ENGINEERING TRANSACTION • Engng. Trans. • 48, 1, 25-42, 2000
Polish Academy of Sciences • Institute of Fundamental Technological Research
10.24423/engtrans.606.2000

STRESSES IN VISCOELASTIC SPHERE DRIED CONVECTIVELY

J. BANASZAK

INSTITUTE OF FUNDAMENTAL TECHNOLOGICAL RESEARCH
POLISH ACADEMY OF SCIENCES
e-mail: Jacek.Banaszak@fct.put.poznan.pl

and

S. J. KOWALSKI

INSTITUTE OF TECHNOLOGY AND CHEMICAL ENGINEERING,
POZNAŃ UNIVERSITY OF TECHNOLOGY
e-mail: Stefan J.Kowalski@fct.put.poznan.pl

The deformations and the drying-induced stresses in a saturated porous elastic and viscoelastic sphere dried convectively are analysed. The considerations are confined to the constant drying rate period. The solution of the problem is obtained using both the Laplace transformations and the numerical finite difference method. The drying experiment was performed on spheres made of three different clay sorts in order to validate the results obtained by numerical analysis. The results obtained are presented in graphical form.

NOTATIONS

σ_{ij}	stresses	N/m^2
ε_{ij}	strains	1
u_{ij}	displacement	m
M	shear modulus of elastic deformations	N/m^2
A	bulk modulus of elastic deformations	N/m^2
K	$3K = 2M + 3A$	N/m^2
α_Θ	moisture expansion coefficient	1
Θ	moisture content	kg/kg
μ	moisture potential	J/kg
Λ_m	moisture transport coefficient	$kg\ s/m^2$

c_Θ	moisture content of the moisture potential	J/m^3
ρ_0	dry body mass density	kg/m^3
ξ	bulk modulus of viscoelastic deformations	N/m^2
η	shear modulus of elastic deformations	N/m^2
χ	$3\chi = 2\eta + 3\xi$	N/m^2

1. Introduction

The main subject of this paper is to study how the assumed constitutive model of saturated porous body influences the numerically estimated deformations and stresses induced during drying. We will study this subject solving the problem of convective drying of a sphere in which the evolution of deformations and shrinkage stresses is analysed. It is known that heating of the material during the drying processes involves its expansion, and removal of moisture – its contraction. The expansion and contraction will induce non-uniform strains (depending on the geometry, material properties, etc.) what, in turn, results in a complex stress state.

The material of the sphere is assumed to be both elastic and viscoelastic. There exists a significant difference between the results obtained for the elastic sphere and viscoelastic one, particularly for stresses. The classic problem of an elastic sphere subjected to the radial temperature distribution has been solved by several authors, as shown by Timoshenko and Goodier [21]. Morland and Lee [16] have analysed the stress in material with temperature – dependent characteristics. Muki and Sternberg [16] studied the thermal stress distribution within infinite and finite linear viscoelastic spheres. Rao, Hamman and Hammerle [19] have investigated experimentally the stresses in viscoelastic sphere.

The thermodynamical background of the model used in this paper is presented in Kowalski [9] and Kowalski and Strumiłło [10]. The solution of the problem was obtained making use of both the Laplace transformations and the finite difference method.

The considerations are confined to the constant drying rate period in which the temperature of the saturated body is constant in the whole cross-section and equal to the wet-bulb temperature. Therefore, the thermal stresses are absent in this stage of drying and the internal stresses are caused only by the moisture changes. Phase transitions inside the dried material are ignored and the whole evaporation of the moisture is assumed to proceed on the boundary of the drying material. The evolution of the distribution of shrinkage stresses and the displacements of the sphere are presented in the form of graphs.

2. Model presentation

The considered sphere is shown in Fig. 1.

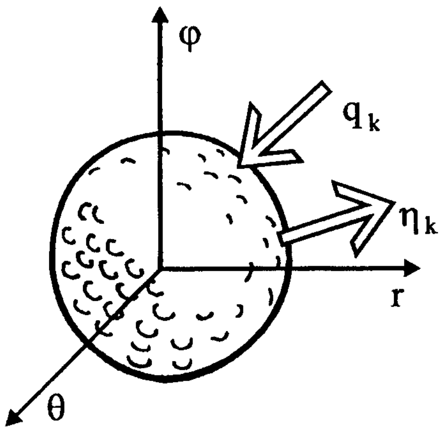

FIG. 1. A sphere dried convectively.

The following assumptions are to be satisfied:

1) The sphere consists of a porous elastic or viscoelastic material (Maxwellian model), whose pores are filled with water.

2) The drying of the sphere proceeds symmetrically with respect to the middle point, so that only the displacement in radial direction is different from zero, that is $u_r \neq 0$, $u_\varphi = 0$, $u_\theta = 0$.

3) The external surface $r = R$ is free of external loading.

4) The body force is neglected.

5) The analysis is confined to the constant drying rate period, which is characterised by uniform temperature in the whole body, equal to the wet-bulb temperature.

6) The sphere is assumed to be isotropic and continuous.

The equation of equilibrium for the moist porous sphere subjected to convective drying process is:

$$\frac{\partial \sigma_{rr}}{\partial r} + 2\frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = 0. \tag{2.1}$$

In spherical co-ordinates the strains are:

$$\varepsilon_{rr} = \frac{\partial u_r}{\partial r}, \qquad \varepsilon_{\theta\theta} = \varepsilon_{\varphi\varphi} = \frac{u_r}{r}. \tag{2.2}$$

The mechanical properties of a linear viscoelastic material are simulated by Maxwell's model that consists of a spring and dashpot connected in series. In this model, all the energy goes initially into stretching of the spring, and the dashpot gradually dissipates energy by its constant rate movement, resulting in an exponential decay of stress in the system. This model reflects well the behaviour of moist materials during drying.

