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LAMINATE PLY STACKING SEQUENCE OPTIMIZATION WITH FIBERS ORIENTATION IMPERFECTIONS

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The paper discusses the fiber orientations imperfections effect on the optimum design of a laminate plate exposed to compressive load. It is assumed that fibers angular imperfection for each design variable can not exceed maximum allowable deviation from variable's nominal value. These maximal accepted deviations are called tolerances. The incorporation of tolerances into the design algorithm is achieved by diminishing the limiting values of state variables by the product of assumed tolerances and appropriate sensitivities. Therefore, the given method allows to introduce tolerances into the design in a relatively simple way and ensures safe results. The paper is illustrated by examples of the rectangular laminate plate minimum thickness design. Numerical results show the reliability-based design to be important for structural safety compared to the approach where tolerances are not taken into account.

Key words: manufacturing tolerances, imperfections, optimum design, composite plates, buckling optimization

1. Introduction

Because of high strength/weight and stiffness/weight ratios, composite elements are applied in light-weight structures, especially as in-plane loaded structural components. Therefore their optimum design for maximum buckling load has been studied extensively in recent years. One of the most often considered problems of laminate plates optimization are ply stacking sequence designs, where optimum orientation of fibers in each ply should be determined – e.g. [3].

Unfortunately, most of the research papers deal with that problem under the assumption that design variables are not subject to imperfections arising from manufacturing processes. Following this approach, optimal solutions obtained for perfect design variables can lead to violation of the constraints imposed on state variables – i.e. buckling load – while considering real structures. This is because of both the high sensitivity of an optimum structure to variations of design variables [2] and the fact, that at least one constraint in the optimal structure comes to its limit value (i.e. is active).

The circumstances mentioned above force an engineer to introduce design variables' imperfections directly into the optimization algorithms. As a result of this approach, the state variables in new optimal design are less sensitive to variations of the design variables, and the structure remains safe even though the design variables in the problem vary due to the manufacturing tolerances.

One of the ideas to deal with the uncertainties in laminate plate optimization is given by B.P. KRISTINSDOTTIR *et al.* in Ref. [7]. This approach is based on changing the right-hand side of the inequality constraints and replacing the initial zero value in original problem by a small positive number called a safety margin. Next the structure is re-optimized and checked, if new design parameters have the specified earlier tolerances as compared to the design variables in the original problem. If not, the safety margin is increased and the redesign procedure is run again. The process is repeated until the assumed tolerances of all design parameters are achieved. In the discussed paper, the presented method is illustrated by an example of a hat-stiffened composite panel optimum design.

In the present paper, a more formal approach is proposed to optimize the laminate plates considering tolerances of fibers orientations. The method presented is a further development of an original concept worked out by W. GUTKOWSKI and J. BAUER in [4]. In the discussed paper, the authors incorporate dimensional imperfections (i.e. manufacturing tolerances) directly into the optimum design search. The main idea of this method is based on sensitivity analysis. First, the sensitivities of state variables to all design parameters are determined, and next the original limit values of state variables in constraints are reduced by products of the derived sensitivities and manufacturing tolerances. In the mentioned paper, the authors illustrated the proposed method by an example of truss optimum design with constraints imposed on stresses and displacements.

2. STATEMENT OF THE PROBLEM

A sandwich panel that will be considered in the present study is given in Fig. 1. The plate is assumed to be rectangular and simply supported at all four edges. It is symmetric and consists of N plies, each of the equal thickness t. In each ply k a nominal fibers' orientation is denoted by θ_k. The laminate is assumed to be composed of layers with 0°, 90° and ±45° fibers only. Additionally, it is assumed that the sandwich plate is balanced – the number of plies having +45° fibers is equal to the number of plies with −45° fibers.

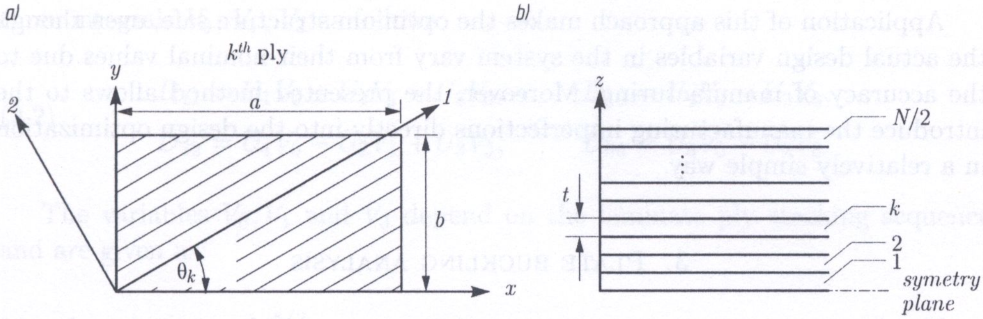

FIG. 1. Laminate sandwich plate.

