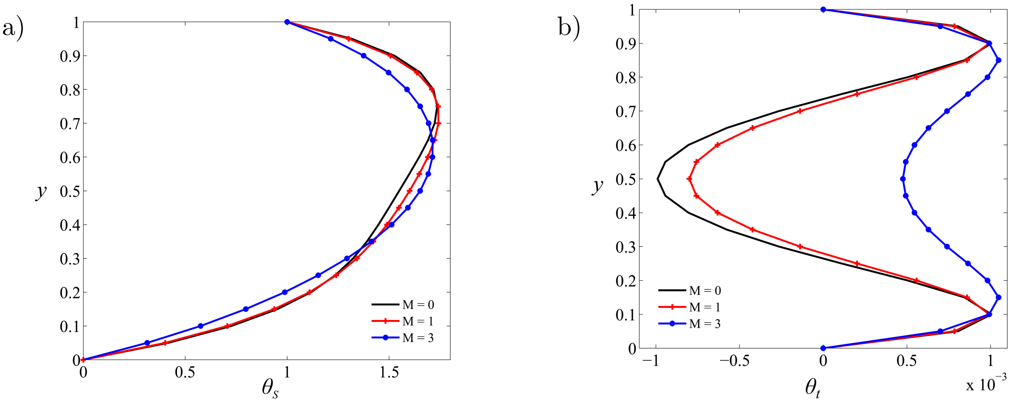

FIG. 6. Effect of M on temperature distribution when $\varepsilon = 0.01$, $\beta = 2$, $t = \pi/4$, $H = 8$, $\mathrm{Ec} = 0.5$, $\mathrm{Pr} = 0.71$, $\mathrm{Rd} = 2$, $\mathrm{Da} = 0.5$.

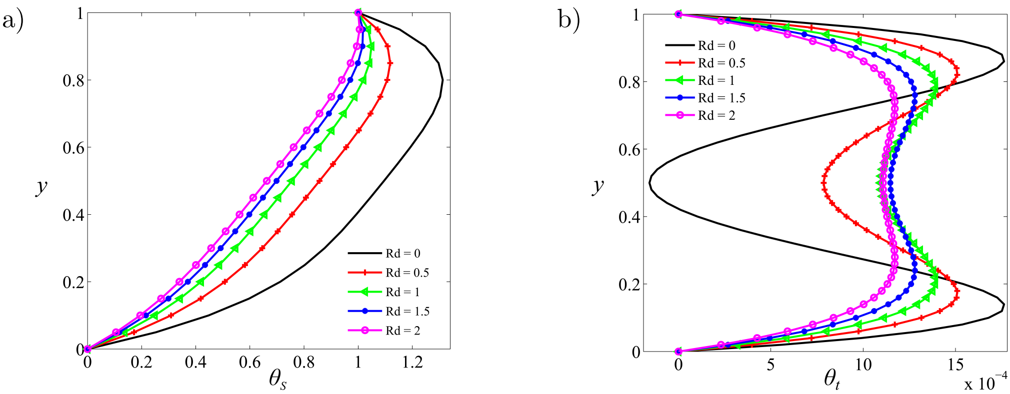

FIG. 7. Effect of Rd on temperature distribution when $\varepsilon = 0.01$, $\beta = 2$, $t = \pi/4$, $H = 5$, $\mathrm{M} = 0.5$, $\mathrm{Pr} = 0.71$, $\mathrm{Ec} = 0.5$, $\mathrm{Da} = 0.5$.

temperature oscillates with an increasing M and the amplitude decreases with an increasing M. Figure 7 depicts the variation of steady and unsteady temperature profiles for different values of radiation parameter. It is noticed that the steady temperature decreases for a given increase in radiation parameter (see Fig. 7a). From Fig. 7b one can see that the unsteady temperature oscillating with an increasing Rd and the maximum is shifted to the near the walls. The effect of t on unsteady temperature distribution is shown in Fig. 8. One can notice that the unsteady temperature profiles oscillate with increasing t.

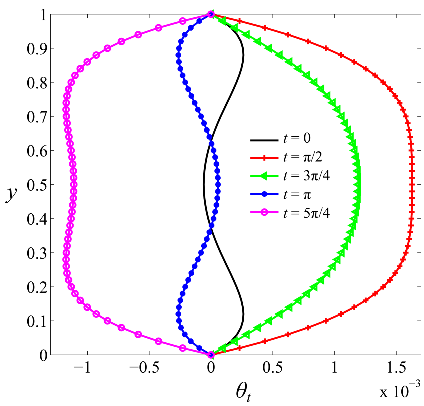

FIG. 8. Effect of t on unsteady temperature distribution when $\varepsilon = 0.01$, $\beta = 2$, $\mathrm{Rd} = 2$, $H = 5$, $\mathrm{M} = 0.5$, $\mathrm{Pr} = 0.71$, $\mathrm{Ec} = 0.5$, $\mathrm{Da} = 0.5$.

