ENGINEERING TRANSACTIONS • *Engng. Trans.* • **53**, 2, 133–146, 2005
Polish Academy of Sciences • Institute of Fundamental Technological Research
10.24423/engtrans.437.2005

JOINT DOAs-TIMINGS ESTIMATION FOR UPLINK DS/CDMA COMMUNICATION SYSTEM BASED ON ESPRIT-MUSIC ALGORITHM

W e i – C h i a n g W u

Department of Electrical Engineering, Da Yeh University
ADDR: 112, Shan-Jiau Rd.,Da-Tsuen, Changhua, Taiwan 515, R.O.C.
e-mail: wcwm53@mail.dyu.edu.tw

This paper proposes a blind algorithm to jointly estimate the direction-of-arrivals (DOAs) and timings (time-delays) in asynchronous DS/CDMA multiuser communication system. Making use of the space-time characteristics of an antenna-array DS/CDMA model, it is shown that the multiple signal classification (MUSIC) algorithm and the estimation of signal parameters via rotational invariance (ESPRIT) technique that are widely used in array signal processing, can be applied to extract the direction-of-arrival (DOA) and timing information. Multiuser timing estimation is based on a MUSIC-like algorithm while ESPRIT is applied to estimate the DOA for each user. More specifically, the proposed algorithm is computationally efficient since it reduces the multiuser parameters' estimation problem to a set of single-user's parameter estimation problems. It requires only two eigendecomposition (EVD) and several (depending on the number of subscribers) one-dimensional searches. Computer simulations under different scenarios not only show the accuracy but also demonstrate that the proposed ESPRIT-MUSIC based DOA-timing estimator is near-far resistant.

Key words: ESPRIT, MUSIC, DOA, timing estimator, near-far resistant.

1. Introduction

Code division multiple access (CDMA) and smart antenna technologies play a major role in the 3G wireless communication system. Quite a few papers have been presented to use multiple antennas for multiuser detection in a wireless communication system [1–3]. It is shown in these papers that by exploiting the additional spatial diversity, the capacity, coverage, and quality can be considerably improved. With multiple antennas applied in the CDMA system, an integration of temporal diversity which is provided by the process gain of the spreading sequence and spatial diversity which is provided by array of sensors (antennas) can be exploited for multiuser detection. Miller and Schwartz [4] proposed the optimum and suboptimal realizations of the multi-sensor's detection for single-path asynchronous Gaussian multiple-access channels. It premises on accurate knowledge of DOA for each user and timing offset (propagation delay) encountered in asynchronous transmission. In other words, both the spatial and temporal parameters sets are essential for the base station receiver to effectively isolate, separate, and demodulate the incoming superimposed asynchronous DS/CDMA multiuser signals.

Recently, mushrooming studies of subspace-based channel parameters estimation algorithms have been proposed. The MUSIC-based algorithm [5] has first been applied to estimate propagation delays in [6–7]. While as antenna array is applied, not only the temporal parameters (propagation delays) but also the spatial signatures (DOAs) should be estimated. VANDERVEEN *et al.* proposed a joint angle and delay estimation algorithm [8] for multipath signal impinging on an antenna array. However, since it premises on a 2-D exhaustive search on the DOA-delay plane, the computation load is prohibitively expensive. To reduce the complexity, VANDERVEEN *et al.* also exploited the shift-invariant property of the estimated channel matrix and applied the ESPRIT algorithm [9] for the estimation of the DOAs and delays of multipath signal [10]. A low complexity and high resolution MUSIC-based algorithm which is referred to as TST-MUSIC, is presented to jointly estimate the DOAs and delays in multiray channel [11]. The core idea of the TST-MUSIC is to perform spectral search on the spatial domain (S-MUSIC) for the DOAs estimation and temporal domain (T-MUSIC) for the timing estimation.

In this paper, we jointly estimate DOAs and timings for each user in uplink (asynchronous) DS/CDMA communication system. In the considered model, the front end of the base station receiver is composed of a uniform linear array (ULA) of antennas. Collecting data samples both from the spatial and temporal domains, it is evident that all the users' DOAs and timings' information are conveyed in the data matrix (with size $M \times N$, where M is the array size and N is the signature sequence length or processing gain). In what follows, we observe from the data matrix that any two row vectors can be regarded as identical subarrays. Therefore, the ESPRIT algorithm that is widely applied in array signal processing can be exploited in our data model for DOA estimation. We then apply the MUSIC algorithm to perform time-delay estimation. Note that no further EVD is required when performing the MUSIC method, the noise subspace is available as long as ESPRIT is undertaken. The proposed algorithm is computationally feasible (two EVD and several one-dimensional searches) and both the ESPRIT and MUSIC are algorithms with high accuracy. Moreover, the proposed scheme is blind since reliable parameters' estimation can be attained without any training sequences.

The rest of this paper is organized as follows. The data model for the considered asynchronous DS/CDMA signal is presented in Sec. 2. Section 3 describes the rationale of the proposed ESPRIT-MUSIC algorithm. Section 4 provides examples to evaluate the performance of the proposed algorithm. Concluding remarks are finally made in Sec. 5.

