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A MODIFIED FLOW FIELD IN THE EXTRUSION OF BIMETALLIC SYSTEMS

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The aim of the paper is to propose a new kinematically admissible velocity field corresponding to experimental results of the co-extrusion of various materials. A modified mathematical description of the plastic flow in the extrusion of the bi-metallic composite including boundaries of the plastic zone described by appropriate functions has been proposed. Velocities, grid distortion and strain rate distribution have been calculated by taking into account information on the plastic zone boundaries and their forms. Excellent agreement between the analytical and the experimental results of the plastic flow of longitudially oriented metal composite has been established.

1. Introduction

In the paper [11] a kinematically admissible velocity field of the bimaterial extrusion has been proposed and discussed to analyze such a type of the extrusion process. In that case, the boundaries of the plastic zone are approximated by ellipsoidal, hyperboloidal and sinusoidal surfaces.

In the paper [10] the previous model has been generalized for an arbitrary form of plastic zone boundaries. For this aim a special parametrization of the boundaries has been applied. Internal consistency of the proposed kinematically admissible velocity field has been estimated and the analytical and new experimental results concerning velocities, strain rate and stress fields obtained in [6, 7, 8] have been compared. In both the papers mentioned, the flow lines are described by straight lines in a plastic domain. Unfortunately, such approach leads to the existence of velocity discontinuity boundaries on the entry and exit of the plastic zone.

However, the results of other papers [e.g. [4, 9]] show that plastic zone may be limited by sufficient arbitrary shape of the surfaces in such a way that flow lines are not straight lines, in general. Moreover, velocity discontinuity boundary is observed only between the plastic zone and the arising dead metal zone.

One way to eliminate this mentioned inconsistency is to use a special procedure for smoothing flow lines. Such procedure cannot be an arbitrary one, because the incompressible condition should be satisfied for the seeking of the corrected flow field, too. There are different techniques to smooth flow fields. As an example, note a sufficiently simple smoothing procedure proposed in the paper [5]. Let us also note the other approaches to the problem of bimetal extrusion applied in papers [1, 2, 3].

In this paper a modified flow field which is smooth immediately from modelling is presented. On the other hand, the process of construction of the flow field is as simple as the solution with the straight lines assumption [10].

2. Conception of the flow field

Different geometry of the deformation boundaries between various materials deformed together (the core and the sleeve) and between the entry and exit surfaces of the each material is considered basing on the results of the papers [6, 7, 9].

The following additional assumptions mentioned below are applied. Most of them are similar to the assumptions presented in the paper [10], but point Sec. 5 below is completely different.

1. There is no friction between the materials as well as along the material-tool interface at the entry of the plastic zone. Hence, relative initial velocity (V/V_0) is the same for every component of the composite.

2. Both materials of the core and the sleeve demonstrate plastic zones ($ABDC$ and $ABFE$, respectively) depended on the geometry and mechanical properties of the core and the sleeve (see Fig. 1). In fact, the plastic regions depend essentially on friction along the metal-metal and metal-tool interfaces.

3. Co-extrusion of different materials lead to the existence of a dead metal zone formation. Its form depends on the geometry of extrusion tools and the properties of materials extruded.

4. The materials are assumed to be incompressible and the stress-strain state is prescribed by the rigid-plastic model.

5. The flow lines are smooth. They are described by appropriate trigonometric functions for both the sleeve and the core materials. In the previous paper [10] it was assumed that flow lines are straight lines in the plastic zone.

6. The degree of deformation is different for the core and the sleeve:

$$\lambda_c^* = R_c^2/r_c^2, \quad \lambda_s^* = (R_o^2 - R_c^2)/(r_o^2 - r_c^2), \quad \lambda_c^* \neq \lambda_s^* \neq \lambda_{\text{global}}. \tag{2.1}$$

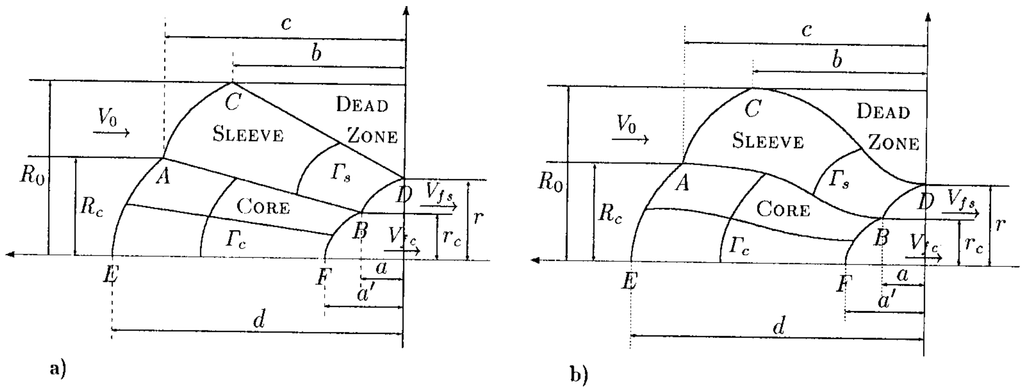


FIG. 1. General concept of the flow field construction in extrusion of bimaterial system: flow lines are straight a) or smooth b).