The mathematical expression for this model is

$$\begin{aligned} \dot{s}_{ij} + \frac{M}{\eta} s_{ij} &= 2M\dot{e}_{ij}, \\ \dot{\sigma} + \frac{K}{\chi}\sigma &= 3K(\dot{\varepsilon} - \dot{\phi}), \end{aligned} \tag{2.3}$$

where s_{ij} is the stress deviator and $\sigma\delta_{ij}$ – the spherical part of the total stress tensor σ_{ij}, e_{ij} is the strain deviator and $\varepsilon\delta_{ij} = \frac{1}{3}\varepsilon_{kk}\delta_{ij}$ – the spherical part of the total strain ε_{ij}, $3K = 2M + 3A$ and $3\chi = 2\eta + 3\xi$ are the material constants with M and A being the shear and bulk elastic moduli, and η and ξ being the shear and bulk viscous moduli.

The physical relations (2.3) are decomposed into a pure shear term and a pure bulk compression term. It is known that the stress deviator (s_{ij}) is responsible for the shape change of the material, while the isotropic stress (σ) is responsible for the change of volume. The function

$$\phi = \alpha_\vartheta \vartheta + \alpha_\Theta \Theta \tag{2.4}$$

expresses the volumetric deformation caused by the temperature and moisture contents, with $\vartheta = T - T_r$ and $\Theta = X - X_r$ being the relative temperature and the relative moisture content, respectively, α_ν is the coefficient of thermal expansion, and α_Θ – the coefficient of shrinkage (or swelling).

The following relation can express the general form of the Maxwellian model:

$$\dot{\sigma}_{ij} + \frac{M}{\eta}\sigma_{ij} = 2M\dot{\varepsilon}_{ij} + \left(\frac{A}{K}\dot{\sigma} + \frac{\xi}{\chi}\frac{M}{\eta}\sigma\right)\delta_{ij} - 2M\dot{\phi}\delta_{ij}. \tag{2.5}$$

This relation is simplified in further considerations through an assumption that the ratio of volumetric moduli for elastic and viscoelastic materials is equal to the ratio of shear moduli of the respective materials, that is $\frac{K}{\chi} = \frac{M}{\eta}$. Using this simplification, one obtains volumetric changes of the drying body. Thus, the physical relation (2.5) is reduced to

$$\dot{\sigma}_{ij} + \frac{M}{\eta}\sigma_{ij} = 2M\dot{\varepsilon}_{ij} + 3A\dot{\varepsilon}\delta_{ij} - 3K\dot{\phi}\delta_{ij}. \tag{2.6}$$

Let us now consider the porous sphere subjected to a radially symmetric temperature and moisture content fields. The conditions of spherical symmetry are fulfilled only if the shear stresses (σ_{ij} $(i \neq j)$) and tangential displacements (u_θ, u_φ) equal zero,

$$\sigma_{r\theta} = \sigma_{\theta\varphi} = \sigma_{\varphi r} = 0 \quad \text{and} \quad u_\theta = u_\varphi = 0. \tag{2.7}$$

The radial displacement u_r is a function of radius r and time t

$$u_r = u_r(r, t). \tag{2.8}$$

The variation of the moisture content in the dried body is described by the mass balance equation (KOWALSKI [6]):

$$\rho_0 \dot{\Theta} = -\eta_{k,k} \tag{2.9}$$

and by the moisture mass transport equation, which relates the moisture flux η_k to the gradient of moisture potential:

$$\eta_k = -\Lambda_m \mu_k, \tag{2.10}$$

where $\Lambda_m \geq 0$ is the moisture transport coefficient. The moisture potential μ is a function of the temperature ϑ, the body volume deformation ε and the moisture contents Θ (KOWALSKI [6]):

$$\mu = \mu(\vartheta, \varepsilon, \Theta) = \left[c_\vartheta \vartheta - \gamma_\Theta \varepsilon + c_\Theta(\Theta - \Theta_r)\right]/\rho_0, \tag{2.11}$$

where $c_\vartheta = \rho_0(\partial\mu/\partial\vartheta)_{\varepsilon,0}$ is termed the thermal coefficient of the moisture potential, $\gamma_\Theta = -\rho_0(\partial\mu/\partial\vartheta)_{\vartheta,\Theta} = \alpha_\Theta(2M + 3A)$ can be termed as the volumetric stiffness and $c_\Theta = \rho_0(\partial\mu/\partial\Theta)_{\vartheta,\varepsilon}$ – an averaged "Leverett function" connected with capillary rise of a wetting fluid in porous medium (SCHEIDEGGER [20]) or (KIRKHAM and POWERS [6]).

As the temperature ϑ of the dried body does not alter during the constant drying rate period, the gradient of moisture potential is:

$$\rho_0 \mu_{,k} = -\gamma_\Theta \varepsilon_{,k} + c_\Theta \Theta_{,k}. \tag{2.12}$$

The mass transport equation is of the form

$$\dot{\Theta} = \frac{\Lambda_m}{\rho_0^2} \nabla^2 \left[-\gamma_\Theta \varepsilon + c_\Theta(\Theta - \Theta_r)\right], \tag{2.13}$$

where $\nabla^2 \equiv \dfrac{\partial^2}{\partial r} + \dfrac{2}{r}\dfrac{\partial}{\partial r} = \dfrac{1}{r^2}\dfrac{\partial}{\partial r}\left(r^2 \dfrac{\partial}{\partial r}\right)$ is the Laplace operator in spherical coordinates, and $\varepsilon = \dfrac{1}{3}\left(\varepsilon_{rr} + \varepsilon_{\varphi\varphi} + \varepsilon_{\theta\theta}\right)$ – spherical part of the strain tensor.