2. Data model

In uplink DS/CDMA communication system, users transmit their information asynchronously. Each user is assigned a unique signature waveform $\{c_k(t)\}_{k=1}^K$ with finite support, $c_k(t) = 0,\ t \notin [0,T],\ k = 1,\ldots,K$, where K is the number of active users and T is the bit (symbol) duration. The information bits, $b_k(i),\ k = 1,\ldots,K,\ i = 1,\ldots,P$, are stationary white sequences. That is, $\{b_k(i)\}_{k=1}^K$ are i.i.d. and $E\{b_k(i)b_k(j)\} = \delta(i-j)$. Under BPSK modulation, $b_k(i) \in \{+1,-1\}$. The receiving front end is composed of an array of M antennas (sensors). In mobile communication system, the time taken for the wavefront to pass through the array is much smaller than the chip interval T_c and therefore, the narrowband assumption is valid. Hence, the response of the antenna elements to an arbitrary source is characterized by an array response vector (or steering vector). The steering vector can also be regarded as the spatial signature uniquely specified for each source that is emitted from a different direction. It should be noted that the steering vector depends on DOA of the received signal, array geometry and carrier frequency. Assuming that ULA is applied and the spacing between adjacent antenna elements is $\lambda/2$, then the normalized steering vector can be expressed in the Vandermonde form

$$\mathbf{a}_k(\theta_k) = \frac{1}{\sqrt{M}} \begin{bmatrix} 1 \\ \exp(j\pi \sin\theta_k) \\ \vdots \\ \exp(j(M-1)\pi\sin\theta_k) \end{bmatrix}; \quad k = 1,2,\ldots,K, \tag{2.1}$$

where θ_k is the k-th user's DOA. By the aid of antenna, the M-by-1 normalized steering vector $\{\mathbf{a}_k(\theta_k)\}_{k=1,\ldots,K}$ that is referred to as the spatial signature, is unique and user-specific. The baseband data model of the antenna array output can then be written as

$$\mathbf{x}(t) = \sum_{k=1}^{K}\sum_{i=1}^{P} \sqrt{w_k(i)}\, b_k(i)\, c_k(t - iT - \tau_k)\mathbf{a}_k + \mathbf{v}(t), \tag{2.2}$$

where $\mathbf{x}(t) = [x_1(t),\ \ldots,\ x_M(t)]^T$ stands for the sensor array's output vector process. $\tau_k \in [0,T]$, is the k-th user's associated timing offset, k-th user's amplitude during i-th bit is $\sqrt{w_k(i)}$. The zero-mean background noise $\mathbf{v}(t)$ is assumed to be uncorrelated with the signal and is both temporally and spatially white. Sampling the antenna array output data chip by chip, then for one bit interval, the discrete-time model is

$$\mathbf{X}(i) = \sum_{k=1}^{K} \left(\sqrt{w_k(i-1)}\, b_k(i-1)\mathbf{a}_k \mathbf{c}_{k1}^T + \sqrt{w_k(i)}\, b_k(i)\mathbf{a}_k \mathbf{c}_{k0}^T\right) + \mathbf{V}(i), \tag{2.3}$$

where we have assumed that the delay spread is small enough, i.e., $\{\tau_k\}_{k=1,..K} \ll T$, such that only the effects of the most recent two bits are considered. τ_k is assumed to be positive multiples of chip time to simplify the analysis. The matrix $\mathbf{X}(i)$ is with dimension $M \times N$.

$$\mathbf{c}_{k1} = [\mathbf{c}_k(\tau_k)\ \mathbf{c}_k(\tau_k+1)\ \cdots\ \mathbf{c}_k(N-1)\ 0\ 0\ \cdots\ 0]^T,$$
$$\mathbf{c}_{k0} = [\ 0\ 0\ \cdots\ 0\ \ \mathbf{c}_k(0)\ \mathbf{c}_k(1)\ \cdots\ \mathbf{c}_k(\tau_k-1)]^T$$

is the signature sequence vector associated to the k-th user that $b_k(i-1)$ and $b_k(i)$ is modulated onto, respectively. Hence, $\mathbf{c}_{k1}+\mathbf{c}_{k0}$ is equivalent with the k-th user's signature sequence vector $\mathbf{c}_k$ permuted by the time offset τ_k. In this paper, we attempt to extract (estimate) the multiuser parameters of DOA $\{\theta_k\}_{k=1,\dots,K}$ and time delays $\{\tau_k\}_{k=1,\dots,K}$ without the training sequence. Obviously, from the data model of (2.3), $\{\theta_k\}_{k=1,\dots,K}$ are contained in the spatial signature $\{\mathbf{a}_k\}_{k=1,\dots,K}$ while $\{\tau_k\}_{k=1,\dots,K}$ are carried by the temporal signature $\{\mathbf{c}_{k1}, \mathbf{c}_{k0}\}_{k=1,\dots,K}$.

Theoretically, the multiuser parameters can be estimated using the maximum likelihood (ML) method [12]. In ML method, data $\mathbf{X}(i)$ are expressed as a function of parameters to be estimated as depicted in (2.3). The ML estimate of $\{\theta_k\}_{k=1,..K}$ and $\{\tau_k\}_{k=1,..K}$ given $\mathbf{X}(i)$ is determined by the following rule:

$$\left\{\hat{\theta}_k, \hat{\tau}_k\right\}_{k=1,\dots,K} = \arg \max_{\{\theta_k,\tau_k\}_{k=1,\dots K}} f\left(\mathbf{X}(i) \,\middle|\, \{\theta_k, \tau_k\}_{k=1,\dots K}\right), \tag{2.4}$$

where $f\left(\mathbf{X}(i) \,\middle|\, \{\theta_k, \tau_k\}_{k=1,\dots K}\right)$ is the probability density function (pdf) of $\mathbf{X}(i)$ conditioned on $\{\theta_k, \tau_k\}_{k=1,\dots K}$. Though the performance of ML estimator has been proven to approach theoretically the optimum result, it still suffers from some difficulties in implementation:

1. It turns out that $f\left(\mathbf{X}(i) \,\middle|\, \{\theta_k, \tau_k\}_{k=1,\dots K}\right)$ is a highly nonlinear function of the parameters to be estimated; and $\{\theta_k, \tau_k\}_{k=1,\dots K}$ in $f\left(\mathbf{X}(i) \,\middle|\, \{\theta_k, \tau_k\}_{k=1,\dots K}\right)$ is hard to separate what leads to a difficult optimization problem.
2. To solve the problem of (2.4) generally involves multi-dimensional iterative searching. The computation load is prohibitively expensive.

