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YIELD SURFACES AND CRITERIA OF PLASTIC YIELDING FOR A STRAIN HARDENING MATERIAL PART 1. EXPERIMENTAL STUDY

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The paper presents the results of experimental investigations of yield surfaces and plastic yield criteria for an aluminium alloy under complex loading. Experiments were performed on flat specimens (cut out of a sheet of metal) under uniaxial tension or compression. The proposed methodology has allowed to determine simultaneously two characteristic sections of the yield load surfaces. The first of these sections lies in the region of tensile stress while the second one is within the compressive stress area. Then, these two sections have enabled us to describe the position and dimensions of the whole yield surface in the stress space. Tests were carried out at first for the material in the initial state and then in three other states of this material obtained under different plastic prestrains. Such a procedure allowed us to determine the quantitative changes in the yield surfaces dimensions and the position of their centres as a function of plastic prestrains.

1. Introduction

Transition of the material to the plastic state under complex stress is usually defined by certain relations between such components of stress as the Huber-Mises-Hencky yield condition, e.g. a constant intensity of shear stress condition or the condition of maximum shear stress called also the Tresca yield condition.

Experimental investigations of the yield conditions and criteria of plastic yield under complex loads are usually performed in a plane state of stress when one of the principal stresses is equal to zero (the test methodology has been discussed in detail in the extensive survey paper [3]).

The Huber-Mises-Hencky yield criterion for a plane stress has the form:

$$\sigma_x^2 + \sigma_y^2 - \sigma_x \sigma_y + 3\tau_{xy}^2 = 3k^2. \tag{1.1}$$

In this criterion $k = \sigma_{pl}/\sqrt{3}$, where k is the yield point under pure sear stress and σ_{pl} is the yield point under uniaxial tension.

The ellipsoid (Fig. 1), with one axis coinciding with the τ_{xy}-axis, is the geometrical representation of the expression (1.1) in the stress space σ_x, σ_y, τ_{xy}. Two other axes of the ellipsoid lie in the $\sigma_x\sigma_y$-plane and they are bisectors of the angles between the axes σ_x and σ_y. Some particular ellipses may be distinguished on the surface of the ellipsoid. They correspond to various loadings of the plane and tubular specimens used in the tests. These ellipses PER and P'E'R', lying on the ellipsoid in the regions of positive and negative stress respectively, also belong to these specific ellipses. These ellipses are created in result of intersection of the ellipsoid by the planes $\pm(\sigma_x + \sigma_y) = \sigma_{pl}$.

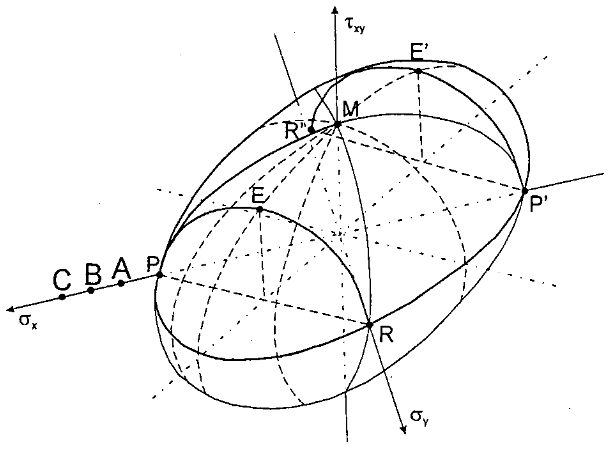

FIG. 1.

Substituting the expressions $(\sigma_x + \sigma_y)$ or $(-\sigma_x - \sigma_y)$ for σ_{pl} to the yield condition (1.1) we obtain the relation $\sigma_x * \sigma_y = \tau_{xy}$ which has to be satisfied on the ellipses PER and P'E'R'. It results from the Mohr diagram that this relation is satisfied only when one of the principal stresses is equal to zero. It means that the points lying on the PER and P'E'R' ellipses correspond to the states of uniaxial tension or compression in different directions with respect to the x-axis [2].

