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SEMI-EMPIRICAL MODEL OF TIRE-PAVEMENT CONTACT

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Mathematical model of the tire-pavement interaction is presented in the paper. One of the main features of the model is easy identification of the parameter values by means typical testing equipment. In the model the contact zone of the tire is divided into the adhesion zone and the slip zone. Influence of the water wedge has been examined. Some typical tire characteristics are shown as a result of application of the proposed model.

Key Words: tire model, pavement, contact zone and water wedge.

1. INTRODUCTION

The forces acting between the vehicle tires and the pavement (so-called tangential forces) belong to more important parameters, decisive for the safety of drive.

Values and directions of these forces can be estimated using mathematical models of tyres. There are many well-known excellent empirical or semi-empirical tire models (BABBEL [1]; PACEJKA and BAKKER [6], PACEJKA and BESSELINK [7]). These models have one inconvenience: they require estimation of the parameter values difficult for identification. These parameters are usually obtained by means of specialistic, not easily accessible, testing equipment (BAKKER and OOSTEN [2]; LEISTER [3]).

One of the main aims of preparation of the new tire model was to avoid these difficulties and to construct the new model with easy identification of the parameters. The values of these parameters should be obtained by means of a typical testing equipment. Simultaneously, the model should take into account phenomena occurring on the tire-pavement contact under difficult exploitation conditions, such as driving on wet pavement during which the appearing water wedge reduces the force of adhesion.

The following assumptions have been introduced:

a) the tire plane is perpendicular to the pavement plane,

b) geometry of the contact zone depends on the vertical force between tire and pavement,

c) distribution of pressure under the tire in the direction perpendicular to the tire plane is constant,

d) static deflection of tires is approximately equal to the dynamic deflection.

In the general case, the contact zone can be divided into smaller areas (Fig. 1): adhesion zone (where there is no relative motion between the tire and pavement), slip zone, and – on wet pavement – zone of water wedge (water zone).

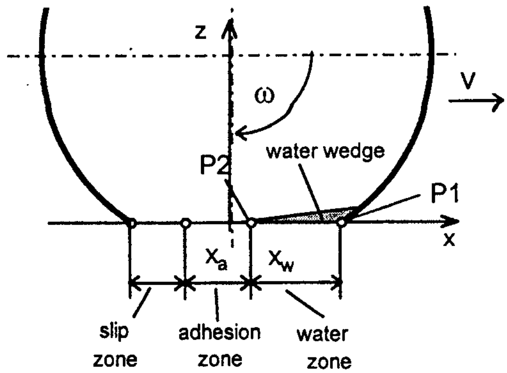

FIG. 1. Contact zones of the tire model.

2. Distribution of pressure between tire and pavement

The vertical load F_z of the wheel is transferred mostly by inflation pressure p inside the tire. We assume that, in normal circumstances, the vertical forces transferred by the tire are relatively small. Thus the area A_{SK} of contact zone

$$A_{SK} = \frac{F_z}{p}. \tag{2.1}$$

The shape of the contact zone can be approximated by two extreme figures: the rectangle $A_{SK\mathrm{MAX}} = B \cdot L$ and the ellipse $A_{SK\mathrm{MIN}} = \pi \cdot B \cdot L$. The real zone of contact lies between these both figures (Fig. 2). Introducing the coefficient $k_{SK} = A_{SK}/A_{SK\mathrm{MAX}}$, where: $\pi < k_{SK} < 1$, the contact zone area A_{SK} can be written as:

$$A_{SK} = k_{SK} \cdot B \cdot L, \tag{2.2}$$

whence the length of the contact zone is

$$L = \frac{A_{SK}}{k_{SK} \cdot B} = \frac{F_z}{k_{SK} \cdot B \cdot p}. \tag{2.3}$$

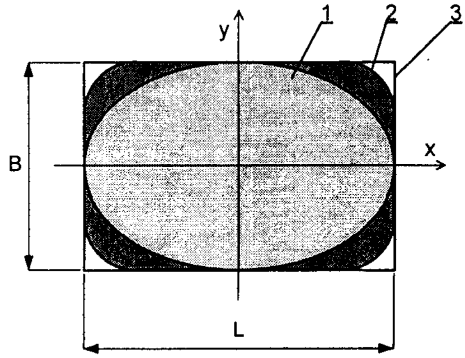

FIG. 2. Shape of the contact zone; 1 – eliptic; 2 – real; 3 – rectangular.