In what follows, it is desirable to develop a realizable approach with lower complexity and acceptable performance.

3. Proposed algorithm

3.1. *ESPRIT-based DOAs estimator*

Based on the data model of (2.3), we exploit any two consecutive rows of the matrix $\mathbf{X}(i)$ to perform DOA estimation. The first row of $\mathbf{X}(i)$ can be expressed as

$$\text{(3.1)}\quad \mathbf{x}_1(i) = \sum_{k=1}^{K}\left(\sqrt{w_k(i-1)}b_k(i-1)\mathbf{c}_{k1} + \sqrt{w_k(i)}b_k(i)\mathbf{c}_{k0}\right) + \mathbf{v}_1(i)$$

$$= [\mathbf{C}_1\ \mathbf{C}_0]\begin{bmatrix}\mathbf{W}^{1/2}(i-1) & \mathbf{0}\\ \mathbf{0} & \mathbf{W}^{1/2}(i)\end{bmatrix}\begin{bmatrix}\mathbf{b}(i-1)\\ \mathbf{b}(i)\end{bmatrix} + \mathbf{v}_1(i)$$

$$= \mathbf{C}\mathbf{W}^{1/2}\mathbf{b} + \mathbf{v}_1(i)$$

where $\mathbf{C}_1, \mathbf{C}_0$ are both N-by-K matrices. $\mathbf{C_1} = [\mathbf{c}_{11}\ \dots\ \mathbf{c}_{K1}]$, $\mathbf{C}_0 = [\mathbf{c}_{10}\ \dots\ \mathbf{c}_{K0}]$, $\mathbf{C} = [\mathbf{C}_1\quad \mathbf{C}_0]$. $\mathbf{W}^{1/2}(i-1), \mathbf{W}^{1/2}(i)$ are K-by-K diagonal matrices with diagonal elements given by the amplitudes of $(i-1)$ and i-th bits of K users, respectively. $\mathbf{b}(i-1)$, $\mathbf{b}(i)$ are both K-by-1 vectors that account for the $(i-1)$ and i-th data bits vectors, respectively. We can also express the second row of $\mathbf{X}(i)$ as

$$\text{(3.2)}\quad \mathbf{x}_2(i) = \sum_{k=1}^{K}\left(\sqrt{w_k(i-1)}b_k(i-1)\exp\left(j\pi\sin\theta_k\right)\mathbf{c}_{k1}\right.$$

$$\left. + \sqrt{w_k(i)}b_k(i)\exp(j\pi\sin\theta_k)\mathbf{c}_{k0}\right) + \mathbf{v}_2(i)$$

$$= [\mathbf{C}_1\ \ \mathbf{C}_0]\begin{bmatrix}\mathbf{W}^{1/2}(i-1) & \mathbf{0}\\ \mathbf{0} & \mathbf{W}^{1/2}(i)\end{bmatrix}\begin{bmatrix}\boldsymbol{\Theta} & \mathbf{0}\\ \mathbf{0} & \boldsymbol{\Theta}\end{bmatrix}\begin{bmatrix}\mathbf{b}\,(i-1)\\ \mathbf{b}\,(i)\end{bmatrix} + \mathbf{v}_2(\mathbf{i})$$

$$= \mathbf{C}\mathbf{W}^{1/2}\boldsymbol{\Phi}\,\mathbf{b} + \mathbf{v}_2\,(i)\,,$$

where $\boldsymbol{\Theta}$ and $\boldsymbol{\Phi}$ are K-by-K and K-by-$2K$ diagonal matrices, respectively. $\boldsymbol{\Theta} = \text{diag}\left(\exp\left(j\pi\sin\theta_1\right)\ \exp\left(j\pi\sin\theta_2\right)\ \cdots\ \exp\left(j\pi\sin\theta_K\right)\right), \boldsymbol{\Phi} = \text{diag}(\boldsymbol{\Theta}\ \boldsymbol{\Theta})$. Note that $\mathbf{x}_1(i)$ and $\mathbf{x}_2(i)$ can be regarded as data vectors received from different subarrays, respectively. Consider the $2N \times 1$ augmented data "snapshot" vector

$$\text{(3.3)}\qquad \mathbf{z}(i) = \begin{bmatrix}\mathbf{x}_1(i)\\ \mathbf{x}_2(i)\end{bmatrix} = \begin{bmatrix}\mathbf{C}\\ \mathbf{C}\boldsymbol{\Phi}\end{bmatrix}\mathbf{W}^{1/2}\mathbf{b} + \begin{bmatrix}\mathbf{v}_1(i)\\ \mathbf{v}_2(i)\end{bmatrix},$$

where the $2N$-by-$2K$ matrix $\begin{bmatrix}\mathbf{C}\\ \mathbf{C}\boldsymbol{\Phi}\end{bmatrix}$ is referred to as the augmented spatial-temporal signature matrix. The correlation matrix of $\mathbf{z}(i)$, $E\left\{\mathbf{z}(i)\mathbf{z}(i)^H\right\}$, can then be obtained from (3.2)

$$\text{(3.4)}\qquad \mathbf{R}_{zz} = \begin{bmatrix}\mathbf{C}\\ \mathbf{C}\boldsymbol{\Phi}\end{bmatrix}\mathbf{W}\begin{bmatrix}\mathbf{C}^H & \boldsymbol{\Phi}^H\mathbf{C}^H\end{bmatrix} + \sigma^2\mathbf{I}_{2N},$$

where $\mathbf{I}_{2N}$ is the identity matrix with dimension $2N$. From (3.1)–(3.4), several observations can be made:

1. The parameters of DOAs to be estimated are contained in the diagonal elements of $\mathbf{\Phi}$.
2. Since $\mathbf{\Phi}$ is a full-rank diagonal matrix, hence, the column (range) space of $\mathbf{C}$ is equivalent to the column (range) space of $\mathbf{C\Phi}$.