It is well known that the complex processes take place in real materials undergoing plastic deformations and that the initial yields surface changes its shape and location.

The new surface of the plastically prestrained material is called the secondary yield surface, the loading surface or the yield surface. In scientific literature there are many papers presenting the effect of the prestrains on the yield surface [e.g. 4 – 9]. However, in most cases, just a fraction of its outline is determined which allows to draw only some qualitative conclusions.

In this paper, two different cross-sections of the ellipsoid (the first one being in the zone of tensile stresses and the second one – in the zone of compressive stresses) have been verified experimentally (the PER and P'E'R' ellipses in Fig. 1). The results of such investigations have enabled us to determine the dimensions as well as the positions of the whole yield ellipsoid in the stress space. They have also made possible the quantitative assessment of the effect of prestrains on the dimensions and a position of the investigated yield surfaces in the stress space. The program and the research methodology seems to be an original contribution to the problem considered.

2. Program, methodology and experimental investigation results

The research program included the precise determination of the shape of the yield ellipsoid in the initial state of material and also in three other states of the material subjected to various plastic strains.

In order to determine the yield ellipsoid, it was necessary to determine the points lying on its surface in the sections situated in the zone of tensile and compressive stresses.

To execute this program, four big specimens were cut out of the metal sheet. Three of these specimens were subject to the quasi-static tension, with various maximum loading (the points A, B and C in Fig. 2). As a result of the prestress (tension), the specimens were extended by 1.74%, 3.28% and 5.88%, respectively. Then, fourteen small specimens were cut, at different angles, out of the three large specimens and the fourth one without prestressing. When seven of these fourteen specimens were subject to tension and the other seven – to compression, then the points were determined lying on the yield surface in the zone of tensile stress (the PER ellipse) and in the zone of compressive stress (the P'E'R' ellipse). Thus, the yield surfaces were obtained in the plane state of stress (the ellipsoid) for four different states of the material.

This paper presents many yield surfaces defined in defined manners in order to increase the possibilities of comparison between the investigation results obtained by various authors. Besides the surface corresponding to the proportionality limit, the surfaces were determined corresponding to the stress which produced in the material the plastic strains equal to: 0.01; 0.02; 0.03; 0.04; 0.05; 0.1; 0.2; 0.3; 0.4; and 0.5 %.

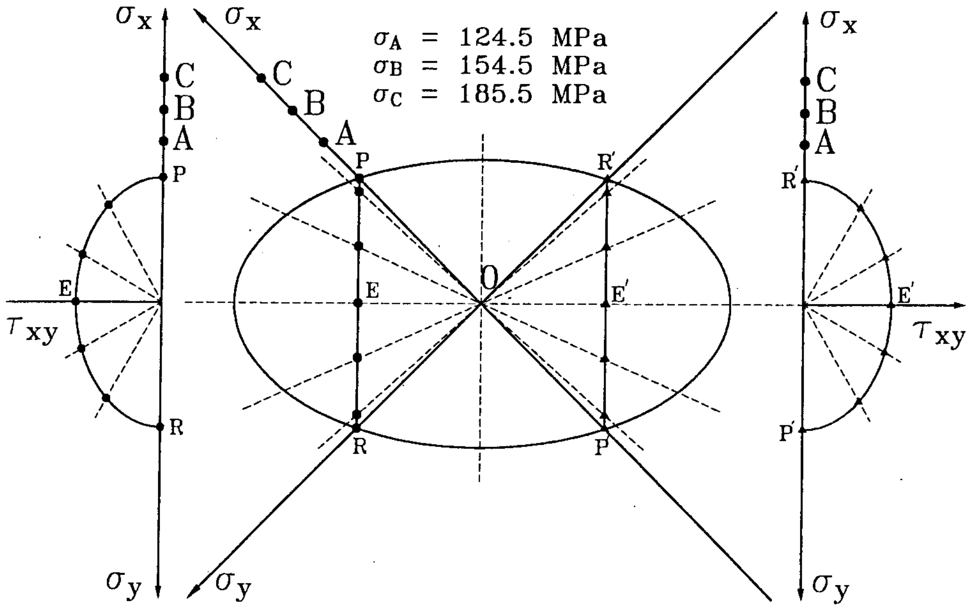

FIG. 2.