According to the assumption that distribution of pressure is constant in the direction perpendicular to the tire plane (Fig. 3), the model of the tire on smooth pavement can be reduced to a two-dimensional model.

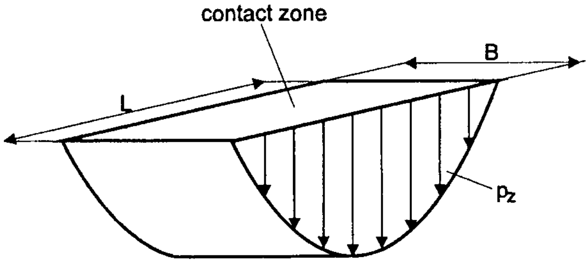

FIG. 3. Two-dimensional distribution of pressure.

Reaction of the pavement $R_z = -F_z$ for the zero longitudinal velocity of the tire ($v = 0$) and zero tangential forces is applied in the center of the contact zone. When the longitudinal forces act, the reaction R_z is displaced, but in the model described, this is not taken into account.

Distribution of pressures can be described by a polynomial, which satisfies the following conditions:

$$p_z(-L/2) = 0, \tag{2.4}$$

$$p_z(L/2) = 0. \tag{2.5}$$

Comparing the vertical force and the distribution of pressures, we obtain the following condition:

$$\int_{-L/2}^{L/2} p_z(x) \cdot B dx = F_z. \tag{2.6}$$

Conditions (2.4) – (2.6) are satisfied by the polynomial of the second order:

$$p_z(x) = p_A x^2 + p_B x + p_C. \tag{2.7}$$

To determine the coefficients p_A, p_B and p_C, one should substitute Eq. (2.7) into the Eqs. (2.4), (2.5) and (2.6):

$$\frac{1}{4} p_A \cdot L^2 - \frac{1}{2} p_B \cdot L + p_C = 0, \tag{2.8}$$

$$\frac{1}{4} p_A \cdot L^2 + \frac{1}{2} p_B \cdot L + p_C = 0, \tag{2.9}$$

$$\frac{1}{12} p_A \cdot L^2 + p_C = F_z/(B \cdot L). \tag{2.10}$$

The matrix form of this system of equations is the following:

$$\begin{bmatrix} \frac{1}{4}L^2 & -\frac{1}{2}L & 1 \\ \frac{1}{4}L^2 & \frac{1}{2}L & 1 \\ \frac{1}{12}L^2 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} p_A \\ p_B \\ p_C \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ F_z/(B \cdot L) \end{bmatrix}. \tag{2.11}$$

3. Tire deformation

Circumferential tire deformation Δx (connected with the longitudinal slip s_x) and lateral deformation Δy (connected with the slip angle α) can be estimated in the following manner. During the time period t_0, axis of the tire will be shifted by distance

$$x_k = v_x \cdot t_0 + \frac{a_x \cdot t_0^2}{2}, \tag{3.1}$$

where v_x i a_x denote longitudinal velocity and acceleration. We have assumed that, during the time period t_0, acceleration a_x is approximately constant and $x_k < x_a$ (see Fig. 1). At the same time, the tire point which entered in contact with the pavement is displaced by the distance:

$$x_0 = \left(\omega \cdot t_0 + \frac{\omega \cdot t_0^2}{2}\right) \cdot (R - z_0) = \omega \cdot (R - z_0) \cdot t_0 + \omega \cdot (R - z_0) \cdot \frac{t_0^2}{2}, \tag{3.2}$$

where ω is the rotational speed of the tire.