Since the signal subspaces on the two subarrays, $\mathbf{x}_1(i)$ and $\mathbf{x}_2(i)$, are related by a unitary transformation $\mathbf{\Phi}$, it is plausible to exploit the ESPRIT algorithm to estimate $\mathbf{\Phi}$. Performing EVD on $\mathbf{R}_{zz}$, we have

$$\mathbf{R}_{zz} = \sum_{i=1}^{2N} \lambda_i \mathbf{e}_i \mathbf{e}_i^H, \tag{3.5}$$

where $\{\lambda_i\}_{i=1}^{2N}$, $\{\mathbf{e}_i\}_{i=1}^{2N}$ are the eigenvalues and eigenvectors, respectively. The $2K$ eigenvectors corresponding to the largest eigenvalues span the column space of the augmented spatial-temporal signature matrix. Construct the $2N$-by-$2K$ signal matrix $\mathbf{E}_s = \begin{bmatrix} \mathbf{e}_1 & \mathbf{e}_2 & \cdots & \mathbf{e}_{2K} \end{bmatrix}$ and $2N$-by-$(2N-2K)$ noise matrix $\mathbf{E}_n = \begin{bmatrix} \mathbf{e}_{2K+1} & \mathbf{e}_{2K+2} & \cdots & \mathbf{e}_{2N} \end{bmatrix}$, then we may diagonalize $\mathbf{R}_{zz}$ and rewrite (3.4) as

$$\mathbf{R}_{zz} = \begin{bmatrix} \mathbf{E}_s & \mathbf{E}_n \end{bmatrix} \begin{bmatrix} \mathbf{\Lambda}_s & \mathbf{0} \\ \mathbf{0} & \mathbf{\Lambda}_n \end{bmatrix} \begin{bmatrix} \mathbf{E}_s^H \\ \mathbf{E}_n^H \end{bmatrix}, \tag{3.6}$$

where $\mathbf{\Lambda}_s = \mathrm{diag}\{\lambda_1, \ldots, \lambda_{2K}\}$ and $\mathbf{\Lambda}_n = \mathrm{diag}\{\lambda_{2K+1}, \ldots, \lambda_{2N}\}$ are $2K$-by-$2K$ and $(2N-2K)$-by-$(2N-2K)$ diagonal matrices, respectively. From the signal subspace theory [13], we have

$$R\{\mathbf{E}_s\} = R\left\{ \begin{bmatrix} \mathbf{C} \\ \mathbf{C\Phi} \end{bmatrix} \right\}, \tag{3.7}$$

where $R\{\}$ means range space of the matrix inside $\{\}$. From (3.7), we can partition $\mathbf{E}_s$ according to two subarrays:

$$\mathbf{E}_s = \begin{bmatrix} \mathbf{E}_{s1} \\ \mathbf{E}_{s2} \end{bmatrix} = \begin{bmatrix} \mathbf{C} \\ \mathbf{C\Phi} \end{bmatrix} \mathbf{T}, \tag{3.8}$$

where $\mathbf{T}$ is a unique $2K$-by-$2K$ matrix and the submatrices $\mathbf{E}_{s1}, \mathbf{E}_{s2}$ are of dimension N-by-$2K$. Note that $\mathbf{T}$ is nonsingular from the fact of (3.7). In what follows, $R\{\mathbf{E}_{s1}\} = R\{\mathbf{C}\} = R\{\mathbf{C\Phi}\} = R\{\mathbf{E}_{s2}\}$, or equivalently, $\mathbf{E}_{s1}, \mathbf{E}_{s2}$ share a common column space. It can be easily derived from (3.8) that $\mathbf{E}_{s1}, \mathbf{E}_{s2}$ admits the following equality:

$$\mathbf{E}_{s1} \mathbf{T}^{-1} \mathbf{\Phi} \mathbf{T} = \mathbf{E}_{s2}. \tag{3.9}$$

Define matrix $\mathbf{Q} = \mathbf{T}^{-1}\mathbf{\Phi}\mathbf{T}$, then $\mathbf{Q}$ can be obtained by solving the least-squares solution of (3.9)

$$\mathbf{Q} = \left(\mathbf{E}_{s1}^H \mathbf{E}_{s1}\right)^{-1} \mathbf{E}_{s1}^H \mathbf{E}_{s2}. \tag{3.10}$$

Furthermore, the fact that $\mathbf{T}$ is nonsingular implies that the diagonal matrix $\mathbf{\Phi}$ is the eigenvalue matrix associated with $\mathbf{Q}$. Performing EVD on $\mathbf{Q}$, we can obtain its eigenvalues $\{\mu_i\}_{i=1}^{2K}$. Based on the data model of (3.2), the DOA for each user can be obtained:

$$\hat{\theta}_k = \sin^{-1}\left(\frac{1}{j\pi}\ln \mu_k\right); \quad k = 1, \ldots, 2K. \tag{3.11}$$