Figures 9–11 presents the effects of the chemical reaction parameter (γ), the Schmidt number (Sc) and the Soret number (Sr) on steady and unsteady concentration distributions. Figure 9 shows the effect of γ on steady and unsteady concentration distributions. It is observed that the steady and unsteady concentration decrease with increasing γ. This is due to fact that for a given increase

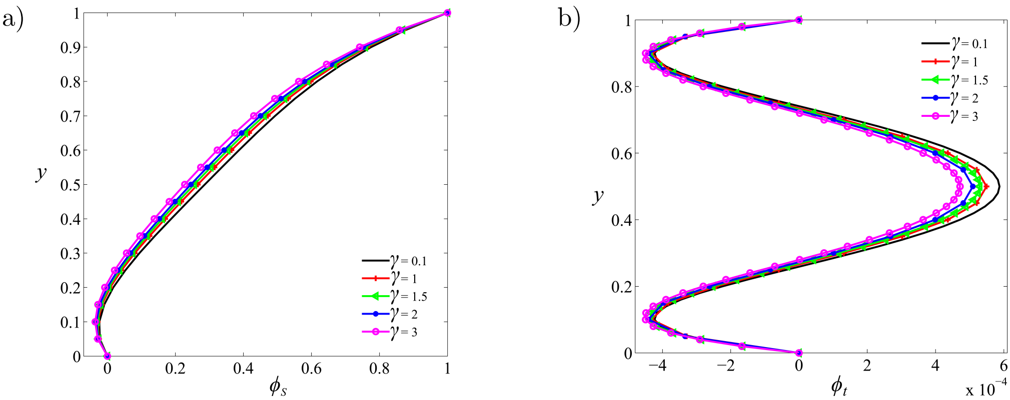

FIG. 9. Effect of γ on concentration distribution when $\varepsilon = 0.01$, $\beta = 2$, $t = \pi/4$, $H = 5$, $\mathrm{M} = 0.5$, $\mathrm{Pr} = 0.71$, $\mathrm{Ec} = 0.5$, $\mathrm{Da} = 0.5$, $\mathrm{Rd} = 2$, $\mathrm{Sc} = 0.65$, $K_1 = 0.001$, $\mathrm{Sr} = 2$.

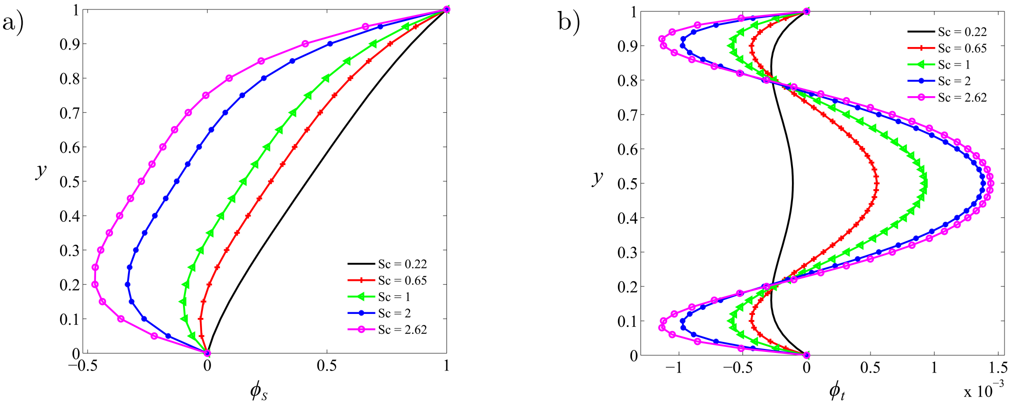

FIG. 10. Effect of Sc on concentration distribution when $\varepsilon = 0.01$, $\beta = 2$, $t = \pi/4$, $H = 5$, $\mathrm{M} = 0.5$, $\mathrm{Pr} = 0.71$, $\mathrm{Ec} = 0.5$, $\mathrm{Da} = 0.5$, $\mathrm{Rd} = 2$, $\gamma = 1$, $\beta = 2$, $K_1 = 0.001$.