Using the definition of longitudinal slip (for the case of braking):

$$s_x = \frac{v_x - \omega \cdot (R - z_0)}{v_x}, \tag{3.3}$$

we have:

$$\omega \cdot (R - z_0) = (1 - s_x) \cdot v_x. \tag{3.4}$$

After differentiation:

$$\omega \cdot (R - z_0) = a_x \cdot (1 - s_x) - s_x \cdot v_x. \tag{3.5}$$

Substituting formulas (3.4) and (3.5) into (3.2), we have:

$$x_0 = (1 - s_x) \cdot v_x \cdot t_0 + [(1 - s_x) \cdot a_x - s_x \cdot v_x]\frac{t_0^2}{2}. \tag{3.6}$$

Thus the circumferential tire deformation is:

$$\Delta x_0 = x_k - x_0 = v_x \cdot s_x \cdot t_0 + (s_x \cdot a_x + s_x \cdot v_x) \cdot \frac{t_0^2}{2}. \tag{3.7}$$

Time t_0 is very short, eg. $t_0 < 0.02$ s for the velocity 5 m/s and the length of contact zone 0.1 m. So, after omission of the last element of Eq. (3.7), we obtain:

$$\Delta x_0 = v_x \cdot s_x \cdot t_0. \tag{3.8}$$

Since time t_a of the tire axis displacement by distance x_a (length of the contact zone) is equal

$$t_a = \frac{x_a}{v_x}, \tag{3.9}$$

the total circumferential tire deformation in the adhesion zone:

$$\Delta x_a = x_a \cdot s_x. \tag{3.10}$$

Assuming that the circumferential deformation in the adhesion zone can be described by a linear function, we have:

$$\Delta x(x) = \frac{\Delta x_a}{x_a} \cdot \left(\frac{L}{2} - x_w - x\right) = s_x \cdot \left(\frac{L}{2} - x_w - x\right). \tag{3.11}$$

This assumption is true only in the adhesion zone, where no relative motion between the tire elements and pavement appears.

Lateral tire deformation can be obtained in a similar manner. Recording lateral velocity of the first tire contact point (for slip angle α), we obtain:

$$v_y = v_x \cdot \tan\alpha, \tag{3.12}$$

so that the total lateral tire deformation in the adhesion zone is

$$\Delta y_a = v_y \cdot t_a = x_a \cdot \tan\alpha. \tag{3.13}$$

In accordance with the assumption on linear deformation, we obtain:

$$\Delta y(x) = \frac{\Delta y_a}{x_a} \cdot \left(\frac{L}{2} - x_w - x\right) = \tan\alpha \cdot \left(\frac{L}{2} - x_w - x\right). \tag{3.14}$$

4. Length of the water wedge

We assume that pavement is covered by a layer of water of constant thickness h_0. Rolling tire on a wet pavement causes "pumping" of water from the contact zone – mostly in the direction perpendicular to the direction of rolling. The linear speed v_w of outflow of the water can be determined from the Bernoulli law:

$$p_w(x) = \frac{\rho \cdot [v_w(x)]^2}{2} \Rightarrow v_w(x) = \sqrt{\frac{2 \cdot p_w(x)}{\rho}}, \tag{4.1}$$

where: p_w – pressure of water, ρ – density of water.

Elementary discharge of water is

$$dQ = v_w \cdot dS, \tag{4.2}$$

where dS – element of the surface area. Thus, the total discharge of water flowing out from the water wedge zones is

$$Q = 2 \cdot \int_0^{x_w} h_w(x) \cdot v_w(x) dx = \sqrt{\frac{8}{\rho}} \cdot \int_0^{x_w} \sqrt{p_w(x)} \cdot h_w(x) dx, \tag{4.3}$$

where $h_w(x)$ – average thickness of the water film in the water wedge zone.