In further considerations we assume that the actual fiber orientation angle in each k-th ply ($k = 1 \ldots N$) may be varied from its nominal value θ_k and stays within a range from $(\theta_k - \Delta\theta_k)$ to $(\theta_k + \Delta\theta_k)$ – see Fig. 2. In the above relation the summand $\Delta\theta_k$ represents the maximum allowable deviation of actual fiber direction in ply k – i.e. it corresponds to the accuracy of manufacturing. It is assumed that the admissible tolerances are constant and equal for each laminate ply. Moreover, it is assumed that the thicknesses of all the layers remain nominal and are not subject to any imperfections.

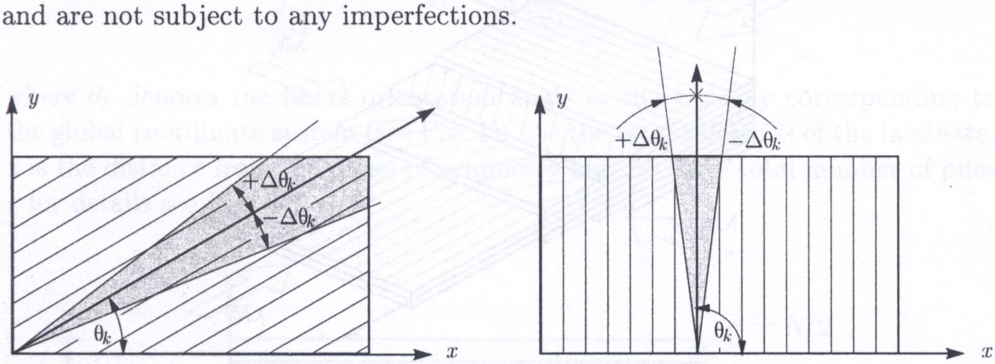

FIG. 2. Tolerances of fibers' orientations. The actual fibers' orientation stays within the allowable range $(\theta_k - \Delta\theta_k,\ \theta_k + \Delta\theta_k)$.

Allowing for manufacturing tolerances, where only the maximal acceptable deviations are given, makes the exact values of design variables to be unknown. To deal with that problem, the following approach to optimum design is proposed. The equality constraints are solved for nominal (average) values of design variables and the imperfections (manufacturing tolerances) are introduced into the inequality constraints imposed on state variables. This is done by diminishing the limiting values of these variables by the absolute value of a product of admissible imperfections and appropriate sensitivities.

Application of this approach makes the optimum structure safe, even though the actual design variables in the system vary from their nominal values due to the accuracy of manufacturing. Moreover, the presented method allows to the introduce the manufacturing imperfections directly into the design optimization in a relatively simple way.

3. Plate buckling analysis

The laminate plate optimization capability presented herein is based upon the classical buckling analysis for a simply supported equivalent orthotropic plate subject to inplane loading conditions. The plate material is assumed to be linearly elastic, with given longitudinal E_1 and transversal E_2 elasticity moduli, and Poison's ratios ν_{12}, ν_{21} respectively.

The uniform longitudinal stress resultants λN_x and λN_y are applied at the edges of the panel, where λ is the amplitude parameter – see Fig. 3. No shear forces are considered in the present research.

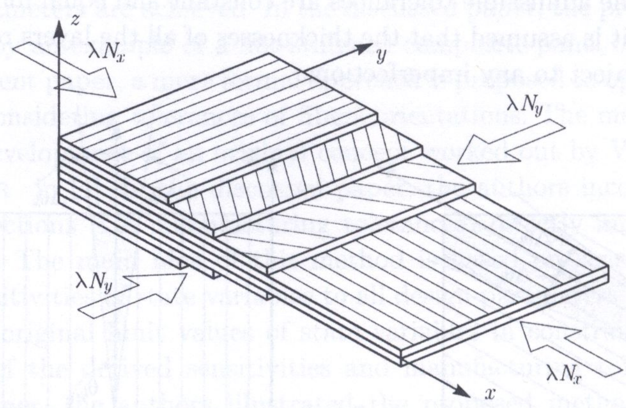

Fig. 3. Plate loading scheme.