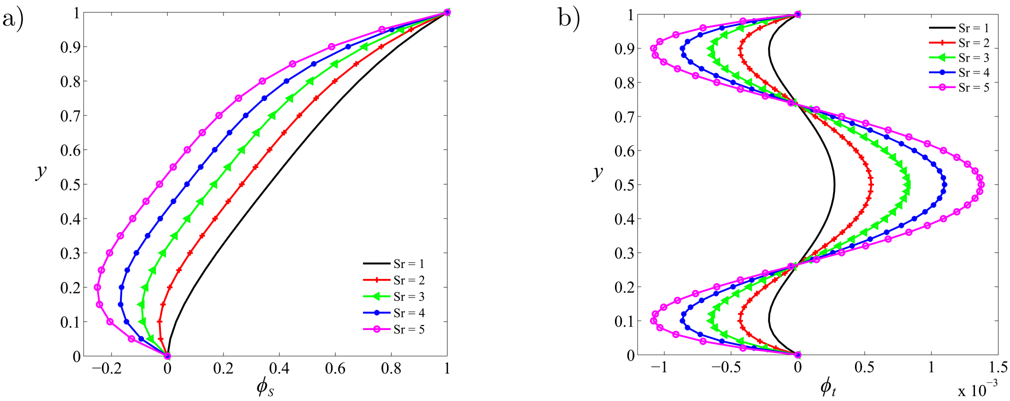

FIG. 11. Effect of Sr on concentration distribution when $\varepsilon = 0.01$, $\beta = 2$, $t = \pi/4$, $H = 5$, $\mathrm{M} = 0.5$, $\mathrm{Pr} = 0.71$, $\mathrm{Ec} = 0.5$, $\mathrm{Da} = 0.5$, $\mathrm{Rd} = 2$, $\gamma = 1$, $\mathrm{Sc} = 0.65$, $\beta = 2$, $K_1 = 0.001$.

in chemical reaction there is a decrease in the concentration boundary layer because the destructive chemical reaction reduces the solutal boundary layer thickness and increases the mass transfer. Moreover, the unsteady concentration exhibits oscillating character and the minimum is shifted to the boundary layers near the walls (see Fig. 9b). The similar behaviour can be observed from Figs. 10 and 11 by varying Sc and Sr.

Figure 12 demonstrates the effects of M and Rd on Nusselt number distribution (Nu) against H. From Fig. 12a it is noticed that for a given increase in Hartmann number, Nu increases at the lower wall while it decreases at the upper wall. The similar behaviour can be found by varying Rd (see Fig. 12b). The influence of Sc and Sr on Sherwood number distribution (Sh) against H is shown in Fig. 13. From this figure one can infer that Sh is an increasing function of Sc and Sr at the lower wall while it is a decreasing function at the upper wall.

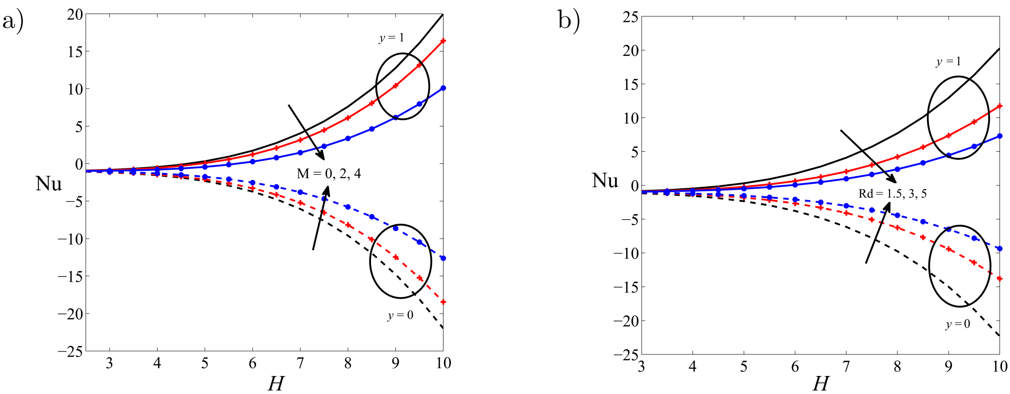

FIG. 12. Nusselt number distribution for $\varepsilon = 0.01$, $t = \pi/4$, $\beta = 2$, $\mathrm{Ec} = 0.5$, $\mathrm{Da} = 0.5$, $\mathrm{Pr} = 0.71$: a) effect of M when $\mathrm{Rd} = 2$, b) effect of Rd when $\mathrm{M} = 2$.