The boundary conditions for the moisture mass transfer stipulate the symmetry of the moisture potential at the centre point of the sphere and convective exchange of moisture on the external surface of the moist body during drying, that is:

$$
\begin{aligned}
-\Lambda_m \left.\frac{\partial \mu}{\partial r}\right|_{r=0} &= 0, \\
-\Lambda_m \left.\frac{\partial \mu}{\partial r}\right|_{r=R} &= \alpha_m \left(\mu|_{r=R} - \mu_a\right),
\end{aligned}
\tag{2.14}
$$

where α_m is the convective mass transfer coefficient and μ_a – the potential of the vapour in the air (drying medium).

3. Solution for wet elastic sphere

First, the solution for wet sphere with elastic skeleton will be presented. The physical relations for elastic sphere are

$$
\begin{aligned}
\sigma_{rr} &= 2M\varepsilon_{rr} + 3(A\varepsilon - K\phi)\delta_{ij} \\
\sigma_{\varphi\varphi} &= \sigma_{\theta\theta} = 2M\varepsilon_{\varphi\varphi} + 3(A\varepsilon - K\phi)\delta_{ij}
\end{aligned}
\tag{3.1}
$$

The shear stresses (the difference between radial and circumferential stresses) are as follows:

$$
\sigma_{rr} - \sigma_{\varphi\varphi} = 2M(\varepsilon_{rr} - \varepsilon_{\varphi\varphi}) = 2M\left(\frac{\partial \mu_r}{\partial r} - \frac{u_r}{r}\right) = 2Mr\frac{\partial}{\partial r}\left(\frac{u_r}{r}\right). \tag{3.2}
$$

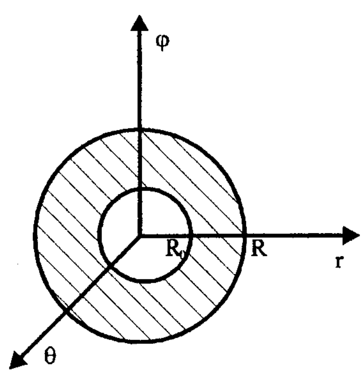

FIG. 2. Cross-section of the analysed sphere.

Substituting Eq. (3.2) into equilibrium Eq. (2.1), one obtains

$$\frac{\partial \sigma_{rr}}{\partial r} + 4M \frac{\partial}{\partial r}\left(\frac{u_r}{r}\right) = 0. \tag{3.3}$$

Integrating the above equation, one obtains the relation between the radial stress and displacement

$$\sigma_{rr} = c_1 - 4M\frac{u_r}{r}. \tag{3.4}$$

To find the radial displacement one should use Eqs. (3.1) and (3.4) and (2.2),

$$(2M + A)\left(\frac{\partial u_r}{\partial r} + \frac{2u_r}{r}\right) = 3K\phi + c_1. \tag{3.5}$$

After integrating, the radial displacements are

$$u_r = c_1 r + \frac{c_2}{r^2} + \frac{3K}{(2M + A)r^2}\int_{R_0}^{r} \phi r^2 dr. \tag{3.6}$$

To find the constants of integration c_1 and c_2, the following boundary conditions are used

$$\begin{aligned} \sigma_{rr}|_{r=R} &= 0, \\ u_r|_{r=R_0} &= 0. \end{aligned} \tag{3.7}$$

After some transformations one finds

$$\begin{aligned} c_1 &= \frac{4M3K}{(2M + A)(3KR^3 + 4MR_0^3)}\int_{R_0}^{R} \phi r^2 dr, \\ c_2 &= -\frac{4M3KR_0^3}{(2M + A)(3KR^3 + 4MR_0^3)}\int_{R_0}^{R} \phi r^2 dr. \end{aligned} \tag{3.8}$$

The final form of the radial displacement is

$$u_r = \frac{3K}{2M + A}\left(\frac{4Mr}{3KR^3 + 4MR_0^3}\int_{R_0}^{R} \phi r^2 dr - \frac{4MR_0^3}{(3KR^3 + 4MR_0^3)r^2}\int_{R_0}^{R} \phi r^2 dr + \frac{1}{r^2}\int_{R_0}^{r} \phi r^2 dr\right). \tag{3.9}$$

Knowing the displacements, one can find the radial and circumferential stresses

$$(3.10)\quad \begin{aligned} \sigma_{rr} &= \frac{4M3K}{2M+A}\left(\frac{4MR_0^3}{(3KR^3+4MR_0^3)r^3}\int_{R_0}^{R}\phi r^2 dr \right. \\ &\qquad \left. + \frac{3K}{3KR^3+4MR_0^3}\int_{R_0}^{R}\phi r^2 dr - \frac{1}{r^3}\int_{R_0}^{r}\phi r^2 dr\right), \\ \sigma_{\theta\theta} = \sigma_{\varphi\varphi} &= \frac{2M3K}{2M+A}\left(\frac{1}{r^3}\int_{R_0}^{r}\phi r^2 dr + \frac{6K}{3KR^3+4MR_0^3}\int_{R_0}^{R}\phi r^2 dr \right. \\ &\qquad \left. + \frac{4MR_0^3}{(3KR^3+4MR_0^3)r^3}\int_{R_0}^{R}\phi r^2 dr - \phi\right), \end{aligned}$$