Taking into account the above assumptions and following the results achieved by other authors (i.e. [1, 6]), the equilibrium equation for the compressed plate is given by the differential relation

$$D_{11}\frac{\partial^4 w}{\partial x^4} + 2\left(D_{12} + 2D_{66}\right)\frac{\partial^4 w}{\partial x^2 \partial y^2} + D_{22}\frac{\partial^4 w}{\partial y^4} = \lambda N_x \frac{\partial^2 w}{\partial x^2} + \lambda N_y \frac{\partial^2 w}{\partial y^2}, \tag{3.1}$$

where w is lateral deflection and D_{11}, D_{12}, D_{22} and D_{66} are flexural stiffnesses. These can be expressed in terms of material invariants U_i ($i = 1, \ldots, 5$) and three integrals V_0, V_1, V_3 as follows:

$$D_{11} = U_1V_0 + U_2V_1 + U_3V_3, \qquad D_{12} = U_4V_0 - U_3V_3,$$
(3.2)
$$D_{22} = U_1V_0 - U_2V_1 + U_3V_3, \qquad D_{66} = U_5V_0 - U_3V_3.$$

The variables V_0, V_1 and V_3 depend on the laminate ply stacking sequence and are given as:

$$V_0 = \int_{-h/2}^{h/2} z^2 dz = \frac{1}{3}\sum_{k=1}^{N}(z_k^3 - z_{k-1}^3),$$

(3.3)
$$V_1 = \int_{-h/2}^{h/2} z^2 \cos 2\theta \, dz = \frac{1}{3}\sum_{k=1}^{N} \cos 2\theta_k \, (z_k^3 - z_{k-1}^3),$$

$$V_3 = \int_{-h/2}^{h/2} z^2 \cos 4\theta \, dz = \frac{1}{3}\sum_{k=1}^{N} \cos 4\theta_k \, (z_k^3 - z_{k-1}^3),$$

where θ_k denotes the fibers orientation angle in the k-th ply corresponding to the global coordinate system (see Fig. 1), h is the total thickness of the laminate, z is the distance from the plane of symmetry and N is the total number of plies – for details see Fig. 4.

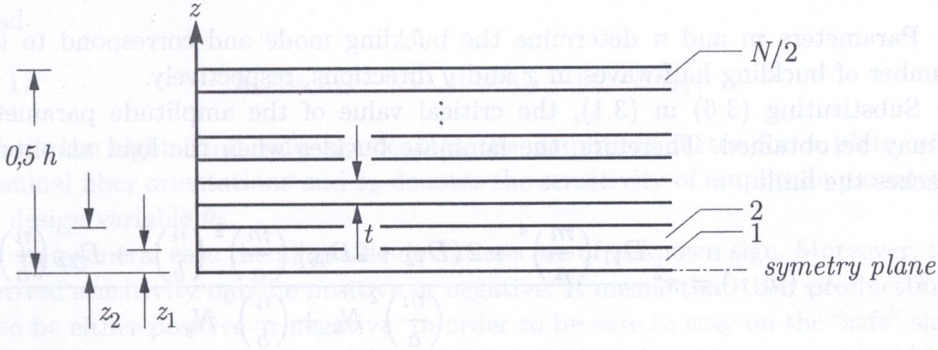

FIG. 4. Laminate cross-section.

Material invariants U_i $(i = 1, \ldots, 5)$ appearing in (3.2) are given by the relations:

$$U_1 = \frac{1}{8}(3Q_{11} + 3Q_{22} + 2Q_{12} + 4Q_{66}),$$

$$U_2 = \frac{1}{2}(Q_{11} - Q_{22}),$$

$$U_3 = \frac{1}{8}(Q_{11} + Q_{22} - 2Q_{12} - 4Q_{66}), \tag{3.4}$$

$$U_4 = \frac{1}{8}(Q_{11} + Q_{22} + 6Q_{12} - 4Q_{66}),$$

$$U_5 = \frac{1}{8}(Q_{11} + Q_{22} - 2Q_{12} + 4Q_{66}),$$

where:

$$Q_{11} = \frac{E_1}{1 - \nu_{12}\nu_{21}},$$

$$Q_{12} = \frac{\nu_{12}E_2}{1 - \nu_{12}\nu_{21}} = \frac{\nu_{21}E_1}{1 - \nu_{12}\nu_{21}}, \tag{3.5}$$

$$Q_{22} = \frac{E_2}{1 - \nu_{12}\nu_{21}},$$

$$Q_{66} = G_{12}.$$

The solution of the given differential plate equilibrium equation (3.1) is

$$w = W \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right), \qquad m, n \in N. \tag{3.6}$$

Parameters m and n determine the buckling mode and correspond to the number of buckling half-waves in x and y directions, respectively.