During the time $t_w = x_w/v$, the tire is displaced by a distance equal to the length of the water wedge. In total time, volume V of water is squeezed out from under the tires,

$$V = K_{01} \cdot B \cdot \left(h_0 \cdot x_w - \int_0^{x_w} h_w(x) dx\right), \tag{4.4}$$

where K_{01} – coefficient depending on the geometry of tire tread. For a smooth tire $K_{01} = 1$, for a real tread $K_{01} < 1$. Let us assume the height of tread elements is h_B. If the water film thickness $h_0 < h_B$, then

$$K_{01} = \frac{V}{V_{PL}} = \frac{A_{WY}}{A_{WY} + A_{WR}}, \tag{4.5}$$

where: V_{PL} – volume of water pressed out by a smooth tire, A_{WY} – total area of protruding elements of the tread in the contact zone, A_{WR} – total area of hollowed elements of the tread in the contact zone ($A_{WR} = A_{SK} - A_{WY}$).

Time t_w is very short. For example, for $v = 5$ m/s and $w_x = 0.05$ m time $t_w = 0.01$ s. One can so assume that discharge of water in that time

$$Q = \frac{V}{t_w} = K_{01} \cdot B \left(h_0 \cdot x_w - \int_0^{x_w} h_w(x)dx \right) \frac{v}{x_w}. \tag{4.6}$$

Comparing (4.6) and (4.3) one can calculate the length of the water wedge.

In the paper of MOONEY and WOOD [4], hydraulic pressure between the tire and pavement is examined similarly to the method applied for estimation of the pressure in the slide bearing. So the pressure $p_w(x)$ in the water wedge can be determined in the following manner:

$$p_w(x) = K_{02} \cdot \eta_w \cdot \frac{v}{h_w(x)^2}, \tag{4.7}$$

where: K_{02} – coefficient describing texture of pavement and tire geometry, η_w – coefficient of dynamic viscosity of water ($\eta_w \approx 1$ kPa·s at temperature 20° C). For the linear function describing variability of the water wedge thickness

$$h_w(x) = \frac{h_0}{x_w} \cdot \left(x + x_w - \frac{L}{2} \right) \tag{4.8}$$

one can derive the following formula for the length of the water wedge:

$$x_w = K_w \cdot h_0 \cdot \sqrt{v}, \tag{4.9}$$

where K_w – coefficient taking into account the tire and the pavement parameters:

$$K_w = \frac{K_{01} \cdot B \cdot \sqrt{\rho}}{\sqrt{32 \cdot K_{02} \cdot \eta_w}}. \tag{4.10}$$

The formula for computing of critical tire speed on a thin water film has the form (NAVIN [5]):

$$v_{kr} = 0.056\sqrt{p} + 3.33\frac{h_B}{h_0} + 16.67 \exp(9 - 3000h_0 + 429h_B), \tag{4.11}$$

where: p – inflation pressure, h_B – tire tread height, h_0 – thickness of water film.

If we assume, that at a speed equal to the critical speed, the length of water wedge is equal to the length of the contact zone, i.e. $x_{w(v=vkr)} = L$, then:

$$K_w = \frac{L}{h_0 \cdot \sqrt{v_{kr}}}. \tag{4.12}$$

5. Boundaries of the adhesion and slide zones

If deformations of a tire occur in a linear area, then the circumferential stress τ_x and lateral stress τ_y are proportional to the deformations Δx and Δy:

$$\tau_x = K_x \cdot \Delta x, \tag{5.1}$$

$$\tau_y = K_y \cdot \Delta y, \tag{5.2}$$

where K_x, K_y – coefficient of circumferential and lateral stress referred to a unitary area.

Total shearing stress is limited by the maximum adhesive forces. According to the well known dependence between the coefficients μ_x i μ_y of static friction in the longitudinal and the lateral directions, we have

$$\left(\frac{\mu_x}{\mu_{x\mathrm{MAX}}}\right)^2 + \left(\frac{\mu_y}{\mu_{y\mathrm{MAX}}}\right)^2 = 1, \tag{5.3}$$

As a result of this formula we obtain the following inequality satisfied in the adhesion zone:

$$\sqrt{\left(\frac{\tau_x(x)}{\mu_x}\right)^2 + \left(\frac{\tau_y(x)}{\mu_y}\right)^2} \leq p_z(x). \tag{5.4}$$

Solving this inequality, one can calculate the length of the adhesion zone.