The above solution is valid for a hollow sphere with internal radius R_0. Going with this radius to zero, one can obtain the solution for a full sphere, that is:

$$(3.11)\qquad u_r = \frac{3K}{2M+A}\left(\frac{4Mr}{3KR^3}\int_0^R \phi r^2 dr + \frac{1}{r^2}\int_0^r \phi r^2 dr\right),$$

$$(3.12)\qquad \sigma_{rr} = \frac{4M3K}{2M+A}\left(\frac{1}{R^3}\int_0^R \phi r^2 dr - \frac{1}{r^3}\int_0^r \phi r^2 dr\right),$$

$$(3.13)\qquad \sigma_{\theta\theta} = \sigma_{\varphi\varphi} = \frac{2M3K}{2M+A}\left(\frac{1}{r^3}\int_0^r \phi r^2 dr + \frac{2}{R^3}\int_0^R \phi r^2 dr - \phi\right).$$

For the centre of the sphere ($r = 0$), the radial displacement is zero, $u(0,t) = 0$. With the help of the De Hospital rule, one determines the stresses at the centre of the sphere

$$(3.14)\qquad \sigma_{rr} = \sigma_{\theta\theta} = \sigma_{\varphi\varphi} = \frac{4M3K}{2M+A}\left(\frac{1}{R^3}\int_0^R \phi r^2 dr - \frac{\phi}{3}\right).$$

4. Stresses and displacements in viscoelastic sphere

Let us consider a viscoelastic sphere that satisfies Maxwell's physical relation (2.6). This relation in terms of Laplace transforms takes the form

$$\overline{\sigma}^V_{ij} = \frac{s}{s+\dfrac{M}{\eta}}\left[2M\overline{\varepsilon}_{ij} + 3(A\overline{\varepsilon} - K\overline{\phi})\delta_{ij}\right]. \tag{4.1}$$

The initial values for the stresses and strains were assumed to be zero. Note that the physical relation for viscoelastic body expressed in Laplace transforms, is proportional to that of the elastic one, that is:

$$\overline{\sigma}^V_{ij} = \frac{s}{s+\dfrac{M}{\eta}}\overline{\sigma}^E_{ij} = 2M^V\overline{\varepsilon}_{ij} + 3(A^V\overline{\varepsilon} - K^V\overline{\phi})\delta_{ij}, \tag{4.2}$$

where

$$\overline{\sigma}^E_{ij} = 2M\overline{\varepsilon}_{ij} + 3(A\overline{\varepsilon} - K\overline{\phi})\delta_{ij} \tag{4.3}$$

and

$$M^V = M\frac{s}{s+\dfrac{M}{\eta}}, \qquad A^V = A\frac{s}{s+\dfrac{M}{\eta}}, \qquad K^V = K\frac{s}{s+\dfrac{M}{\eta}}, \tag{4.4}$$

are the viscoelastic material constants represented in Laplace transforms by their elastic counterparts, (see Alfrey (1944), Lee (1955)). The equation of internal equilibrium of forces expressed in Laplace transforms is

$$\overline{\sigma}^V_{ij,j} = 0. \tag{4.5}$$

Substituting physical relation (4.1) into the above equilibrium equation, one obtains

$$2M^V\overline{\varepsilon}_{ij,j} + 3(A^V\overline{\varepsilon}_{,i} - K^V\overline{\phi}_{,i}) = 0. \tag{4.6}$$

Similarly, the boundary condition for viscoelastic body expressed in Laplace transforms

$$\overline{\sigma}^V_{ij}n_j|_{r=R} = 0, \tag{4.7}$$

or

$$\left[2M^V\overline{\varepsilon}_{ij} + 3(A^V\overline{\varepsilon} - K^V\overline{\phi})\delta_{ij}\right]n_j|_{r=R} = 0. \tag{4.8}$$

The above statements justify the construction of solution for the viscoelastic sphere with the use of the solution for the elastic sphere. The solutions are similar to those given by (3.11), (3.12) and (3.13),

$$(4.9) \qquad \overline{u}_r^V = \frac{3K^V}{2M^V + A^V}\left(\frac{4M^V r}{3K^V R^3}\int\limits_0^R \overline{\phi} r^2 dr + \frac{1}{r^2}\int\limits_0^r \overline{\phi} r^2 dr\right),$$

$$(4.10) \qquad \overline{\sigma}_{rr}^V = \frac{4M^V 3K^V}{2M^V + A^V}\left(\frac{1}{R^3}\int\limits_0^R \overline{\phi} r^2 dr - \frac{1}{r^3}\int\limits_0^r \overline{\phi} r^2 dr\right),$$

$$(4.11) \qquad \overline{\sigma}_{\varphi\varphi}^V = \overline{\sigma}_{\theta\theta}^V = \frac{2M^V 3K^V}{2M^V + A^V}\left(\frac{1}{r^3}\int\limits_0^r \overline{\phi} r^2 dr + \frac{2}{R^3}\int\limits_0^R \overline{\phi} r^2 dr + -\overline{\phi}\right).$$

It is easy to notice that radial displacement of the viscoelastic model is the same as that of the elastic one, that is

$$(4.12) \qquad \overline{u}_r^V = \overline{u}_r^E \quad \text{and} \quad u_r^V = u_r^E.$$

The inverse transformation for stresses gives

$$(4.13) \qquad \sigma_{rr}^V(r,t) = \sigma_{rr}^E(r,t) - a\int\limits_0^t e^{-a(t-\tau)}\sigma_{rr}^E(r,\tau)d\tau,$$

$$(4.14) \qquad \sigma_{\varphi\varphi}^V(r,t) = \sigma_{\varphi\varphi}^E(r,t) - a\int\limits_0^t e^{-a(t-\tau)}\sigma_{\varphi\varphi}^E(r,\tau)d\tau,$$

$$(4.15) \qquad \sigma_{\varphi\varphi}^V(r,t) = \sigma_{\theta\theta}^V(r,t),$$

where $a = \dfrac{M}{\eta}$ is termed as the viscosity coefficient. Note that for $\eta \to \infty$ or $a \to 0$, one obtains the solution for the elastic sphere.