Substituting (3.6) in (3.1), the critical value of the amplitude parameter λ may be obtained. Therefore, the laminate buckles when the load amplitude reaches the limit

$$\lambda_{\mathrm{cr}}(m, n) = \pi^2 \frac{D_{11}\left(\frac{m}{a}\right)^4 + 2(D_{12} + 2D_{66})\left(\frac{m}{a}\right)^2\left(\frac{n}{b}\right)^2 + D_{22}\left(\frac{n}{b}\right)^4}{\left(\frac{m}{a}\right)^2 N_x + \left(\frac{n}{b}\right)^2 N_y}. \tag{3.7}$$

In further research is is assumed that an eigenmode corresponding to a lower eigenvalue is more critical than that corresponding to a higher one. If so, then the lowest λ value – over parameters (m, n) – must be taken into account.

The problem of plate optimum design with discrete design variables may be solved by the method proposed by R.T. HAFTKA and J.L. WALSH in [5]. In this approach the original design problem is described in terms of zero-one variables.

Plies stacking sequence is defined in terms of N sets of four fibers-orientation-identity variables o_k, n_k, f_k^p, f_k^m, $k = 1, \ldots, N$ that are of Boolean type. The variable o_k, n_k, f_k^p or f_k^m is equal to one if there are 0°, 90°, +45°, −45° fibers respectively in the k-th layer; otherwise it is equal to zero. Finally, recalling that the plate is symmetric and only the plies above the plane of symmetry can be considered, the stacking sequence variables (3.3) are given as:

$$V_0 = \frac{2}{3} t^3 \sum_{k=1}^{N/2} [k^3 - (k-1)^3](o_k + n_k + f_k^p + f_k^m),$$

$$V_1 = \frac{2}{3} t^3 \sum_{k=1}^{N/2} [k^3 - (k-1)^3](o_k - n_k), \tag{3.8}$$

$$V_3 = \frac{2}{3} t^3 \sum_{k=1}^{N/2} [k^3 - (k-1)^3](o_k + n_k - f_k^p - f_k^m).$$

The above given relations allow to determine the critical buckling load of a laminate plate in terms of $(4 \cdot N/2)$ fibers-orientation variables.

4. State variables considering manufacturing tolerances

As it was stated in the second section, it is assumed that the actual fibers orientation angle in every k-th ply may be varied from its nominal value θ_k by the tolerance $\pm\Delta\theta_k$. This imperfection will cause a change in plate critical buckling load

$$\lambda_{\text{cr}} := \lambda_{\text{cr}} - |\Delta\lambda_{\text{cr}}| = \lambda_{\text{cr}} - |s_k \cdot \Delta\theta_k| \,, \tag{4.1}$$

where the right-hand side λ_{cr} denotes the amplitude factor for a plate with nominal fiber orientations and s_k denotes the sensitivity of amplitude parameter to design variable θ_k.

In a general case the allowable deviations are of unknown sign. Moreover, the derived sensitivity may be positive or negative. It means that their product may also be either positive or negative. In order to be sure to stay on the "safe" side, the absolute value of the $s_k \cdot \Delta\theta_k$ product must be taken into account in (4.1).