6. Shearing forces in the contact zone

Total shearing forces acting in the contact zone are the sum of the shearing forces acting in the adhesion zone and the forces acting in the slip zone.

Shearing forces acting in the water wedge zone are negligibly small compared to the forces acting in other zones. Elementary longitudinal force f_{Ax} acting on area $d\Omega$ in the adhesion zone equals:

$$f_{Ax} = \tau_x \cdot d\Omega, \tag{6.1}$$

so that the total force F_{Ax}:

$$(6.2)\qquad F_{Ax} = \iint_A \tau_x \cdot d\Omega = \int_{L/2-x_w-x_a}^{L/2-x_w} K_x \cdot \Delta x(x) d\Omega$$

$$= K_x \cdot B \cdot \int_{L/2-x_w-x_a}^{L/2-x_w} \Delta x(x) dx.$$

In a similar manner one can write the forces acting in the lateral direction:

$$(6.3)\qquad f_{Ay} = \tau_y \cdot d\Omega,$$

$$(6.4)\qquad F_{Ay} = \iint_A \tau_y \cdot d\Omega = \int_{L/2-x_w-x_a}^{L/2-x_w} K_y \cdot \Delta y(x) d\Omega$$

$$= K_y \cdot B \cdot \int_{L/2-x_w-x_a}^{L/2-x_w} \Delta y(x) dx.$$

Unitary shearing forces acting in the slip zone are equal:

$$(6.5)\qquad f_{Sx} = p_z(x) \cdot \mu^s_{x\mathrm{MOD}} \cdot d\Omega,$$

$$(6.6)\qquad f_{Sy} = p_z(x) \cdot \mu^s_{y\mathrm{MOD}} \cdot d\Omega,$$

where, instead of static adhesion coefficients μ^s_x and μ^s_y, the modified coefficients $\mu^s_{x\mathrm{MOD}}$ and $\mu^s_{y\mathrm{MOD}}$ are applied, resulting from the motion in the longitudinal and lateral directions.

It was assumed, that the adhesive coefficients of friction μ^s_x and μ^s_y fulfil the following dependence:

$$(6.7)\qquad \left(\frac{\mu^s_{x\mathrm{MOD}}}{\mu^s_x}\right)^2 + \left(\frac{\mu^s_{y\mathrm{MOD}}}{\mu^s_y}\right)^2 = 1.$$

Additionally, the condition into account was taken:

$$(6.8)\qquad \frac{\mu^s_{y\mathrm{MOD}}}{\mu^s_{x\mathrm{MOD}}} = \tan\alpha_F,$$

where α_F – the angle between the direction of total tangential force and the 0X axis.
Solving the system of Eqs. (6.7) – (6.8) one can compute the coefficients $\mu^s_{x\mathrm{MOD}}$ and $\mu^s_{y\mathrm{MOD}}$. Assuming that direction of the total tangential force activity complies with the direction of relative motion of the tire and of pavement, angle α_F can be determined from the following formula:

$$\tan\alpha_F = \frac{\tan\alpha}{s_x}, \tag{6.9}$$

Finally:

$$\mu^s_{x\mathrm{MOD}} = \frac{\mu^s_x \cdot \mu^s_y \cdot s_x}{\sqrt{(\mu^s_y)^2 \cdot s^2_x + (\mu^s_x)^2 \cdot (\tan\alpha)^2}}, \tag{6.10}$$

$$\mu^s_{y\mathrm{MOD}} = \frac{\mu^s_x \cdot \mu^s_y \cdot \tan\alpha}{\sqrt{(\mu^s_y)^2 \cdot s^2_x + (\mu^s_x)^2 \cdot (\tan\alpha)^2}}. \tag{6.11}$$