5. Calculation of the moisture potential

One can obtain the moisture distribution across the sphere using the mass balance Eq. (2.9) and the moisture mass transport Eq. (2.10). The physical relation for moisture potential (2.13) contains the body volume deformation ε, which now can be expressed as:

$$\varepsilon = \frac{1}{3}\varepsilon_{kk} = \frac{1}{3}\left(\frac{\partial u_r}{\partial r} + \frac{2u_r}{r}\right) = \frac{3K}{2M + A}\left(\frac{4M}{3KR^3}\int_0^R \phi r^2 dr + \frac{\phi}{3}\right). \tag{5.1}$$

Thus, the moisture potential is a function of the moisture contents and the temperature (here being constant):

$$\mu\rho_0 = c_\vartheta \vartheta - \gamma_\Theta \frac{3K}{2M + A}\left(\frac{4M}{3KR^3}\int_0^R \phi r^2 dr + \frac{\phi}{3}\right) + c_\Theta(\Theta - \Theta_r). \tag{5.2}$$

The above relation allows to express the boundary conditions (2.14) by the moisture contents function only.
Finally, using Eqs. (2.9) and (2.10), the equation of diffusion type for the moisture contents distribution is derived,

$$\dot{\Theta} = L\nabla^2\Theta, \tag{5.3}$$

with the coefficient

$$L = \frac{\Lambda_m}{\rho_0^2}\left(c_\Theta - \gamma_\Theta \alpha_\Theta\right), \tag{5.4}$$

containing material constants. The temperature did not appear in Eq. (5.3) since the constant drying period is considered here.

The drying process begins with constant moisture potential distribution in the whole sphere

$$\mu(r, 0) = \mu_0, \tag{5.5}$$

where μ_0 is calculated from relation (5.2) for the initial values of moisture contents and temperature.

The moisture contents is determined from Eq. (5.3), using the boundary conditions (2.14) and the initial condition (5.5). To solve Eq. (5.3) numerically, the partial derivatives are replaced by the central finite-difference approximations (the Crank-Nicolson method), that is:

$$\left.\frac{\partial^2\Theta}{\partial r^2}\right|_{\substack{r=r_i\\ t=t_j}} = \frac{\Theta_{i+1}^j - 2\Theta_i^j + \Theta_{i-1}^j}{\Delta r^2}, \qquad \left.\frac{\partial\Theta}{\partial r}\right|_{\substack{r=r_i\\ t=t_j}} = \frac{\Theta_{i+1}^j - \Theta_{i-1}^j}{2\Delta r}, \tag{5.6}$$

and for the time derivative:

$$\left.\frac{\partial^2\Theta}{\partial t}\right|_{\substack{r=r_i\\ t=t_j}} = \frac{\Theta_i^{j+1} - \Theta_i^j}{\Delta t}, \tag{5.7}$$

where subscripts are used to denote the position and superscripts – the time. For a one-dimensional problem in spherical coordinates, the Crank-Nicolson formula has the form:

$$(5.8)\qquad -D\left(1-\frac{\Delta r}{r}\right)\Theta_{i-1}^{j+1}+(2+2D)\Theta_i^{j+1}-D\left(1+\frac{\Delta r}{r}\right)\Theta_{i+1}^{j+1} = D\left(1-\frac{\Delta r}{r}\right)\Theta_{i-1}^{j}+(2-2D)\Theta_i^{j}+D\left(1+\frac{\Delta r}{r}\right)\Theta_{i+1}^{j},$$

$$(5.9)\qquad D=\frac{\Delta t}{L\Delta r^2}.$$

The stability and convergence for the Crank-Nicolson method is true for any positive D, although small values are more accurate. The moisture contents distribution is calculated from Eq. (5.8). Note that the new moisture contents Θ^{j+1} is not given directly in terms of the known moisture contents one time step earlier, but is a function of the unknown moisture contents at adjacent positions as well. It is therefore termed an implicit method. This method requires a simultaneous solution of Eq. (5.8) at each time step. The first and the last of these equations are connected with the boundary conditions (2.14).

6. Experimental technique

A drying experiment was carried out to validate the results obtained in the theoretical analysis. Figure 3 shows the configuration of the experimental equipment. The length of drying tunnel was 1000 mm. The inside size of the cross-section was 150 × 150 mm. The test section was located 500 mm far the air inlet. To make the air flow in the whole cross-section uniform, the system of wire meshes was installed. The drying medium (hot air) was supplied into the duct through an electric heater from a blower. The hot air velocity was constant and equal to 1 m/s. The sample was made of clay in the form of a sphere of 30 mm in diameter. Three kinds of clay (used in producing flowerpots) were tested. Five samples for each kind of clay were prepared and dried convectively. The sample was placed on a small mesh table, connected with the electronic reading balance (Radwag WPS-720). To secure antisymmetrical drying, the mesh table was rotated after each weight measure through 120° around its axis, and after each diameter measure, the sphere was rotated through 120° in the plane perpendicular to this table. Dry and wet thermometers were installed to control the air and sample temperatures. To measure the changes of the sphere size, the sample was taken out of the test section every three minutes.