The details concerning the preparation of large and small specimens as well as the investigation methodology were presented in the paper [1].

Figure 3 presents, as an example, the experimental investigation results of several yield surfaces for a material under preloading along the OAO path. The conventional stress in the specimen, equal to 124.5 MPa, and the permanent extension of the large specimen, equal to 1.74%, corresponded to the maximum loading at point A.

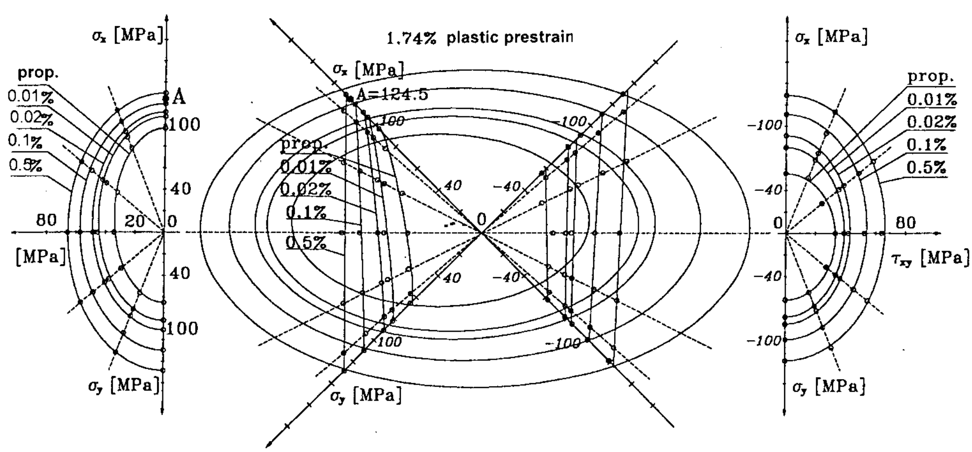

FIG. 3.

3. The analysis of the yield surface behaviour

On the basis of our original experimental investigations as well as the results known from the literature, it may be stated that the form of yield surface depends mainly on the magnitude of plastic strain introduced in the material. It is usually observed that the yield surface dimension and its displacement in the direction of preload grow with increase of permanent prestrains. Under specified load the yield surface may be rotated [4, 6]. The above changes take place simultaneously, but a share of particular components in these changes is often different for respective phases of the strain process. It has to be stressed that the shift value of the yield surface depends not on the Bauschinger effect and the plastic strain but also on the accepted definition of the yield point. It may be clearly visible and it will be proved later, that yield surfaces defined by high values of the permanent strain intensity are subject to a smaller shift than the surfaces defined by low values of plastic strains intensity.

Knowing that there are various factors affecting the yield surfaces, in this paper the attempt has been made to assess the quantitative changes in the yield ellipsoid dimensions and their location in the stress space as a function of permanent prestrains under the accepted definition of the yield point. The above assessment was performed assuming that the yield surfaces remain similar to the Huber-Mises-Hencky ellipsoid, they do not undergo a rotation and their centres displace in the $\sigma_x\sigma_y$-plane only.