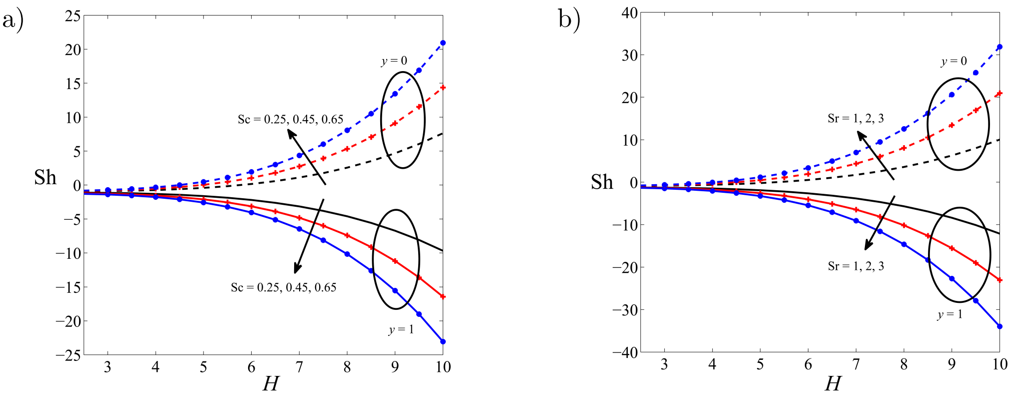

FIG. 13. Sherwood number distribution for $\varepsilon = 0.01$, $t = \pi/4$, $\beta = 2$, $\mathrm{Ec} = 0.5$, $\mathrm{M} = 2$, $\mathrm{Da} = 0.5$, $\mathrm{Pr} = 0.71$, $\mathrm{Rd} = 2$, $\gamma = 1$, $K_1 = 0.001$: a) effect of Sc when $\mathrm{Sr} = 2$, b) effect of Sr when $\mathrm{Sc} = 0.65$.

5. Conclusion

In the present analysis, the pulsating MHD flow of a Casson fluid in a porous space with thermal radiation, thermal-diffusion, Joule heating and chemical reaction has been investigated. Analytical solutions are obtained for flow variables. The main findings are summarized as follows:

- The velocity decreases for a given increasing Hartmann number while it is increasing with Da and H.
- The temperature distribution increases with an increasing Casson parameter while it is decreases with an increasing Rd.
- The concentration distributions in steady and unsteady cases decreases with an increase in chemical reaction parameter.
- The steady concentration decreases with increasing Sc and Sr while the unsteady concentration distribution oscillates with increasing Sc and Sr.
- Nusselt number distribution increases with an increasing Rd at the lower wall while it decreases at the upper wall.
- Sherwood number is a increasing function of Sc and Sr at the lower wall.

Appendix

$$A_1 = -A_3,$$

$$A_2 = \frac{A_3(\cos\sqrt{B_1} - 1)}{\sin\sqrt{B_1}},$$

$$A_3 = -\frac{H^2}{(1+\frac{1}{\beta})B_1},$$

$$A_4 = -A_6,$$

$$A_5 = \frac{A_6(\cos\sqrt{B_2} - 1)}{\sin\sqrt{B_2}},$$

$$A_6 = -\frac{H^2}{(1+\frac{1}{\beta})B_2},$$

$$A_7 = -A_{10} - A_{12},$$

$$A_8 = 1 - A_9 + A_{10}(1 - \cos\sqrt{B_1}) - A_{11}\sin\sqrt{B_1} + A_{12}(1 - \cos 2\sqrt{B_1}) - A_{13}\sin 2\sqrt{B_1},$$

$$A_9 = D_1 + D_2 + D_3 + D_4 + D_5,$$

$$A_{10} = \frac{2\mathrm{M}^2\,\mathrm{Ec}\,\mathrm{Pr}\,A_1 A_3}{B_1(1+\frac{4}{3}\mathrm{Rd})},$$

$$A_{11} = \frac{2\mathrm{M}^2\,\mathrm{Ec}\,\mathrm{Pr}\,A_2 A_3}{B_1(1+\frac{4}{3}\mathrm{Rd})},$$

$$A_{12} = D_6 + D_7 + D_8 + D_9,$$

$$A_{13} = D_{10} + D_{11},$$

$$A_{14} = -A_{16} - A_{17} - A_{20} - A_{22} - A_{24},$$

$$\begin{aligned} A_{15} = \Big(&(A_{16} + A_{17} + A_{20} + A_{22} + A_{24})\cos(\sqrt{B_3}) - A_{16}\cos(\sqrt{B_1} - \sqrt{B_2}) \\ &- A_{17}\cos(\sqrt{B_1} + \sqrt{B_2}) - A_{18}\sin(\sqrt{B_1} + \sqrt{B_2}) - A_{19}\sin(\sqrt{B_1} - \sqrt{B_2}) \\ &- A_{20}\cos(\sqrt{B_1}) - A_{21}\sin(\sqrt{B_1}) - A_{22}\cos(\sqrt{B_2}) \\ &- A_{23}\sin(\sqrt{B_2}) - A_{24}\Big)/\sin(\sqrt{B_3}), \end{aligned}$$