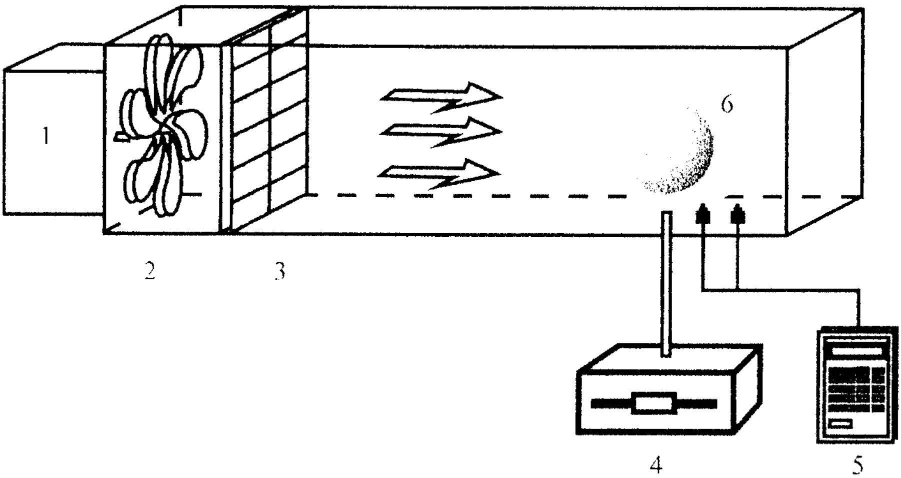

FIG. 3. Experimental equipment. *1* – electric heater, *2* – blower, *3* – heater and wire mesh, *4* – electronic balance, *5* – thermometer, *6* – clay sample.

7. Numerical examples

The computer calculations were carried out for the clay (a ceramic-like material) characterised by the following data:

$$\begin{aligned} A &= 10^3 \text{ [MPa]} & M &= 625 \text{ [MPa]} \\ \alpha_\Theta &= 2.4\times10^{-4} \text{ [1]} & \alpha_\vartheta &= 3\times10^{-8} \text{ [deg}^{-1}\text{]} \\ c_\vartheta &= 2.4 \text{ [J/m}^3\text{ deg]} & c_\Theta &= 6.6\times10^{3} \text{ [kJ/m}^3\text{]} \\ \Lambda_m &= 3\times10^{-5} \text{ [kg s/m}^3\text{]} & \rho_0 &= 1.2\times10^{3} \text{ [kg/m}^3\text{]} \\ a &= M/\eta \text{ [1/s]} & a &= 10^{-3}\div10^{-6} \text{ [1/s]} \end{aligned}$$

Figure 4 presents the distribution of the moisture contents versus the sphere radius at several instants of the drying time.
It is seen that the region close to the surface ($r = 0.03$ m) is being dried relatively quickly but at the centre of the sphere it reaches the reference moisture contents Θ_0 after 2.5 hours of the drying time. There is no difference between the curves $\Theta(r,t)$ for the elastic sphere and the viscoelastic one for the data given above.

Figure 5 presents the radial stresses in the drying sphere as a function of time for various radii. The stresses are the highest at the centre and zero on the surface. The shape of the curve is typical for Maxwell's model. The stresses reach their maxima after three minutes of heating and then relax slowly, approaching stress-free conditions after a long period of time.

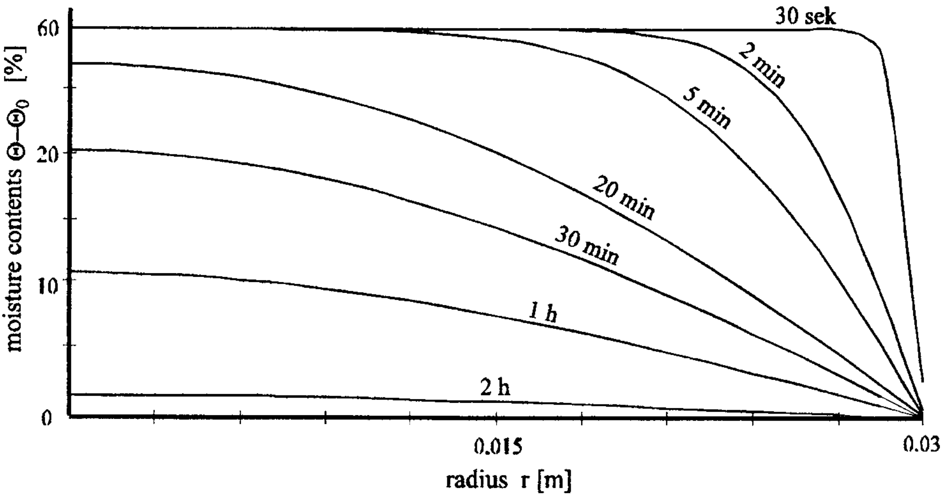

FIG. 4. Moisture contents distribution in the sphere.