Components F_{Sx} i F_{Sy} of the total force F_S acting in the slip zone in the direction x and y are equal:

$$\begin{aligned} F_{Sx} &= \int\!\!\int_S p_z(x) \cdot \mu^s_{x\mathrm{MOD}} d\Omega = \int_{-L/2}^{L/2-x_w-x_a} p_z(x) \cdot \mu^s_{x\mathrm{MOD}} d\Omega \\ &= \mu^s_{x\mathrm{MOD}} \cdot B \cdot \int_{-L/2}^{L/2-x_w-x_a} p_z(x)dx, \end{aligned} \tag{6.12}$$

$$\begin{aligned} F_{Sy} &= \int\!\!\int_S p_z(x) \cdot \mu^s_{y\mathrm{MOD}} d\Omega = \int_{-L/2}^{L/2-x_w-x_a} p_z(x) \cdot \mu^s_{y\mathrm{MOD}} d\Omega \\ &= \mu^s_{y\mathrm{MOD}} \cdot B \cdot \int_{-L/2}^{L/2-x_w-x_a} p_z(x)dx. \end{aligned} \tag{6.13}$$

Total tangential force F_{xy} acting in the contact zone

$$F_{xy} = \sqrt{F^2_x + F^2_y}, \tag{6.14}$$

where:

$$F_x = F_{Ax} + F_{Sx}, \tag{6.15}$$

$$F_y = F_{Ay} + F_{Sy}. \tag{6.16}$$

The stabilizing moment:

$$M_z = M_{Az} + M_{Sz} \tag{6.17}$$

where: M_{Az} – component of moment acting in the adhesion zone, M_{Sz} – component of moment acting in the slip zone.

Moments M_{Az} and M_{Sz} are the results of tire deformation and of unsymmetric distribution of lateral force with respect to the y axis.

Components M_{Az} and M_{Sz} of the stabilizing moment can be calculated from the following dependencies:

$$M_{Az} = \int\limits_{L/2-x_w-x_a}^{L/2-x_w} B \cdot \tau_y \cdot (x - \Delta x) \cdot dx + \int\limits_{L/2-x_w-x_a}^{L/2-x_w} B \cdot \tau_x \cdot \Delta y \cdot dx, \tag{6.18}$$

$$M_{Sz} = \int\limits_{-L/2}^{L/2-x_w-x_a} p_z(x) \cdot \mu^s_{y\mathrm{MOD}} \cdot B \cdot (x - \Delta x) \cdot dx + \int\limits_{-L/2}^{L/2-x_w-x_a} p_z(x) \cdot \mu^s_{x\mathrm{MOD}} \cdot B \cdot \Delta y \cdot dx. \tag{6.19}$$

After substituting the formulae describing $\Delta x(x)$, $\Delta y(x)$, $\tau_x(x)$ and $\tau_y(x)$ – i.e. formulae (3.11), (3.14), (5.1) and (5.2), for pure cornering ($s_x = 0$) we have:

$$M_{Az} = B \cdot K_y \cdot \tan(\alpha) \cdot \int\limits_{L/2-x_w-x_a}^{L/2-x_w} \left(\frac{1}{2}L \cdot L \cdot x - x_w \cdot x - x^2\right) dx, \tag{6.20}$$

$$M_{Sz} = B \cdot \int\limits_{-L/2}^{L/2-x_w-x_a} p_z(x) \cdot \mu^s_y \cdot x \cdot dx. \tag{6.21}$$

7. Examples

To verify the qualitative correctness of the tire characteristics obtained from the proposed model, a few calculations have been done for the following data: $R = 0.3$ m, $h_B = 6$ mm, $K_x = 1 \cdot 10^7$ N/m^3, $K_y = 0.8 \cdot 10^7$ N/m^3, $p = 0.15$ MPa, $B = 0.15$ m, $k_{SK} = 0.9$, $\mu_{x\mathrm{MAX}} = 1.0$, $\mu_{y\mathrm{MAX}} = 0.8$, $\mu_x^s = 0.7$, $\mu_y^s = 0.6$, $F_z = 3$ kN. For the above data, the length of the contact zone $L = 0.15$ m is obtained.