3.2. *MUSIC-based timing estimator*

In this subsection, we develop the MUSIC-based timing estimator by exploiting some of the results in the DOA estimator. Since the signal subspace is orthogonal to noise subspace, we have

$$R\left\{\begin{bmatrix} \mathbf{C} \\ \mathbf{C\Phi} \end{bmatrix}\right\}^{\perp} = \mathrm{CSP}\left\{\mathbf{E}_n\right\}, \tag{3.12}$$

where CSP{} means the column space of the matrix inside { }. From the structure of the augmented spatial-temporal signature matrix $\begin{bmatrix} \mathbf{C} \\ \mathbf{C\Phi} \end{bmatrix}$, we can define the k-th user's augmented signature vectors, with size $2N$-by-1 as:

$$\mathbf{l}_{k1} = \begin{bmatrix} \mathbf{c}_{k1} \\ \mathbf{c}_{k1} \times \exp(j\pi \sin\theta_k) \end{bmatrix},$$

$$\mathbf{l}_{k0} = \begin{bmatrix} \mathbf{c}_{k0} \\ \mathbf{c}_{k0} \times \exp(j\pi \sin\theta_k) \end{bmatrix}.$$

Thus, by constructing the $2N$-by-$2N$ projection matrix:

$$\mathbf{P}_n = \mathbf{E}_n \mathbf{E}_n^H = \left(\mathbf{E}_s \mathbf{E}_s^H\right)^{\perp} = \mathbf{P}_S^{\perp}, \tag{3.13}$$

the timings $\{\tau_k\}_{k=1,\ldots,K}$ for each user can be found by performing "spectral search" on the "MUSIC spectrum" for each user

$$\hat{\tau}_k = \arg\max_{\tau_k}\left\{\frac{1}{\mathbf{l}_{k1}^H(\tau_k)\,\mathbf{P}_n \mathbf{l}_{k1}(\tau_k)} + \frac{1}{\mathbf{l}_{k0}^H(\tau_k)\,\mathbf{P}_n \mathbf{l}_{k0}(\tau_k)}\right\}; \quad k = 1, 2, \ldots, K. \tag{3.14}$$

In summary, the ESPRIT-MUSIC based joint DOA-Timing estimation algorithm can be generalized as follows:

Step 1: Applying eigendecomposition (EVD) on $\mathbf{R}_{zz}$, we have

$$\mathbf{R}_{zz} = \begin{bmatrix} \mathbf{E}_s & \mathbf{E}_n \end{bmatrix} \begin{bmatrix} \mathbf{\Lambda}_s & \mathbf{0} \\ \mathbf{0} & \mathbf{\Lambda}_n \end{bmatrix} \begin{bmatrix} \mathbf{E}_s^H \\ \mathbf{E}_n^H \end{bmatrix}.$$

Step 2: Construct $\mathbf{E}_s = \begin{bmatrix} \mathbf{E}_{s1} \\ \mathbf{E}_{s2} \end{bmatrix}$ and $\mathbf{E}_n$.

Step 3: Solve $\mathbf{E}_{s1}\mathbf{Q} = \mathbf{E}_{s2}$ to obtain $\mathbf{Q}$.

Step 4: Perform EVD on $\mathbf{Q}$, we can obtain its eigenvalues $\{\mu_i\}_{i=1}^{2K}$.

Step 5: The DOA for each user is

$$\hat{\theta}_k = \sin^{-1}\left(\frac{1}{j\pi}\ln \mu_k\right); \quad k = 1, \ldots, 2K.$$

Step 6: Construct $\mathbf{P}_n = \mathbf{E}_n\mathbf{E}_n^H$.

Step 7: Construct $\mathbf{l}_{k1}(\tau_k) = \begin{bmatrix} \mathbf{c}_{k1}(\tau_k) \\ \mathbf{c}_{k1}(\tau_k) \times \exp(j\pi \sin \hat{\theta}_k) \end{bmatrix}$

and $\mathbf{l}_{k0}(\tau_k) = \begin{bmatrix} \mathbf{c}_{k0}(\tau_k) \\ \mathbf{c}_{k0}(\tau_k) \times \exp(j\pi \sin \hat{\theta}_k) \end{bmatrix}$.

Step 8: Search for the peak of "MUSIC spectrum" for each user

$$\hat{\tau}_k = \arg\max_{\tau_k}\left\{\frac{1}{\mathbf{l}_{k1}^H(\tau_k)\mathbf{P}_n\mathbf{l}_{k1}(\tau_k)} + \frac{1}{\mathbf{l}_{k0}^H(\tau_k)\mathbf{P}_n\mathbf{l}_{k0}(\tau_k)}\right\}; \quad k = 1, 2, \ldots, K.$$

4. Performance evaluation and discussion

In this section, several simulations are conducted to demonstrate the performance of the proposed DOA-Timing estimator. In all the simulation examples, three subscribers that are modulated by BPSK and spread by Gold code with $N = 15$ are received by an ULA. We first assume that the arriving angle and timing for each user is $\{\theta_1 = -45°, \tau_1 = 3T_c\}$, $\{\theta_2 = 25°, \tau_2 = 4T_c\}$, $\{\theta_3 = 50°, \tau_3 = 5T_c\}$, respectively; the signal-to-noise-ratio (SNR) for each user is fixed and equals 10 dB. Under ideal conditions, where we use Eq. (3.4) to account for $\mathbf{R}_{zz}$, the DOAs are perfectly estimated and timings estimation are presented in Fig. 1. As depicted in Fig. 1(a)–(c), the MUSIC spectrum show spectral peaks on $\tau_1 = 3T_c, \tau_2 = 4T_c, \tau_3 = 5T_c$, respectively. In Figs. 2–4, $\mathbf{R}_{zz}$ is estimated by performing time-average on the measurements. For a window size of J, $\hat{\mathbf{R}}_{zz}$ is given by

$$\hat{\mathbf{R}}_{zz} = \frac{1}{J} \sum_{j=i}^{i+J-1} \mathbf{z}(j)\mathbf{z}^H(j). \tag{4.1}$$