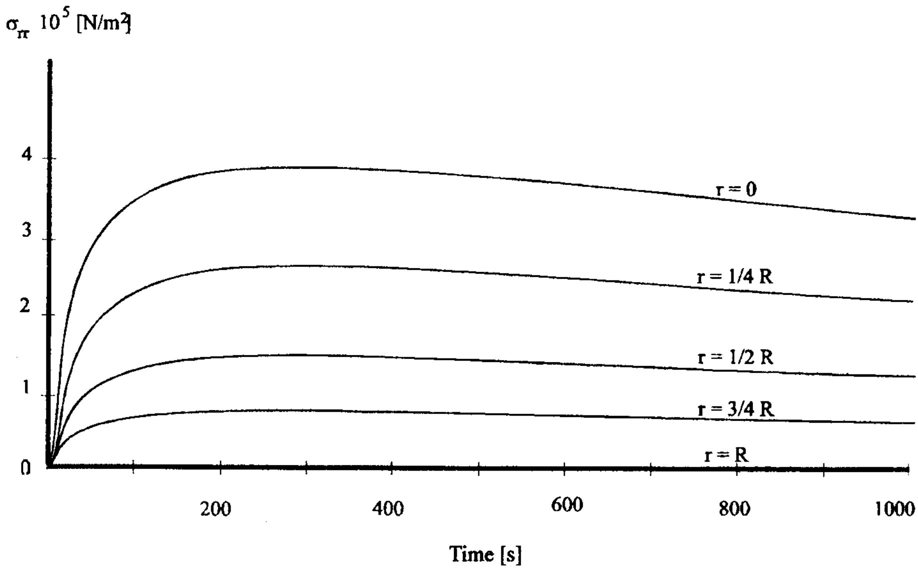

FIG. 5. Radial stresses as a function of time for different radii.

Figure 6 illustrates the changes of the relative volume of the sphere. The experimental curves represent three kinds of clay. It is seen that the behaviour of individual clays is different. One of them (clay number I) is almost incompressible. The third one changes its dimensions slowly but in a continuos way. The failure of the clay number I occurred after a short period of drying. On the contrary, the clay number III has not cracked on the surface during tests.

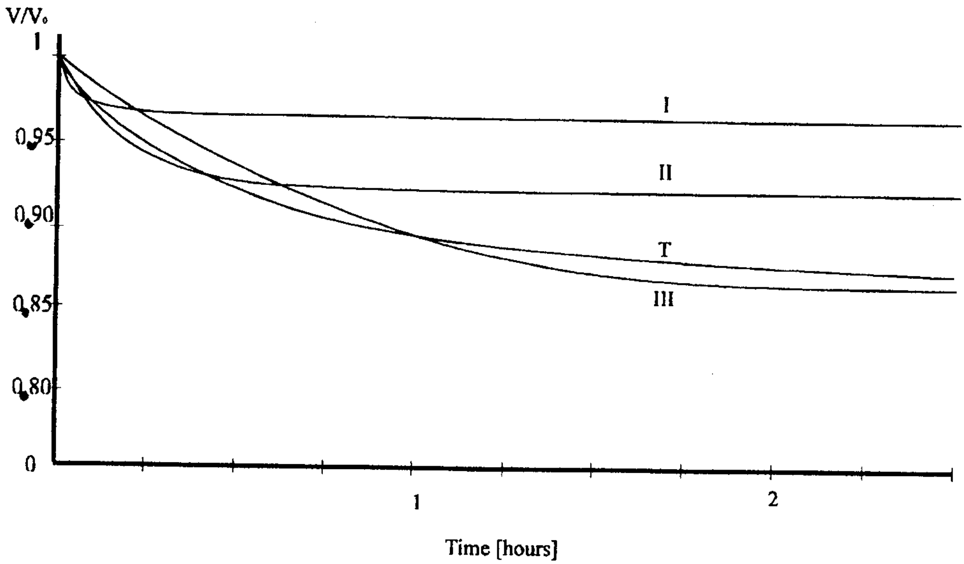

FIG. 6. Changes of the sphere volume: I, II, III – experimental data, T – theoretical curve.

The theoretical curve represents approximately the second kind of the tested clay. It should be noticed that the theoretical model was developed for the first period of drying.

In Figure 7 it is visible that the duration of the constant drying rate period (first period of drying) is about 46 minutes for the sample made of clay II. This experimental curve in Figure 7 allows to divide the drying process into three periods. First period consist of two phases: preheating (about 12 minutes for clay II) and constant drying rate period (about 46 minutes). The second period (overheating) begins after about 46 minutes of the experiment. There are no exact limits between these two periods of drying.

Figure 8 shows the difference between viscoelastic stresses and the elastic ones.

This is seen that viscosity influences significantly the magnitude of stress in dried materials. For great viscosity, represented by parameter a, the stress distribution is more uniform and reaches smaller values.

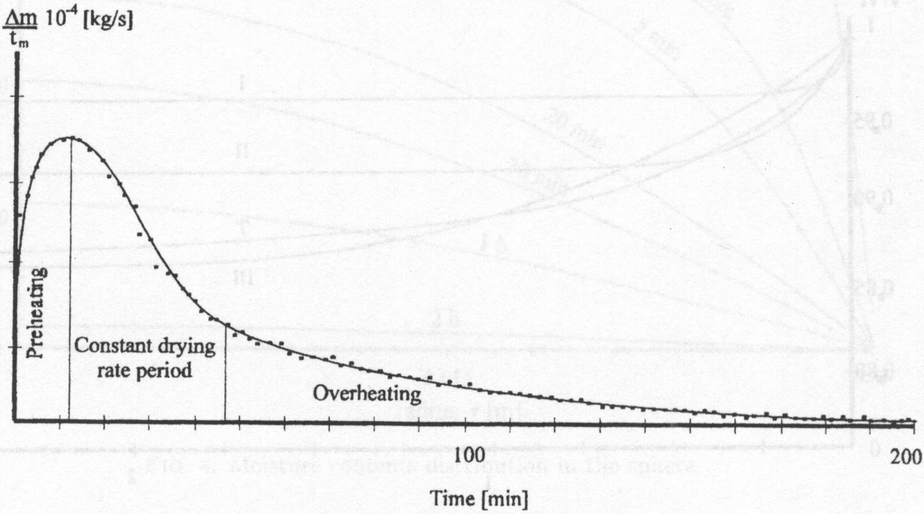

FIG. 7. Experimentally measured mass rate curve for clay number II.