Extending the considerations to all plies in the laminate, the new value of a state variable λ_{cr} is given as follows:

$$\lambda_{\text{cr}} := \lambda_{\text{cr}} - |\Delta\lambda_{\text{cr}}| = \lambda_{\text{cr}} - |\mathbf{s} \cdot \mathbf{\Delta\theta}| \,. \tag{4.2}$$

In the above relation $\mathbf{s}$ denotes the sensitivity vector of λ to all N design variables $\boldsymbol{\theta} = \{\theta_1, \ldots, \theta_N\}^T$

$$\mathbf{s} = \frac{d\lambda_{\rm cr}}{d\boldsymbol{\theta}} = \left\{ \frac{d\lambda_{\rm cr}}{d\theta_1}, \frac{d\lambda_{\rm cr}}{d\theta_2}, \ldots, \frac{d\lambda_{\rm cr}}{d\theta_k}, \ldots, \frac{d\lambda_{\rm cr}}{d\theta_N} \right\}^T, \tag{4.3}$$

whereas $\boldsymbol{\Delta\theta}$ in (4.2) denotes the vector of admissible deviations of fibers orientations from their nominal value – see Fig. 2

$$\boldsymbol{\Delta\theta} = \{\Delta\theta_1, \Delta\theta_2, \ldots, \Delta\theta_k, \ldots, \Delta\theta_{N/2}\}^T. \tag{4.4}$$

Following the Eq. (3.7), individual terms of the sensitivity vector are given

$$\frac{d\lambda_{\rm cr}}{d\theta_k} = \pi^2 \frac{\frac{dD_{11}}{d\theta_k}\left(\frac{m}{a}\right)^4 + 2\left(\frac{dD_{12}}{d\theta_k} + 2\frac{dD_{66}}{d\theta_k}\right)\left(\frac{m}{a}\right)^2\left(\frac{n}{b}\right)^2}{\left(\frac{m}{a}\right)^2 N_x + \left(\frac{n}{b}\right)^2 N_y} + \frac{\frac{dD_{22}}{d\theta_k}\left(\frac{n}{b}\right)^4}{\left(\frac{m}{a}\right)^2 N_x + \left(\frac{n}{b}\right)^2 N_y}. \tag{4.5}$$

Considering the relations (3.4), (3.8) and substituting them, into (3.2), the derivatives $dD_{ij}/d\theta_k$ are:

$$\begin{aligned}
\frac{dD_{11}}{d\theta_k} &= \frac{2}{3}t^3(3k^2 - 3k + 1)\left[U_1(o_k + n_k + f_k^p + f_k^m) + 2U_2(f_k^p - f_k^m)\right],\\
\frac{dD_{22}}{d\theta_k} &= \frac{2}{3}t^3(3k^2 - 3k + 1)\left[U_1(o_k + n_k + f_k^p + f_k^m) - 2U_2(f_k^p - f_k^m)\right],\\
\frac{dD_{12}}{d\theta_k} &= \frac{2}{3}t^3 U_4(3k^2 - 3k + 1)(o_k + n_k + f_k^p + f_k^m),\\
\frac{dD_{66}}{d\theta_k} &= \frac{2}{3}t^3 U_5(3k^2 - 3k + 1)(o_k + n_k + f_k^p + f_k^m),
\end{aligned} \tag{4.6}$$

bearing in mind, that

$$\frac{dV_3}{d\theta_k} = -\frac{2}{3}t^3 \cdot 4 \cdot \sin 4\theta_k \left[k^3 - (k-1)^3\right] \equiv 0, \tag{4.7}$$

since for all admissible values of design variables $\theta_k \in (0°, 90°, +45°, -45°)$ we have $\sin 4\theta_k = 0$.

In practical structural applications, a laminated plate is mainly stacked symmetrically with respect to the mid-plane to avoid bending-extensional coupling. However, allowing for manufacturing tolerances results in the actual values of fiber orientation angles to be different from their nominal values. Thus the actual stacking sequence of the laminated plate will not be symmetric, even though the mean (nominal) stacking sequence is assumed to be symmetric. Consequently, this causes the anisotropic plate behaviour and buckling analysis for an unsymmetric laminate is required.

The importance of anisotropy on analytical predictions of structural response is examined in detail in several papers – for instance by M.P. NEMETH in [8]. It is proved that the anisotropic constitutive terms (D_{12}, D_{26}) appear whenever a ply is stacked with fiber orientation other than 0° or 90° to the reference axes of a plate. Moreover, the importance of these coefficients depend on ply orientation, number of plies and their stacking sequence. Values of D_{12} and D_{26} asymptotically approach zero when the number of plies is increased. On the basis of a criterion presented in the discussed paper, the derived analysis indicates that the buckling loads obtained for laminates with more than 12 plies are within 2% of the loads obtained while neglecting the anisotropic coefficients. Having this in mind, one may assume that the relations (3.7) and (4.1) for critical amplitude parameter remain valid, even if the actual plate stacking sequence is not symmetric, and the influence of anisotrophy may be neglected.