$$A_{16} = D_{12} + D_{13} + D_{14} + D_{15},$$

$$A_{17} = D_{16} + D_{17} + D_{18} + D_{19},$$

$$A_{18} = D_{20} + D_{21} + D_{22} + D_{23},$$

$$A_{19} = D_{24} + D_{25} + D_{26} + D_{27},$$

$$A_{20} = -\frac{2\mathrm{M}^2\,\mathrm{Ec}\,\mathrm{Pr}\,A_1 A_6}{\big(1+\frac{4}{3}\mathrm{Rd}\big)(-B_1 + B_3)},$$

$$A_{21} = -\frac{2\mathrm{M}^2\,\mathrm{Ec}\,\mathrm{Pr}\,A_2 A_6}{\big(1+\frac{4}{3}\mathrm{Rd}\big)(-B_1 + B_3)},$$

$$A_{22} = -\frac{2\mathrm{M}^2\,\mathrm{Ec}\,\mathrm{Pr}\,A_3 A_4}{(1+\frac{4}{3}\mathrm{Rd})(-B_2 + B_3)},$$

$$A_{23} = -\frac{2\mathrm{M}^2\,\mathrm{Ec}\,\mathrm{Pr}\,A_3 A_5}{(1+\frac{4}{3}\mathrm{Rd})(-B_2 + B_3)},$$

$$A_{24} = -\frac{2\mathrm{M}^2\,\mathrm{Ec}\,\mathrm{Pr}\,A_3 A_6}{(1+\frac{4}{3}\mathrm{Rd})(B_3)},$$

$$A_{25} = -A_{27} - A_{30} - A_{31},$$

$$A_{26} = \Big((A_{27} + A_{30} + A_{31}) \cos\sqrt{B_4} - A_{27}\cos 2\sqrt{B_2} - A_{28}\sin 2\sqrt{B_2} - A_{29}\sin\sqrt{B_2} - A_{30}\cos\sqrt{B_2} - A_{31}\Big)/\sin\sqrt{B_4},$$

$$A_{27} = D_{28} + D_{29} + D_{30} + D_{31},$$

$$A_{28} = D_{32} + D_{33},$$

$$A_{29} = -\frac{2\mathrm{M}^2\,\mathrm{Ec}\,\mathrm{Pr}\,A_5A_6}{(1+\frac{4}{3}\mathrm{Rd})(B_4 - B_2)},$$

$$A_{30} = -\frac{2\mathrm{M}^2\,\mathrm{Ec}\,\mathrm{Pr}\,A_4A_6}{(1+\frac{4}{3}\mathrm{Rd})(B_4 - B_2)},$$

$$A_{31} = D_{34} + D_{35} + D_{36} + D_{37} + D_{38},$$

$$A_{32} = -A_{34} - A_{36} - A_{38},$$

$$A_{33} = \Big(1 + (A_{34} + A_{36} + A_{38})\cos\sqrt{B_5} - A_{34}\cos\sqrt{B_1} - A_{35}\sin\sqrt{B_1} + A_{36}\cos 2\sqrt{B_1} - A_{37}\sin 2\sqrt{B_1} - A_{38}\Big)/\sin\sqrt{B_5},$$

$$A_{34} = \frac{A_{10}B_1\mathrm{Sc}\,\mathrm{Sr}}{B_5 - B_1},$$

$$A_{35} = \frac{A_{11}B_1\mathrm{Sc}\,\mathrm{Sr}}{B_5 - B_1},$$

$$A_{36} = \frac{4A_{12}B_1\mathrm{Sc}\,\mathrm{Sr}}{B_5 - 4B_1},$$

$$A_{37} = \frac{4A_{13}B_1\mathrm{Sc}\,\mathrm{Sr}}{B_5 - 4B_1},$$

$$A_{38} = D_{39} + D_{40},$$

$$A_{39} = -A_{41} - A_{43} - A_{44} - A_{47} - A_{49},$$

$$A_{40} = \Big((A_{41} + A_{43} + A_{44} + A_{47} + A_{49})\cos\sqrt{B_6} - A_{41}\cos\sqrt{B_3} - A_{42}\sin\sqrt{B_3} - A_{43}\cos(\sqrt{B_1} - \sqrt{B_2}) - A_{44}\cos(\sqrt{B_1} + \sqrt{B_2}) - A_{45}\sin(\sqrt{B_1} + \sqrt{B_2}) - A_{46}\sin(\sqrt{B_1} - \sqrt{B_2}) - A_{47}\cos\sqrt{B_1} - A_{48}\sin\sqrt{B_1} - A_{49}\cos\sqrt{B_2} - A_{50}\sin\sqrt{B_2}\Big)/\sin\sqrt{B_6},$$

$$A_{41} = \frac{ScSrA_{14}B_3}{-B_3 + B_6},$$

$$A_{42} = \frac{\mathrm{Sc}\,\mathrm{Sr}\,A_{15}B_3}{-B_3 + B_6},$$

$$A_{43} = \frac{\mathrm{Sc}\,\mathrm{Sr}\,A_{16}(\sqrt{B_1} - \sqrt{B_2})^2}{-(\sqrt{B_1} - \sqrt{B_2})^2 + B_6},$$