Figure 4 shows the cross-sections of the $\sigma_{0.02}$ yield surface by the $\sigma_x\sigma_y$-plane, for four different states of the material. They show the surface of the initial material and three material surfaces after permanent prestrains equal to: 1.74%, 3.28%, 5.88%. These sections were determined by means of the method of the least squares in relation to the tests results, assuming a geometrical similarity of

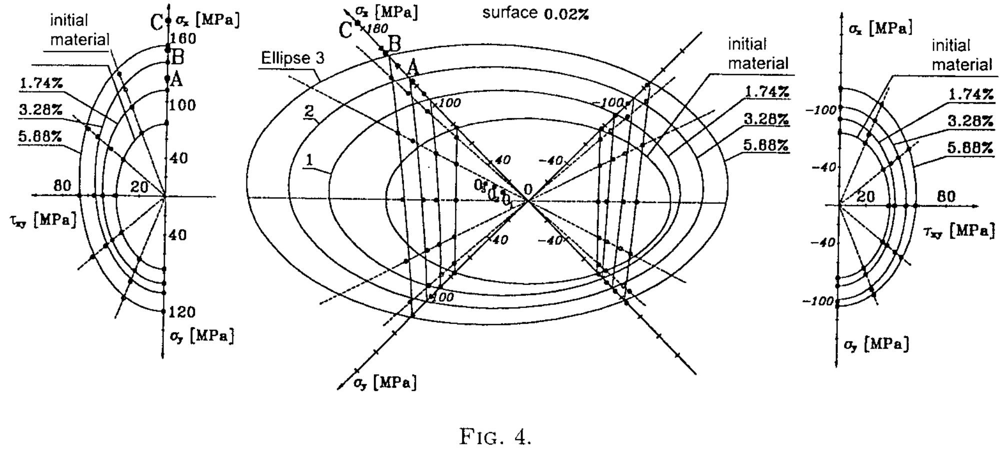

FIG. 4.

the determined surfaces to the corresponding Huber-Mises ellipse. Their axes and location of their centres are marked in the figure. With such a statement of results, the increase of the dimensions of the presented yield surfaces and the displacement of their centres due to the growth of permanent strains is clearly visible.

Figure 5 shows a process of the surfaces changes as a function of the permanent prestrains. Changes of the k_1 parameter describing the size of the particular yield surfaces as a function of prestrains for eleven variously defined surfaces are presented. This parameter was determined on the basis of the ellipses lying in the $\sigma_x\sigma_y$-plane parallel to the σ_x-axis in the zone of tensile stress. Variation of the k_1 parametr is shown in Figs. 5 and 6.

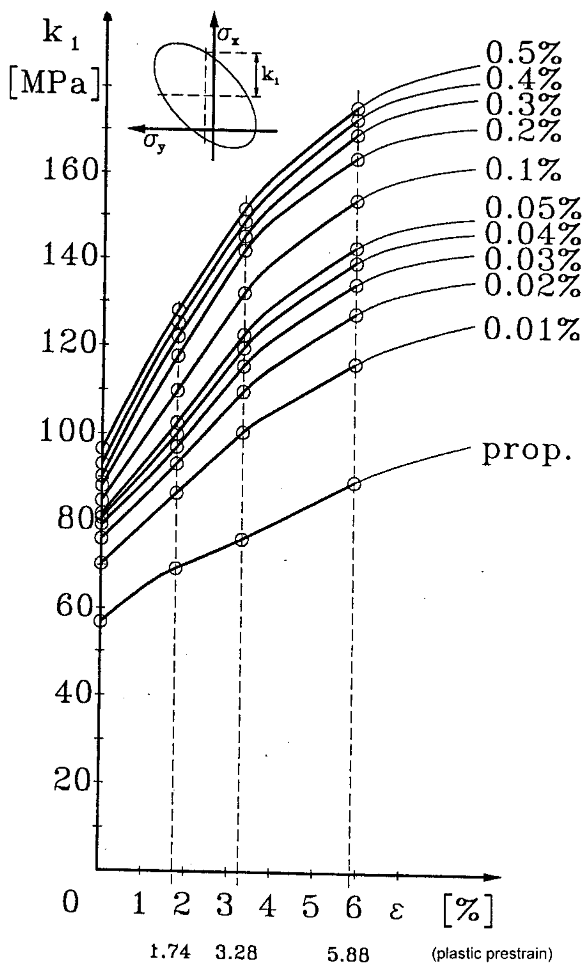

FIG. 5.