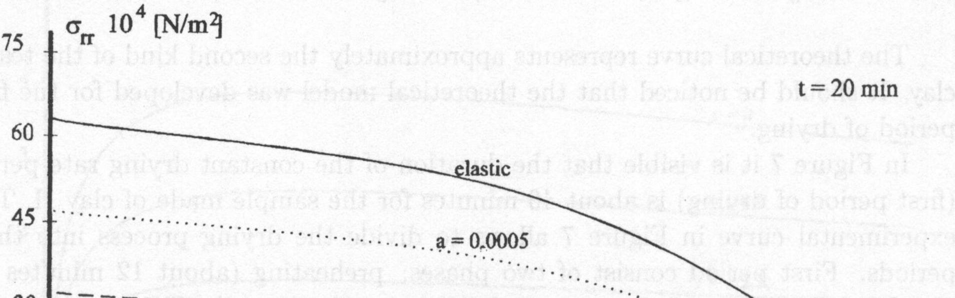

FIG. 8. Comparison of stress distributions in the elastic and viscoelastic sphere.

8. Conclusions

1. The rheological properties of dried material influence considerably the drying-induced stresses and insignificantly – the moisture distribution and the moisture potential.

2. The stresses reach smaller magnitude (depending on viscosity) in viscoelastic material than in the elastic one.

3. The stresses are the highest at the centre and zero on the surface.

4. The stresses reach their maxima in the first period of drying process and then relax slowly, approaching stress-free state after a long period of time.

5. The solution of the viscoelastic problem agrees well with the elastic one if the dependence on time is cancelled.

Acknowledgment

This work was carried out as a part of the research project No 3T09C 005 12 sponsored by the Polish State Committee for Scientific Research in the year 1999.

References

1. T. Alfrey, *Mechanical behaviour of high polymers*, Interscience, New York – London 1948.
2. B. A. Boley and J. H. Weiner, *Theory of thermal stresses*, John Wiley and Sons, New York, London 1960.
3. J. D. Ferry, *Viscoelastic properties of polymers*, John Wiley and Sons, Inc., NY 1970.
4. N. J. Hoff, *Stress distribution in the presence of creep, high temperature effects in aircraft structures*, Pergamon Press, 248–266, 1958.
5. Y. Itaya, S. Mabuchi and M. Hasatani, *Deformation behavior of ceramic slabs by nonuniform drying*, Drying Technology, **13**(3), 801–819, 1995.
6. D. Kirkham, W. L. Powers, *Advanced soil physics*, John Wiley and Sons, Canada 1972.
7. S. J. Kowalski, *Thermomechanics of constant drying rate period*, Arch. Appl. Mech., **39**, 3, 157–176, 1987.
8. S. J. Kowalski, *Drying processes involving permanent deformations of dried materials*, Int. J. Engng. Sci., **34**, 13, 1491–1506, 1966.
9. S. J. Kowalski, *Mathematical modelling of shrinkage by drying*, Drying Technology, **14**, 2, 307–331, 1966.
10. S. J. Kowalski and Cz. Strumiłło, *Moisture transport in dried materials*, Boundary Conditions, Chem. Engng. Sci., **52**, 7, 1141–1150, 1997.

11. S. J. KOWALSKI, G. MUSIELAK, *Mathematical modelling of the drying process of capillary porous media; An example of convectively dried plate*, Engng. Trans., **36**, 2, 239–252, 1988.

12. S. J. KOWALSKI, G. MUSIELAK and A. RYBICKI, *Shrinkage stresses in dried materials*, Engng. Trans., **40**, 1, 115–131, 1988.

13. S. J. KOWALSKI, G. MUSIELAK and A. RYBICKI, *Drying processes – thermomechanical approach* [in Polish], IFTR - PSP, Poznań - Warszawa, pp. 225, 1996.

14. R. W. LEWIS, M. STRADA and G. COMINI, *Drying-induced stresses in porous bodies*, Int. J. Num. Meth. Engng., **11**, 1175–1184, 1996.

15. P. MOON and D. E. SPENCER, *Field theory for engineers* [in Polish], PWN, Warszawa 1996.

16. L. W. MORLAND and E. H. LEE, *Stress analysis for linear viscoelastic materials with temperature variation*, Trans. of the Society of Rheology, IV, 233, 1960.

17. R. MUKI and E. STERNBERG, *On transient thermal stresses in viscoelastic materials with temperature dependent properties*, J. Appl. Mech, **6**, 193, 1961.

18. W. NOWACKI, *Theory of elasticity*, PWN, Warszawa 1970.

19. V. N. M. RAO, D. D. HAMMAN and J. R. HAMMERLE, *Stress analysis of a viscoelastic sphere subjected to temperature and moisture gradients*, J. Agric. Engng Res., **20**, 283–293, 1975.

20. A. E. SCHEIDEGGER, *The physics of flow through porous media*, University of Toronto Press, Toronto 1957.

21. S. P. TIMOSHENKO and J. N. GOODIER, *Theory of elasticity*, McGraw-Hill Book Co., NY. 1970.

Received May 27, 1999; revised version November 5, 1999.