$$A_{44} = \frac{\mathrm{Sc}\,\mathrm{Sr}\,A_{17}(\sqrt{B_1} + \sqrt{B_2})^2}{-(\sqrt{B_1} + \sqrt{B_2})^2 + B_6},$$

$$A_{45} = \frac{\mathrm{Sc}\,\mathrm{Sr}\,A_{18}(\sqrt{B_1} + \sqrt{B_2})^2}{-(\sqrt{B_1} + \sqrt{B_2})^2 + B_6},$$

$$A_{46} = \frac{\mathrm{Sc}\,\mathrm{Sr}\,A_{19}(\sqrt{B_1} - \sqrt{B_2})^2}{-(\sqrt{B_1} - \sqrt{B_2})^2 + B_6},$$

$$A_{47} = \frac{\mathrm{Sc}\,\mathrm{Sr}\,A_{20}B_1}{-B_1 + B_6},$$

$$A_{48} = \frac{\mathrm{Sc}\,\mathrm{Sr}\,A_{21}B_1}{-B_1 + B_6},$$

$$A_{49} = \frac{\mathrm{Sc}\,\mathrm{Sr}\,A_{22}B_2}{-B_2 + B_6},$$

$$A_{50} = \frac{\mathrm{Sc}\,\mathrm{Sr}\,A_{23}B_2}{-B_2 + B_6},$$

$$A_{51} = -A_{53} - A_{55} - A_{58},$$

$$\begin{aligned} A_{52} = \Big(&(A_{53}+A_{55}+A_{58})\cos\sqrt{B_7} - A_{53}\cos\sqrt{B_4} - A_{54}\sin\sqrt{B_4} - A_{55}\cos 2\sqrt{B_2} \\ &- A_{56}\sin 2\sqrt{B_2} - A_{57}\sin\sqrt{B_2} - A_{58}\cos\sqrt{B_2}\Big)/\sin\sqrt{B_7}, \end{aligned}$$

$$A_{53} = \frac{A_{25}B_4\,\mathrm{Sc}\,\mathrm{Sr}}{B_7 - B_4},$$

$$A_{54} = \frac{A_{26}B_4\,\mathrm{Sc}\,\mathrm{Sr}}{B_7 - B_4},$$

$$A_{55} = \frac{4A_{27}B_2\,\mathrm{Sc}\,\mathrm{Sr}}{B_7 - 4B_2},$$

$$A_{56} = \frac{4A_{28}B_2\,\mathrm{Sc}\,\mathrm{Sr}}{B_7 - 4B_2},$$

$$A_{57} = \frac{A_{29}B_2\,\mathrm{Sc}\,\mathrm{Sr}}{B_7 - B_2},$$

$$A_{58} = \frac{A_{30} B_2 \,\mathrm{Sc}\,\mathrm{Sr}}{B_7 - B_2},$$

$$B_1 = -\frac{\mathrm{M}^2 + \frac{1}{\mathrm{Da}}}{1 + \frac{1}{\beta}},$$

$$B_2 = -\frac{\mathrm{M}^2 + \frac{1}{\mathrm{Da}} + iH^2}{1 + \frac{1}{\beta}},$$

$$B_3 = -\frac{iH^2 \,\mathrm{Pr}}{1 + \frac{4}{3}\mathrm{Rd}},$$

$$B_4 = -\frac{2iH^2 \,\mathrm{Pr}}{1 + \frac{4}{3}\mathrm{Rd}},$$

$$B_5 = -\gamma \,\mathrm{Sc},$$

$$B_6 = -(iH^2 \,\mathrm{Sc} + \gamma \,\mathrm{Sc}),$$

$$B_7 = -(2iH^2 \,\mathrm{Sc} + \gamma \,\mathrm{Sc}),$$

$$D_1 = -\frac{(1 + \frac{1}{\beta})\mathrm{Ec}\,\mathrm{Pr}\, B_1 A_1^2}{4(1 + \frac{4}{3}\mathrm{Rd})},$$

$$D_2 = -\frac{(1 + \frac{1}{\beta})\mathrm{Ec}\,\mathrm{Pr}\, B_1 A_2^2}{4(1 + \frac{4}{3}\mathrm{Rd})},$$

$$D_3 = -\frac{\mathrm{M}^2\,\mathrm{Ec}\,\mathrm{Pr}\, A_1^2}{4(1 + \frac{4}{3}\mathrm{Rd})},$$

$$D_4 = -\frac{\mathrm{M}^2\,\mathrm{Ec}\,\mathrm{Pr}\, A_2^2}{4(1 + \frac{4}{3}\mathrm{Rd})},$$