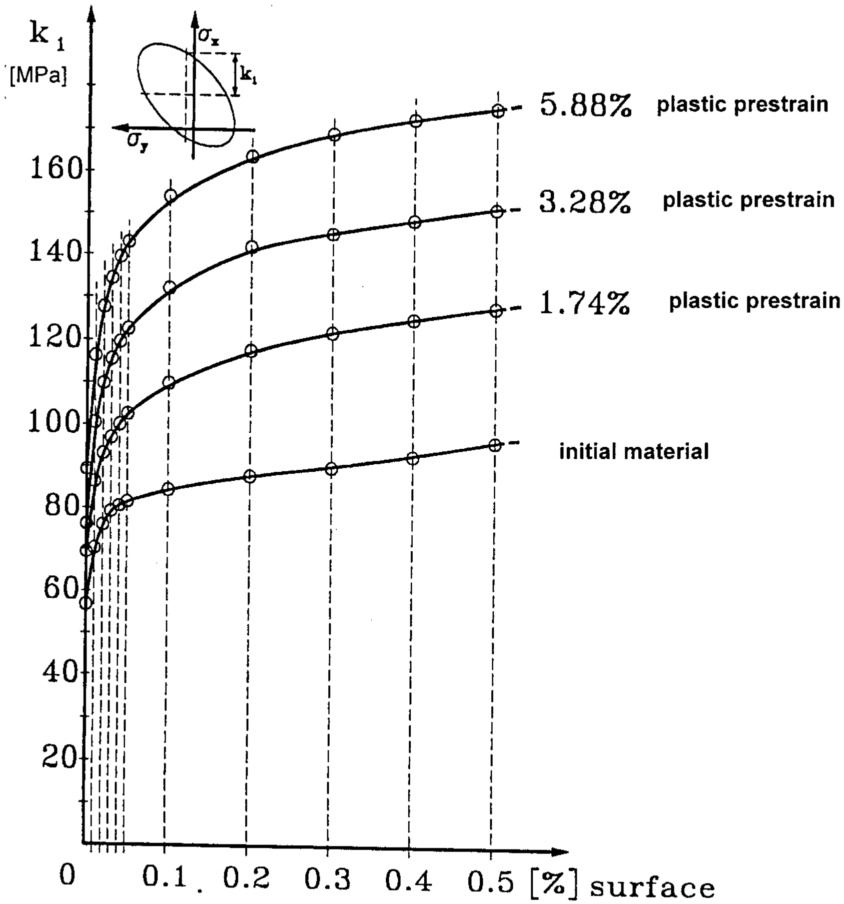

FIG. 6.

It is clearly visible that as the prestrains increase, the yield surfaces enlarge their dimensions. The growth of the surface is the larger, the greater is the value of permanent strains defining this surface.

Figure 6 shows the changes of dimensions as a function of plastic strains defining the surface for the initial material and the three prestrains under investigation.

The results of performed experiments, in form of graphs, describe the quantitative increase of yield surface as a function of plastic prestrains, determined on the basis of three different sections. Besides, it is also visible that the intensity (size) of the yield surface increase depends on the values of plastic prestrains. After a considerable increase for small prestrains the intensity of surface growth decreases. Taking into account the plots in Fig. 5 it may appear that a stabilization of the surface dimensions takes place for considerably larger material prestrains which have not been included in these tests, but only the further research may determine it explicitly.