5. EXAMPLE PROBLEM

To illustrate the proposed method of incorporating the manufacturing tolerances into design the problem of the laminate with minimum thickness for a given buckling load is considered. The presented example comes from the paper [5] by R.T. HAFTKA and J.L. WALSH however it is solved there only for nominal values of the design variables.

For minimum plate weight design, the lowest number (N) of plies is searched for. Since this number is unknown at the beginning of design, a sufficiently large value is initially assumed. This can be done by analyzing any trial design and then scaling the laminate thickness so that it does not buckle – see relation (5.3). Thus an additional constraint is put forward, that if there are any "empty" layers, they are located outermost of the symmetry plane – see relation (5.6).

The discussed problem is as follows:

1) find

$$o_k, n_k, f_k^p, f_k^m, \qquad k = 1, \ldots, N/2; \tag{5.1}$$

2) such that

$$\min\left(\sum_{k=1}^{N/2} o_k + n_k + f_k^p + f_k^m\right); \tag{5.2}$$

3) with constraints

$$\lambda_{\rm cr}(o_k, n_k, f_k^p, f_k^m, m, n) \geq 1, \qquad m = 1, \ldots, m_{\max} \quad n = 1, \ldots, n_{\max} \tag{5.3}$$

$$o_k + n_k + f_k^p + f_k^m = 1, \qquad o_k, n_k, f_k^p, f_k^m \in (0,1) \quad \text{where } k = 1, \ldots, N/2, \tag{5.4}$$

$$\sum_{k=1}^{N/2} (f_k^p - f_k^m) = 0, \tag{5.5}$$

$$o_{k-1}, n_{k-1}, f_{k-1}^p, f_{k-1}^m \leq o_k, n_k, f_k^p, f_k^m, \qquad k = 1, \ldots, N/2. \tag{5.6}$$

The solution of this problem must be an integer and an even number (plate is assumed to be symmetric). Therefore, it is not unique, since there may be many plates having the same number of plies that ensure the structural stability. In the present research, to achieve a unique solution, a plate having the largest possible buckling margin among all plates of the same thickness that ensure stability is chosen. To achieve this goal, an additional analysis is performed and amplitude parameter $\lambda_{\rm cr}$ values are compared.

5.1. *Numerical results*

Computations are performed for the plate consisting of plies, each of the equal thickness $t = 0.127$ mm and having dimensions $a = 50.8$ cm and $b = 25.4$ cm, $(b/a = 0.5)$. Graphite-epoxy laminate material properties are assumed: $E_1 = 128.0$ GPa, $E_2 = 13.0$ GPa, $G_{12} = 6.4$ GPa and $\nu_{12} = 0.3$. The axial stress resultants are constant ($N_x = 175$ N/m), while transverse loading N_y (see Fig. 3) is varied within the range $(0, \ldots, 13140)$ N/m. The problem is solved by a direct enumeration method.