$$D_5 = -\frac{\mathrm{M}^2\,\mathrm{Ec}\,\mathrm{Pr}\, A_3^2}{2(1 + \frac{4}{3}\mathrm{Rd})},$$

$$D_6 = -\frac{(1 + \frac{1}{\beta})\mathrm{Ec}\,\mathrm{Pr}\, B_1 A_1^2}{8B_1(1 + \frac{4}{3}\mathrm{Rd})},$$

$$D_7 = \frac{(1 + \frac{1}{\beta})\mathrm{Ec}\,\mathrm{Pr}\, B_1 A_2^2}{8B_1(1 + \frac{4}{3}\mathrm{Rd})},$$

$$D_8 = \frac{\mathrm{M}^2\,\mathrm{Ec}\,\mathrm{Pr}\, A_1^2}{8B_1(1 + \frac{4}{3}\mathrm{Rd})},$$

$$D_9 = -\frac{\mathrm{M}^2\,\mathrm{Ec}\,\mathrm{Pr}\, A_2^2}{8B_1(1 + \frac{4}{3}\mathrm{Rd})},$$

$$D_{10} = -\frac{(1 + \frac{1}{\beta})\mathrm{Ec}\,\mathrm{Pr}\, B_1 A_1 A_2}{4B_1(1 + \frac{4}{3}\mathrm{Rd})},$$

$$D_{11} = \frac{\mathrm{M}^2\,\mathrm{Ec}\,\mathrm{Pr}\, A_1 A_2}{4B_1(1 + \frac{4}{3}\mathrm{Rd})},$$

$$D_{12} = -\frac{(1 + \frac{1}{\beta})\mathrm{Ec}\,\mathrm{Pr}\, A_1 A_4 \sqrt{B_1}\sqrt{B_2}}{(1 + \frac{4}{3}\mathrm{Rd})(-(\sqrt{B_1} - \sqrt{B_2})^2 + B_3)},$$

$$D_{13} = -\frac{(1 + \frac{1}{\beta})\mathrm{Ec}\,\mathrm{Pr}\, A_2 A_5 \sqrt{B_1}\sqrt{B_2}}{(1 + \frac{4}{3}\mathrm{Rd})(-(\sqrt{B_1} - \sqrt{B_2})^2 + B_3)},$$

$$D_{14} = -\frac{\mathrm{M}^2\,\mathrm{Ec}\,\mathrm{Pr}\, A_1 A_4}{(1 + \frac{4}{3}\mathrm{Rd})(-(\sqrt{B_1} - \sqrt{B_2})^2 + B_3)},$$

$$D_{15} = -\frac{\mathrm{M}^2\,\mathrm{Ec}\,\mathrm{Pr}\, A_2 A_5}{(1 + \frac{4}{3}\mathrm{Rd})(-(\sqrt{B_1} - \sqrt{B_2})^2 + B_3)},$$

$$D_{16} = \frac{(1 + \frac{1}{\beta})\mathrm{Ec}\,\mathrm{Pr}\, A_1 A_4 \sqrt{B_1}\sqrt{B_2}}{(1 + \frac{4}{3}\mathrm{Rd})(-(\sqrt{B_1} + \sqrt{B_2})^2 + B_3)},$$

$$D_{17} = -\frac{(1 + \frac{1}{\beta})\mathrm{Ec}\,\mathrm{Pr}\, A_2 A_5 \sqrt{B_1}\sqrt{B_2}}{(1 + \frac{4}{3}\mathrm{Rd})(-(\sqrt{B_1} + \sqrt{B_2})^2 + B_3)},$$

$$D_{18} = -\frac{\mathrm{M}^2\,\mathrm{Ec}\,\mathrm{Pr}\, A_1 A_4}{(1 + \frac{4}{3}\mathrm{Rd})(-(\sqrt{B_1} + \sqrt{B_2})^2 + B_3)},$$

$$D_{19} = \frac{\mathrm{M}^2\,\mathrm{Ec}\,\mathrm{Pr}\, A_2 A_5}{(1 + \frac{4}{3}\mathrm{Rd})(-(\sqrt{B_1} + \sqrt{B_2})^2 + B_3)},$$

$$D_{20} = \frac{(1 + \frac{1}{\beta})\mathrm{Ec}\,\mathrm{Pr}\, A_1 A_5 \sqrt{B_1}\sqrt{B_2}}{(1 + \frac{4}{3}\mathrm{Rd})(-(\sqrt{B_1} + \sqrt{B_2})^2 + B_3)},$$