The investigation results have also enabled us to carry out the detailed analysis of the phenomenon of displacement of the yield surface as a function of plastic prestrains. Figure 7 presents the plots of the shift, in the direction of preloads, of

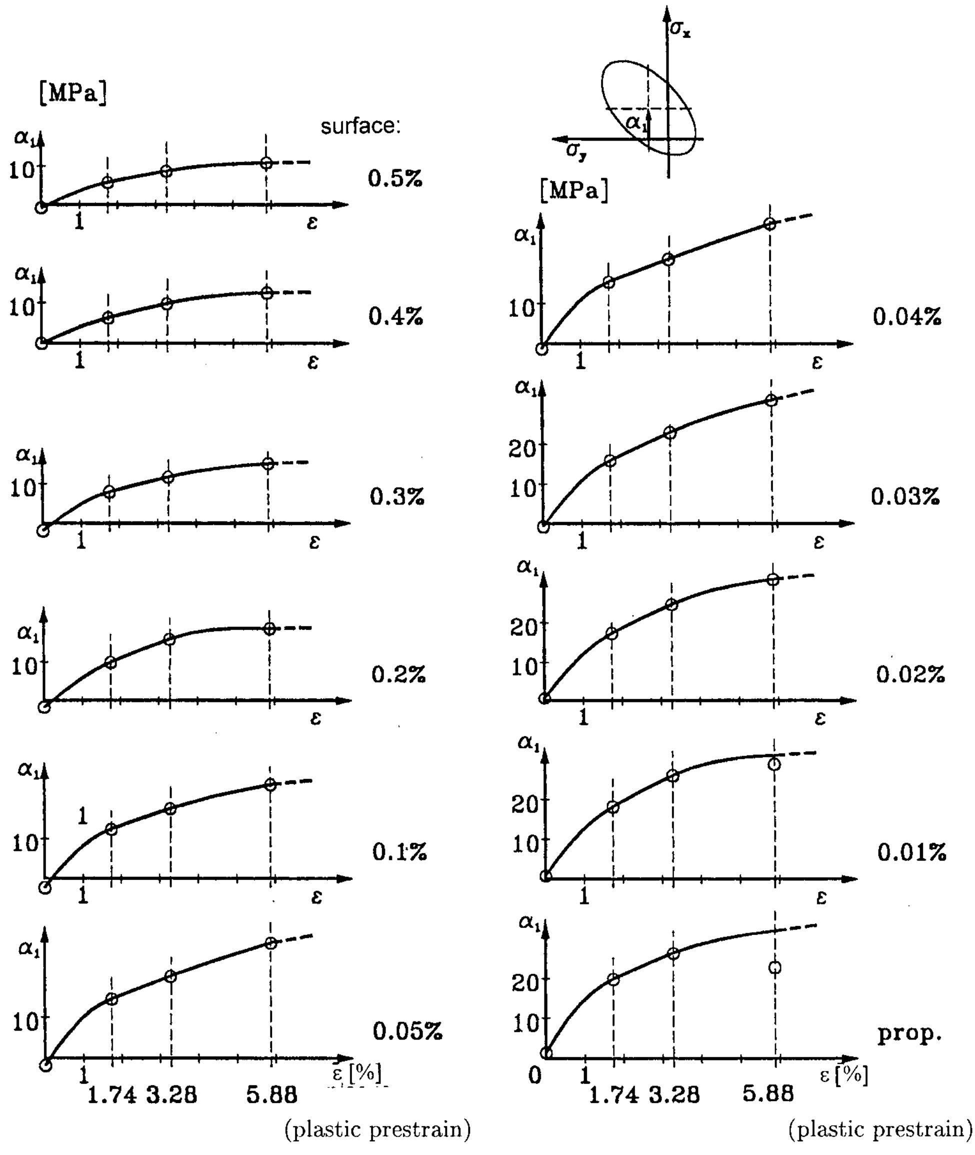

FIG. 7.

section centres of yield surface by the plane $\tau_{xy} = 0$ as a function of prestrains for various yield surfaces, i.e. the shift in the σ_x direction by the α_1 value marked schematically in the figure.