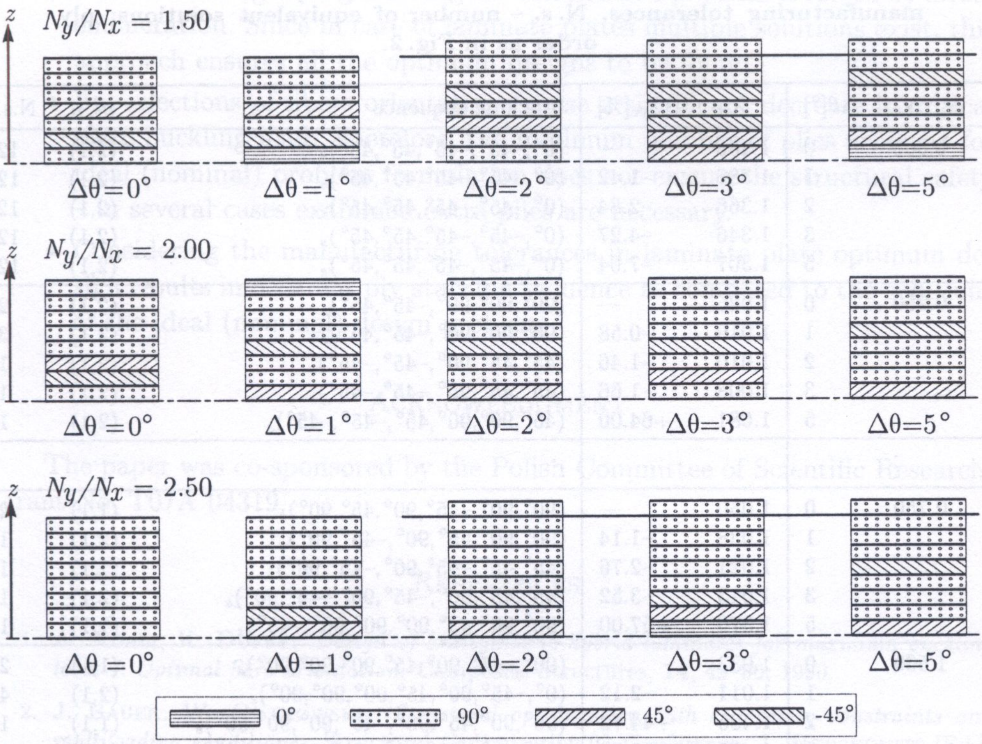

FIG. 5. Examples of ply stacking sequence in optimal laminate plates according to fiber orientation tolerances. For $N_y/N_x = 1.5$ and $N_y/N_x = 2.5$ extra plies are necessary if $\Delta\theta \geq 2°$.

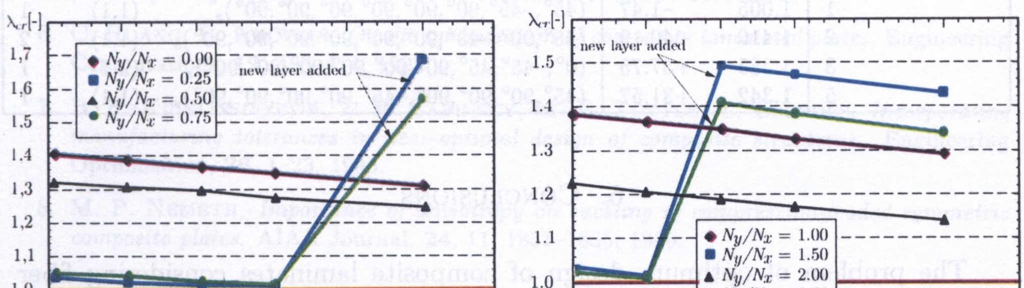

FIG. 6. Amplitude load parameter λ_{cr} for optimal plates considering fiber tolerances $\Delta\theta$.

Table 1. Ply stacking sequence for optimum laminate plates considering manufacturing tolerances, N. s. – number of equivalent solutions; ply order as in Fig. 2.