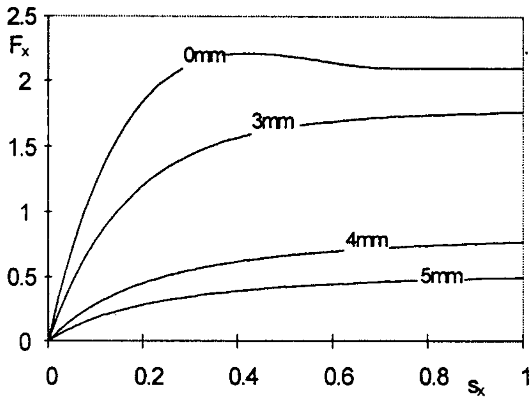

FIG. 4. Longitudinal force [kN] versus slip s_x for thickness of the water film $h_0 = 0, 3, 4$ and 5 mm.

In Figs. 4 and 5 are shown the relations between the longitudinal force F_x versus slip s_x and between the lateral force F_y and slip angle α for various values of thickness of the water film. Shape of these characteristics is such as the characteristics obtained in experimental investigations.

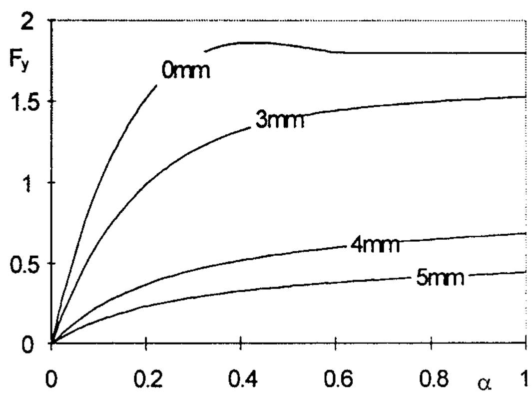

FIG. 5. Lateral force [kN] versus slip angle α for thickness of the water film $h_0 = 0$, 3, 4 and 5 mm.

The same conclusion can be formulated for the examples shown in Figs. 6 and 7, for the relation between longitudinal and lateral forces, and in the case of combined slip ($s_x \neq 0$ and $\alpha \neq 0$).

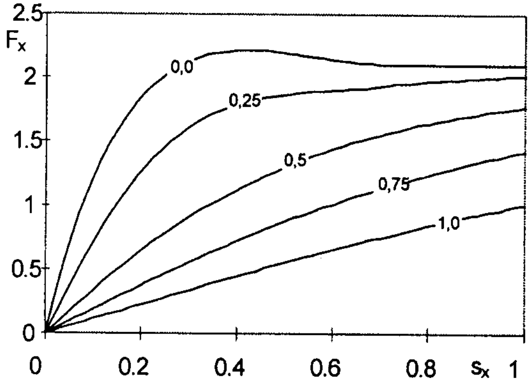

FIG. 6. Longitudinal force [kN] versus slip for slip angle 0, 0.25, 0.5, 0.75 and 1.0 rad.

On the graphs of Fig. 8 is shown the relation between longitudinal and lateral forces for various values of the slip angle. Dependence between the stabilization moment and the slip angle is shown in Fig. 9.

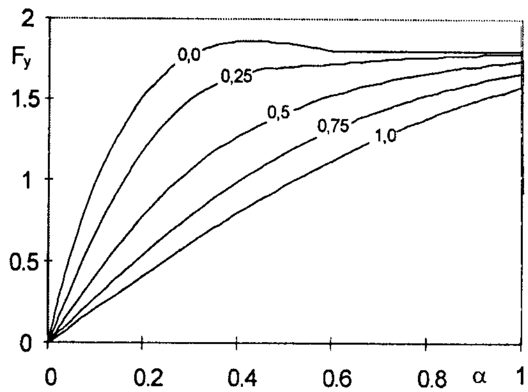

FIG. 7. Lateral force [kN] versus slip angle [rad] for slip $s_x = 0$, 0.25, 0.5, 0.75 and 1.0.

Results presented in Figs. 10 and 11 show that length of the water zone rapidly grows for the water film of 3 mm thickness and simultaneously, the height of tread elements has no significant influence on this parameter.