$$D_{21} = \frac{(1+\frac{1}{\beta})\mathrm{Ec}\,\mathrm{Pr}\,A_2A_4\sqrt{B_1}\sqrt{B_2}}{(1+\frac{4}{3}\mathrm{Rd})(-(\sqrt{B_1}+\sqrt{B_2})^2+B_3)},$$

$$D_{22} = -\frac{\mathrm{M}^2\,\mathrm{Ec}\,\mathrm{Pr}\,A_1A_5}{(1+\frac{4}{3}\mathrm{Rd})(-(\sqrt{B_1}+\sqrt{B_2})^2+B_3)},$$

$$D_{23} = -\frac{\mathrm{M}^2\,\mathrm{Ec}\,\mathrm{Pr}\,A_2A_4}{(1+\frac{4}{3}\mathrm{Rd})(-(\sqrt{B_1}+\sqrt{B_2})^2+B_3)},$$

$$D_{24} = \frac{(1+\frac{1}{\beta})\mathrm{Ec}\,\mathrm{Pr}\,A_1A_5\sqrt{B_1}\sqrt{B_2}}{(1+\frac{4}{3}\mathrm{Rd})(-(\sqrt{B_1}-\sqrt{B_2})^2+B_3)},$$

$$D_{25} = -\frac{(1+\frac{1}{\beta})\mathrm{Ec}\,\mathrm{Pr}\,A_2A_4\sqrt{B_1}\sqrt{B_2}}{(1+\frac{4}{3}\mathrm{Rd})(-(\sqrt{B_1}-\sqrt{B_2})^2+B_3)},$$

$$D_{26} = \frac{\mathrm{M}^2\,\mathrm{Ec}\,\mathrm{Pr}\,A_1A_5}{(1+\frac{4}{3}\mathrm{Rd})(-(\sqrt{B_1}-\sqrt{B_2})^2+B_3)},$$

$$D_{27} = -\frac{\mathrm{M}^2\,\mathrm{Ec}\,\mathrm{Pr}\,A_2A_4}{(1+\frac{4}{3}\mathrm{Rd})(-(\sqrt{B_1}-\sqrt{B_2})^2+B_3)},$$

$$D_{28} = \frac{(1+\frac{1}{\beta})\mathrm{Ec}\,\mathrm{Pr}\,A_4^2B_2}{2(1+\frac{4}{3}\mathrm{Rd})(B_4-4B_2)},$$

$$D_{29} = -\frac{(1+\frac{1}{\beta})\mathrm{Ec}\,\mathrm{Pr}\,A_5^2B_2}{2(1+\frac{4}{3}\mathrm{Rd})(B_4-4B_2)},$$

$$D_{30} = -\frac{\mathrm{M}^2\,\mathrm{Ec}\,\mathrm{Pr}\,A_4^2}{2(1+\frac{4}{3}\mathrm{Rd})(B_4-4B_2)},$$

$$D_{31} = \frac{\mathrm{M}^2\,\mathrm{Ec}\,\mathrm{Pr}\,A_5^2}{2(1+\frac{4}{3}\mathrm{Rd})(B_4-4B_2)},$$

$$D_{32} = \frac{(1+\frac{1}{\beta})\mathrm{Ec}\,\mathrm{Pr}\,A_4A_5B_2}{(1+\frac{4}{3}\mathrm{Rd})(B_4-4B_2)},$$

$$D_{33} = \frac{\mathrm{M}^2\,\mathrm{Ec}\,\mathrm{Pr}\,A_4A_5}{(1+\frac{4}{3}\mathrm{Rd})(B_4-4B_2)},$$

$$D_{34} = -\frac{(1+\frac{1}{\beta})\mathrm{Ec}\,\mathrm{Pr}\,A_4^2B_2}{2B_4(1+\frac{4}{3}\mathrm{Rd})},$$

$$D_{35} = -\frac{(1+\frac{1}{\beta})\mathrm{Ec}\,\mathrm{Pr}\,A_5^2B_2}{2B_4(1+\frac{4}{3}\mathrm{Rd})},$$

$$D_{36} = -\frac{\mathrm{M}^2\,\mathrm{Ec}\,\mathrm{Pr}\,A_4^2}{2B_4(1+\frac{4}{3}\mathrm{Rd})},$$

$$D_{37} = -\frac{\mathrm{M}^2\,\mathrm{Ec}\,\mathrm{Pr}\,A_5^2}{2B_4(1+\frac{4}{3}\mathrm{Rd})},$$

$$D_{38} = -\frac{\mathrm{M}^2\,\mathrm{Ec}\,\mathrm{Pr}\,A_6^2}{B_4(1+\frac{4}{3}\mathrm{Rd})},$$

$$D_{39} = -\frac{2A_9\,\mathrm{Sc}\,\mathrm{Sr}}{B_5},$$

$$D_{40} = \frac{K_1\,\mathrm{Sc}}{B_5}.$$

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