N_y/N_x	$\Delta\theta[^\circ]$	$\lambda_{\rm cr}$	$\Delta\lambda_{\rm cr}[\%]$	Stacking sequence	m,n	N.s.
0.000	0	1.406	—	$(0^\circ,-45^\circ,-45^\circ,45^\circ,45^\circ)_s$	(2,1)	12
	1	1.386	-1.42	$(0^\circ,-45^\circ,-45^\circ,45^\circ,45^\circ)_s$	(2,1)	12
	2	1.366	-2.84	$(0^\circ,-45^\circ,-45^\circ,45^\circ,45^\circ)_s$	(2,1)	12
	3	1.346	-4.27	$(0^\circ,-45^\circ,-45^\circ,45^\circ,45^\circ)_s$	(2,1)	12
	5	1.307	-7.04	$(0^\circ,-45^\circ,-45^\circ,45^\circ,45^\circ)_s$	(2,1)	12
0.250	0	1.025	—	$(90^\circ,90^\circ,90^\circ,-45^\circ,45^\circ)_s$	(2,1)	2
	1	1.019	-0.58	$(90^\circ,90^\circ,90^\circ,-45^\circ,45^\circ)_s$	(2,1)	3
	2	1.010	-1.46	$(45^\circ,45^\circ,90^\circ,-45^\circ,-45^\circ)_s$	(2,1)	1
	3	1.008	-1.66	$(45^\circ,45^\circ,90^\circ,-45^\circ,-45^\circ)_s$	(2,1)	1
	5	1.681	+64.00	$(45^\circ,90^\circ,90^\circ,45^\circ,-45^\circ,-45^\circ)_s$	(2,1)	1
⋮						
0.750	0	1.050	—	$(90^\circ,90^\circ,-45^\circ,90^\circ,45^\circ,90^\circ)_s$	(1,1)	2
	1	1.038	-1.14	$(90^\circ,90^\circ,45^\circ,90^\circ,-45^\circ,90^\circ)_s$	(2,1)	1
	2	1.021	-2.76	$(45^\circ,45^\circ,-45^\circ,90^\circ,-45^\circ,90^\circ)_s$	(1,1)	1
	3	1.013	-3.52	$(45^\circ,45^\circ,90^\circ,-45^\circ,90^\circ,-45^\circ,90^\circ)_s$	(2,1)	1
	5	1.570	+57.00	$(90^\circ,90^\circ,45^\circ,90^\circ,90^\circ,90^\circ,-45^\circ)_s$	(1,1)	1
1.500	0	1.033	—	$(90^\circ,-45^\circ,90^\circ,45^\circ,90^\circ,90^\circ,90^\circ)_s$	(1,1)	2
	1	1.011	-2.12	$(0^\circ,-45^\circ,90^\circ,45^\circ,90^\circ,90^\circ,90^\circ)_s$	(2,1)	4
	2	1.495	+44.72	$(90^\circ,90^\circ,45^\circ,90^\circ,-45^\circ,90^\circ,90^\circ,90^\circ)_s$	(1,1)	1
	3	1.475	+42.78	$(45^\circ,45^\circ,-45^\circ,90^\circ,-45^\circ,90^\circ,90^\circ,90^\circ)_s$	(1,1)	1
	5	1.434	+42.01	$(0^\circ,45^\circ,90^\circ,90^\circ,90^\circ,-45^\circ,90^\circ,90^\circ)_s$	(2,1)	2
2.000	0	1.228	—	$(90^\circ,-45^\circ,45^\circ,90^\circ,90^\circ,90^\circ,90^\circ,90^\circ)_s$	(2,1)	2
	1	1.207	-1.71	$(45^\circ,90^\circ,90^\circ,-45^\circ,90^\circ,90^\circ,90^\circ,45^\circ)_s$	(1,1)	1
	2	1.191	-3.10	$(45^\circ,90^\circ,90^\circ,-45^\circ,90^\circ,90^\circ,90^\circ,90^\circ)_s$	(2,1)	1
	3	1.172	-4.56	$(90^\circ,45^\circ,90^\circ,-45^\circ,90^\circ,90^\circ,90^\circ,90^\circ)_s$	(2,1)	1
	5	1.141	-7.08	$(45^\circ,90^\circ,90^\circ,90^\circ,-45^\circ,90^\circ,90^\circ,90^\circ)_s$	(1,1)	1
2.500	0	1.020	—	$(90^\circ,90^\circ,90^\circ,90^\circ,90^\circ,90^\circ,90^\circ,90^\circ)_s$	(1,1)	1
	1	1.005	-1.47	$(45^\circ,-45^\circ,90^\circ,90^\circ,90^\circ,90^\circ,90^\circ,90^\circ)_s$	(1,1)	1
	2	1.410	+40.19	$(45^\circ,90^\circ,-45^\circ,90^\circ,90^\circ,90^\circ,90^\circ,90^\circ,90^\circ)_s$	(1,1)	2
	3	1.385	+37.75	$(0^\circ,-45^\circ,45^\circ,90^\circ,90^\circ,90^\circ,90^\circ,90^\circ,90^\circ)_s$	(2,1)	1
	5	1.342	+31.57	$(45^\circ,90^\circ,90^\circ,90^\circ,-45^\circ,90^\circ,90^\circ,90^\circ,90^\circ)_s$	(1,1)	1

6. Conclusions

The problem of optimum design of composite laminates considering fiber orientations tolerances is addressed. The theoretical and numerical results allow the following conclusions to be drawn.

- The algorithm presented in this study offers an efficient and safe approach to incorporate manufacturing tolerances into the optimum design.

- The linear integer programming formulation of a problem is solved by direct enumeration. Since in case of laminate plates multiple solutions exist, this approach ensures all the optimum designs to be found.
- Imperfections of fibers orientations cause a significant decrease in critical plate buckling load. Therefore, the minimum number of plies achieved for ideal (nominal) problem formulation does not ensure the structural safety – in several cases examined extra plies are necessary.
- Considering the manufacturing tolerances in laminate plate optimum design results in different ply stacking sequence as compared to the solutions of the ideal (nominal) design problem.

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