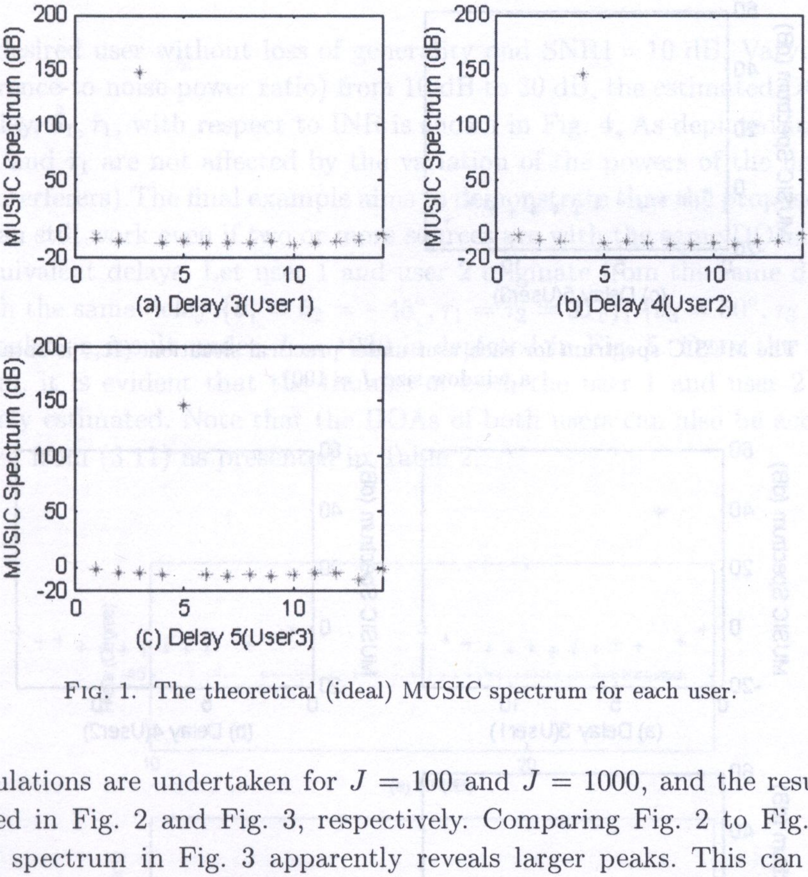

FIG. 1. The theoretical (ideal) MUSIC spectrum for each user.

Simulations are undertaken for $J = 100$ and $J = 1000$, and the results are presented in Fig. 2 and Fig. 3, respectively. Comparing Fig. 2 to Fig. 3, the MUSIC spectrum in Fig. 3 apparently reveals larger peaks. This can be expected since $\hat{\mathbf{R}}_{zz}$ is asymptotically (according to the window size J) approaching the optimum $\mathbf{R}_{zz}$. The DOAs' estimation for both cases are summarized in Table 1. We can observe from Table 1, Fig. 2, and Fig. 3 that, as J and/or desired user's SNR increases, more accurate DOA and timing estimation can be obtained.

The second simulation is devoted to examine the near-far resistant characteristics of the proposed algorithm. We set $J = 1000$, user 1 is assumed to

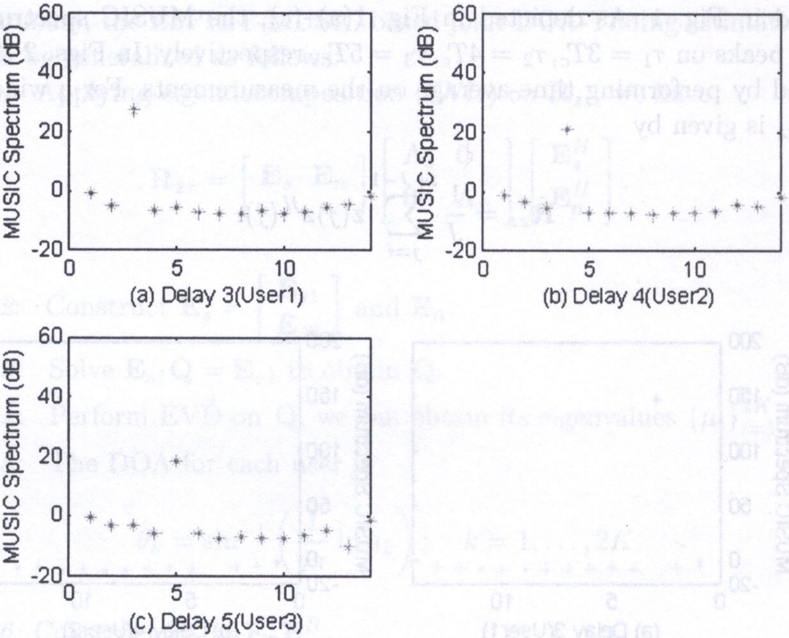

FIG. 2. The MUSIC spectrum for each user under practical situation. ($\hat{\mathbf{R}}_{zz}$ is obtained for a window size $J = 100$).

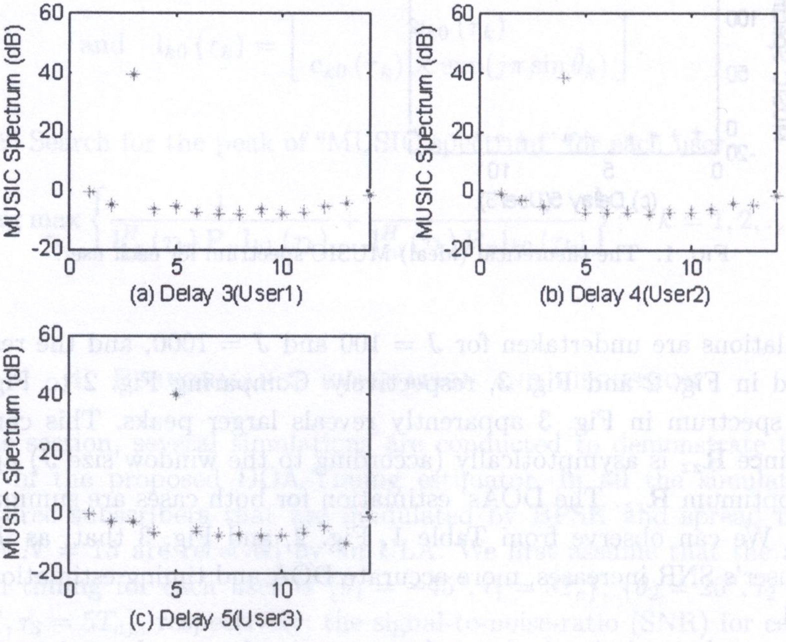

FIG. 3. The MUSIC spectrum for each user under practical situation. ($\hat{\mathbf{R}}_{zz}$ is obtained for a window size $J = 1000$).