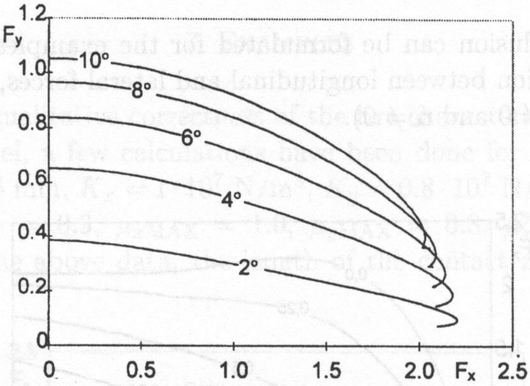

FIG. 8. Lateral force [kN] versus longitudinal force [kN] for slip angle $\alpha = 2°, 4°, 6°, 8°$ and $10°$.

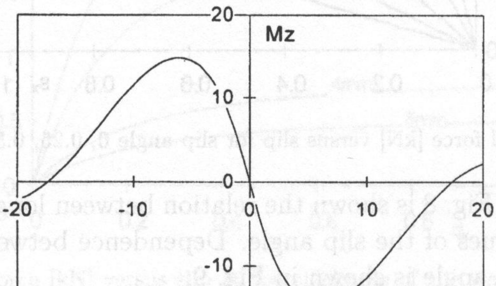

FIG. 9. Self-aligning moment [Nm] versus slip angle [deg] for pure cornering ($s_x = 0$).

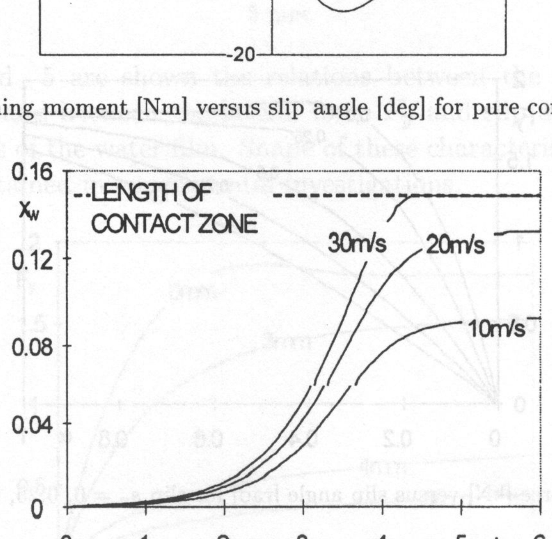

FIG. 10. Length of the water zone [m] versus thickness of the water film [mm] for velocity $v_x = 10$, 20 and 30 m/s.

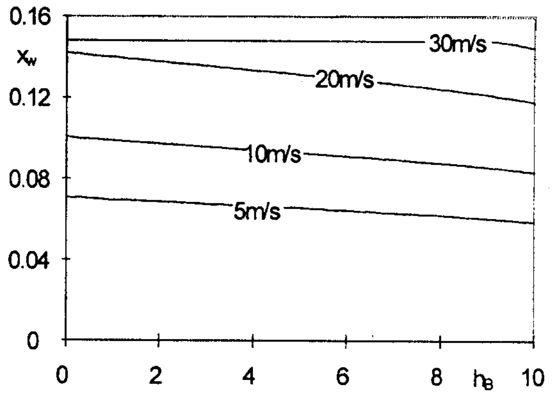

FIG. 11. Length of the water zone [m] versus height of the tread elements [mm] for velocity $v_x = 5$, 10, 20 and 30 m/s.

8. FINAL REMARKS

Correct calculations of the tangential forces which act between the tire and pavement depend on the tire model. There are many excellent tire models known in the literature. However, values of their parameters depend on the type of applied tires and very frequently they are unknown. It means that values of these parameters should be determined experimentally way using, as a rule, special experimental measurement stands. Such stands are not always easily accessible.

All parameters necessary for the presented tire model can be obtained by means of typical testing equipment. Simultaneously the tire model takes into account the influence of water wedge and other effects important from the point of view of safety.

The examples presented confirm the correctness of the estimated tire characteristics and usefulness of the model in computer simulation of vehicles motion.

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