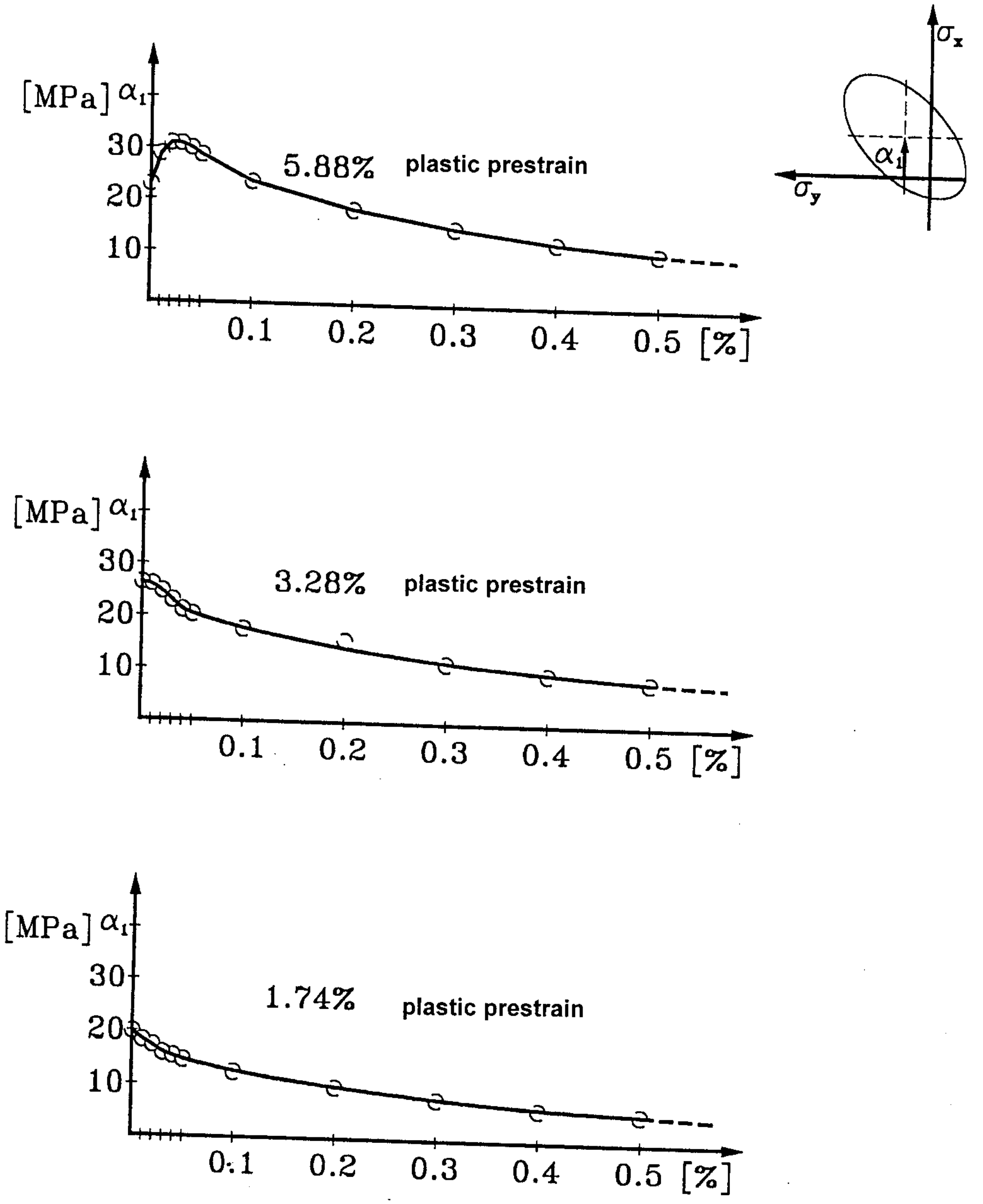

FIG. 8.

As the plastic strains, defining the yield surface, increase the shift of this surface decreases, contrary to the case of the surface dimensions. This effect is marked strongly by presenting the test results in another approach. Figure 8 shows the shift values of the yield surface as a function of plastic strains defining these surfaces. It is visible that the shifts are the largest for the proportionality surfaces and those defined by small plastic strains. For the surfaces defined by larger strains there is a sharp decrease in a shift of the yield surface in the direction of the preloading.

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