Table 1. The DOAs estimation results versus window size and desired user's SNR for $\{\theta_1 = -45°, \theta_2 = 25°, \theta_3 = 50°\}$.

	Source DOA	$J = 100$		$J = 1000$	
		SNR = 10 dB	SNR = 20 dB	SNR = 10 dB	SNR = 20 dB
User #1	−45°	−44.6283°	−45.0913°	−44.8916°	−44.9324°
User #2	25°	24.7580°	24.9141°	25.0622°	25.0207°
User #3	50°	49.8321°	49.8615°	49.9561°	50.0093°

be the desired user without loss of generality and SNR1 = 10 dB. Varying INR (interference-to-noise power ratio) from 10 dB to 30 dB, the estimated DOA and time delay, $\hat{\theta}_1, \hat{\tau}_1$, with respect to INR is shown in Fig. 4. As depicted in Fig. 4, both $\hat{\theta}_1$ and $\hat{\tau}_1$ are not affected by the variation of the powers of the undesired users (interferers).The final example aims to demonstrate that the proposed algorithm can still work even if two or more sources are with the same DOAs and/or with equivalent delays. Let user 1 and user 2 originate from the same direction and with the same delay $\{\theta_1 = \theta_2 = -45°, \tau_1 = \tau_2 = 3T_c\}$, $\{\theta_3 = 50°, \tau_3 = 5T_c\}$. The simulation result under $J = 1000$ is depicted in Fig. 5. From the MUSIC spectrum, it is evident that the timings of both the user 1 and user 2 can be accurately estimated. Note that the DOAs of both users can also be accurately extracted from (3.11) as presented in Table 2.

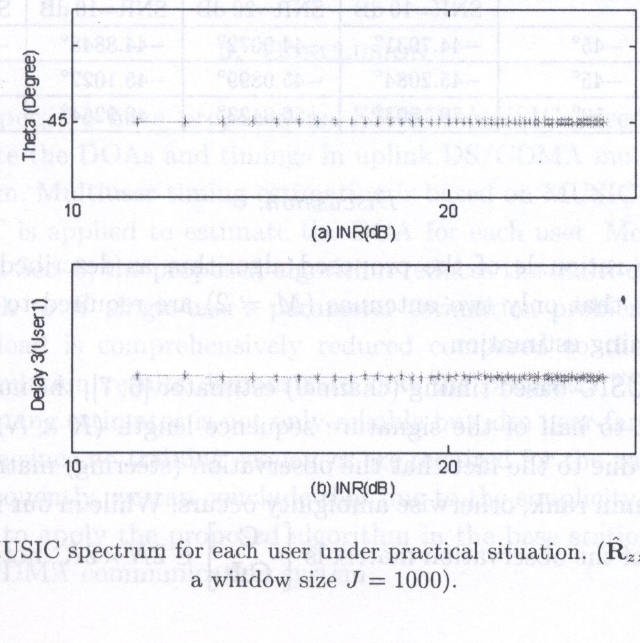

FIG. 4. The MUSIC spectrum for each user under practical situation. ($\hat{\mathbf{R}}_{zz}$ is obtained for a window size $J = 1000$).

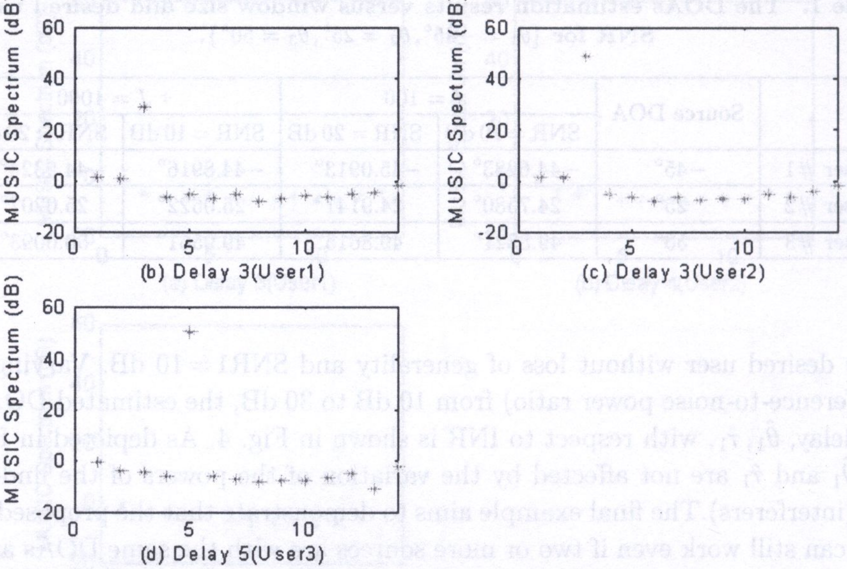

FIG. 5. The MUSIC spectrum for each user under practical situation. ($\hat{\mathbf{R}}_{zz}$ is obtained for a window size $J = 1000$).

Table 2. The DOAs estimation results versus window size and desired user's SNR for $\{\theta_1 = -45°, \theta_2 = -45°, \theta_3 = 50°\}$.

	Source DOA	$J = 100$		$J = 1000$	
		SNR=10 dB	SNR=20 dB	SNR=10 dB	SNR=20 dB
User #1	−45°	−44.7031°	−44.9072°	−44.8848°	−44.9202°
User #2	−45°	−45.2084°	−45.0899°	−45.1022°	−45.0307°
User #3	50°	50.1503°	50.0423°	49.9254°	50.0031°

Discussion:

1. From the rationale of the proposed algorithm as described in Sec. 3, it is evident that only two antennas ($M = 2$) are required to complete the DOA-Timing estimation.
2. In the MUSIC-based timing (channel) estimator [6, 7], the number of users is limited to half of the signature sequence length ($K < N/2$). The constraint is due to the fact that the observation (steering) matrix must be of a full column rank, otherwise ambiguity occurs. While in our model, the dimension of the observation matrix is $\begin{bmatrix} \mathbf{C} \\ \mathbf{C}\mathbf{\Phi} \end{bmatrix} \in 2N \times 2K$, not $\mathbf{C} \in N \times 2K$.

Hence, the limitation of the number of users can be relaxed to $K < N$. Note also that if more than two antennas are exploited, it is obvious that the limitation can further be relaxed.

3. To simplify the notation and analysis, we assume that the delay for each user is integer multiple of chip duration (T_c) in this paper. Thus, the data extracted from the temporal domain is obtained by sampling once in each chip duration. The case for time-delay that is noninteger multiples of T_c can be realized without conceptual difficulty. We can modify the data model by oversampling in the temporal domain and the proposed algorithm can still work. Note that the higher the oversampling factor (number of samples in each chip duration), the higher the resolution of the timing estimator. However, the computation load of the MUSIC spectral search is inevitably increased.
4. In contrast to the inherent limitations of the ESPRIT and MUSIC algorithms, the proposed algorithm can still work even if more than two users are originating from the same direction or (and) with the same delay. This is due to the fact of the additional diversity (code diversity) introduced by CDMA.
5. As simulated in Fig. 4, the proposed multiuser DOA-Timing estimator enjoys the near-far resistant characteristics. In other words, the performance of the desired user's parameters' estimators is independent of the strength of the undesired users.

5. Conclusion

In this paper, we have proposed an ESPRIT-MUSIC based algorithm to jointly estimate the DOAs and timings in uplink DS/CDMA multiuser communication system. Multiuser timing estimation is based on MUSIC-like algorithm while ESPRIT is applied to estimate the DOA for each user. More specifically, as described in Sec. 3, the proposed algorithm reduces the multi-dimensional exhaustive search to K single-user's parameter estimation problems. Hence, the computation load is comprehensively reduced compared to the ML method. Moreover, simulation results demonstrate that the proposed ESPRIT-MUSIC based DOA-timing estimator is not only reliable but also near-far resistant. It is blind in nature since no training sequences are required for the parameters' estimation. Consequently, we can conclude that due to the simplicity and efficiency, it is plausible to apply the proposed algorithm in the base station receiver in a wireless DS/CDMA communication system.

References

1. B. Suard, A. F. Naguib, G. Xu and A. J. Paulraj, *Performance of CDMA mobile communication system using antenna arrays*, IV 153–156, ICASSP 1993.

2. J. H. Winters, J. Salz, and R. D. Gitlin, *The impact of antenna diversity on the capacity of wireless communication system*, IEEE Trans. Commun., **42**, 2/3/4, 1740–1751, 1994.

3. A. J. Paulraj and C. B. Papadias, *Space-time processing for wireless communications*, IEEE SP Magazine, 49–83, 1997.

4. S. Y. Miller and S. C. Schartz, *Integrated spatial-temporal detectors for asynchronous Gaussian multiple-access channels*, IEEE Trans. Commun., **43**, 2/3/4, 396–411, 1995.

5. R. Schmidt, *Multiple emitter location and signal parameter estimation*, IEEE Trans. Ant. Propagation, AP–34, 276–290, March 1986.

6. E. G. Ström, S. Parkvall, S. L. Miller and B. E. Ottersten, *Propagation delay estimation in asynchronous direct-sequence code division multiple access system*, IEEE Trans. Commun., **44**, 1, 84–93, 1996.

7. S. E. Bensley and B. Aazhang, *Subspace-based channel estimation for code division multiple access communication system*, IEEE Trans. Commun., **44**, 8, 1009–1020, 1996.

8. M. C. Vanderveen, C. B. Papadias, and A. J. Paulraj, *Joint angle and delay estimation for multipath signals arriving at an antenna array*, IEEE Commun. Lett., **1**, 12–14, 1997.

9. R. Roy and T. Kailath, *ESPRIT-Estimation of signal parameters via rotational invariance techniques*, IEEE Trans. ASSP, **37**, 7, 984–995, 1989.

10. A. J. Vanderveen, M. C. Vanderveen, and A. J. Paulraj, *Joint angle and delay estimation using shift-invariance properties*, IEEE Trans. Signal Processing, **46**, 405–418, 1998.

11. Y. Y. Wang, J. T. Chen, and W. H. Feng, *TST-MUSIC for joint DOA-Delay estimation*, IEEE Trans. Signal Processing, **49**, 4, 721–729, 2001.

12. J. M. Mendle, *Lessons in estimation theory for signal processing, communications, and control*, Lesson 11, Englewood Cliffs, Prentice Hall, NJ 1995.

13. T. K. Moon and W. C. Stirling, *Mathematical method and algorithms for signal processing*, Chap. 6, Upper Saddle River, Prentice Hall, NJ 2000.

Received September 20, 2004.