As a result, the exit velocities of the components are constant, but not the same.

These assumptions simplify a model of the plastic flow in co-extrusion. Of course, such a model is an approximation of the real plastic flow, and has its own disadvantadges:

- Mechanical parameters are introduced to the model in an indirect way only.
- Velocity at the exit of the plastic zone is not constant in each of the components of the composite.

However, these assumptions are better in comparison with those supposed in the previous papers [10, 11]. Namely, in this new modelling the velocity field is continuous along the entry and exit boundaries of the plastic zone.

3. A KINEMATICALLY ADMISSIBLE FIELD

In view of Assumption 5, the corresponding trajectory in the plastic zone is given by the curve:

$$z = z_1 - \frac{z_1 - z_2}{\pi}\arccos\left[\frac{2r - r_1 - r_2}{r_1 - r_2}\right], \quad r \in [r_2, r_1], \tag{3.1}$$

where (r, z) is the successive position of the point, and $(r_1, z_1) \in \Gamma_{AE}$ (Γ_{AC}) and $(r_2, z_2) \in \Gamma_{BF}$ (Γ_{BD}) are the coordinates of the point at the entry and exit in the core (sleeve).

It is assumed that the boundaries of the plastic zone are prescribed by the following curves determined by their parametric forms:

$$\Gamma_{AE} : \begin{cases} z_1 = f_1^c(t), \\ r_1 = R_c t, \end{cases}, \qquad \Gamma_{BF} : \begin{cases} z_2 = f_2^c(t), \\ r_2 = r_c t, \end{cases}, \quad t \in [0, 1], \tag{3.2}$$

in the core, and

$$
\text{(3.3)} \qquad \begin{array}{l} \Gamma_{AC}: \begin{cases} z_1 = f_1^s(t), \\ r_1 = \sqrt{R_o^2 t^2 + R_c^2(1-t^2)}, \end{cases} \\ \Gamma_{BD}: \begin{cases} z_2 = f_2^s(t), \\ r_2 = \sqrt{r_o^2 t^2 + r_c^2(1-t^2)}, \end{cases} \end{array} \qquad t \in [0,1],
$$

in the sleeve.

In such a way arbitrary forms of these curves can be chosen by determining the functions f_j^c, f_j^s $(j = 1, 2)$. However, these fuctions have to satisfy additional conditions dealing with the geometry of the plastic zone (see Fig. 1a, 1b)

$$
\text{(3.4)} \qquad \begin{array}{ll} f_1^c(0) = c, \; f_1^c(1) = s, & f_2^c(0) = a', \; f_2^c(1) = a, \\ f_1^s(0) = s, \; f_1^s(1) = b, & f_2^s(0) = a, \; f_2^s(1) = 0. \end{array}
$$

Besides, note that for each value of parameter $t \in (0,1)$ the points $(r_1(t), z_1(t))$ and $(r_2(t), z_2(t))$ correspond to entry and exit positions of the material point.

Now, auxilarly curves Γ_C, Γ_S are denoted in the core and the sleeve along which the corresponding extrusion ratio is constant (defferent in core and sleeve, in general):

$$
\text{(3.5)} \qquad r_1^2(t)/r_c^2(t) \equiv \lambda_c, \quad \lambda_c \in [1, \lambda_c^*], \quad [r_1^2(1) - r_1^2(t)]/[r_s^2(1) - r_s^2(t)] \equiv \lambda_s, \quad \lambda_s \in [1, \lambda_s^*],
$$

hence

$$
\text{(3.6)} \qquad r_c(t) = r_* t, \quad r_s(t) = \sqrt{r_D^2 t^2 + r_*^2(1-t^2)}, \quad t \in [0,1],
$$

where r_* is the first coordinate of the intersection point of the curves Γ_C, Γ_S and the interfacial line AB, but $r_D = r_D(r_*)$ is the first coordinate of the intersection point of the curve Γ_S and the boundary EF of "the dead metal zone". As it has been shown in the previous paper [10], the function $r_D = r_D(r_*)$ can be described by the form:

$$
\text{(3.7)} \qquad r_D(r_*) = \sqrt{\frac{R_o^2 - r_o^2}{R_c^2 - r_c^2}(r_*^2 - r_c^2) + r_o^2}.
$$

Finally, note that the second components of the points belong to the curves Γ_C, Γ_S, and are defined by the relation (3.2) with $r = r_c$ or $r = r_s$, respectively:

(3.8) $$\Gamma_c \; : \; z = z(r), \; r \in [0, r_*]$$

$$\Leftrightarrow \begin{cases} z = z_1(t) - \dfrac{z_1(t) - z_2(t)}{\pi} \arccos\left[\dfrac{2r(t) - r_1(t) - r_2(t)}{r_1(t) - r_2(t)}\right], \\ r = r_* t, \end{cases}$$

$$t \in [0, 1],$$

in the core, and

(3.9) $$\Gamma_s \; : \; z = z(r), \; r \in [r_*, r_D]$$

$$\Leftrightarrow \begin{cases} z = z_1(t) - \dfrac{z_1(t) - z_2(t)}{\pi} \arccos\left[\dfrac{2r(t) - r_1(t) - r_2(t)}{r_1(t) - r_2(t)}\right] \\ r(t) = \sqrt{r_D^2 t^2 + r_*^2(1 - t^2)}, \end{cases}$$

$$t \in [0, 1],$$

in the sleeve. Let us note that value $r_* \in [r_c, R_c]$ in the relations (3.9) and (3.10) is a parameter determining position of the curves. Namely, if $r_* = R_c$ then the entry boundaries of the plastic zone (curves Γ_{AE} and Γ_{AC}) are obtained, but in the case $r_* = r_c$ the exit boundaries of the plastic zone (curves Γ_{BF} and Γ_{BD}) are found. The flow lines are defined by relation (3.1) where functions $z_{1(2)}(t)$ and $r_{1(2)}(t)$ are written in (3.2) and (3.3). Here the value of t is a parameter determining which flow line is under consideration. It means that the plastic zone (core and sleeve separately) is parametrized by two parameters: $t \in [0,1], r_* \in [r_c, R_c]$. In order for this fact to be true, the functions f_j^c, f_j^s $(j = 1, 2)$ from (3.2) and (3.3) have to satisfy some additional conditions. When the hypothesis of the straight line is applied [10, 11] it is sufficient to assume that all these functions are monotonic. In the present case we have to check if the Jakobian corresponding to this transformation is not equal to zero in the core as well as in the sleeve:

(3.10) $$\begin{vmatrix} \dfrac{\partial}{\partial t} r_{c(s)} & \dfrac{\partial}{\partial t} z_{c(s)} \\ \dfrac{\partial}{\partial r_*} r_{c(s)} & \dfrac{\partial}{\partial r_*} z_{c(s)} \end{vmatrix} \neq 0, \qquad t \in (0, 1), \; r_* \in (r_c, R_c).$$

Angle ϕ between the OZ axis and the trajectory of each particle of the metal is calculated by taking into account Eq. (3.1), but ϕ_N is the acute angle between the normal to the curve Γ_C in the core (Γ_S – in the sleeve) and the OZ-axis. Functions $\phi = \phi(t, r_*)$ and $\phi_N = \phi_N(t, r_*)$ are determined from the relations:

$$
\begin{aligned}
\operatorname{ctg}\phi &= \frac{dz_{c(s)}}{dr}|_{\Gamma} = \frac{\partial z_{c(s)}}{\partial r_*}\cdot\frac{\partial r_*}{\partial r}, \\
&\qquad\qquad t\in[0,1],\quad r_*\in[r_c,R_c]. \\
\operatorname{tg}\phi_N &= -\frac{dz_{c(s)}}{dr}|_{\Gamma_{c(s)}} = -\frac{dz_{c(s)}}{dt}\cdot\frac{dt}{dr},
\end{aligned}
\tag{3.11}
$$

Here we use the fact that parameter t is constant along the curve Γ, but the next parameter r_* is constant along the curves $\Gamma_{c(s)}$. After simple calculations we obtain:

$$
\operatorname{tg}\phi = \frac{\pi\sqrt{(r_1-r)(r-r_2)}}{z_1-z_2}, \qquad t\in[0,1],\quad r_*\in[r_c,R_c]. \tag{3.12}
$$

$$
\begin{aligned}
\operatorname{tg}\phi_N = &-\frac{z_1-z_2}{\pi(r_1-r_2)\sqrt{(r_1-r)(r-r_2)}}\Bigg[2(r_1-r_2) \\
&+\frac{dt}{dr}\left[\frac{dr_2}{dt}(r-r_1)-\frac{dr_1}{dt}(r-r_2)\right]\Bigg] \\
&-\frac{dt}{dr}\left[\frac{dz_1}{dt}-\frac{1}{\pi}\left(\frac{dz_1}{dt}-\frac{dz_2}{dt}\right)\arccos\left[\frac{2r-r_1-r_2}{r_1-r_2}\right]\right], \\
&\qquad t\in[0,1],\quad r_*\in[r_c,R_c].
\end{aligned}
\tag{3.13}
$$

Let us note, that in the core the first term in (3.13) is equal to zero due to the following identities:

$$
\frac{dr_1}{dr_c} = \frac{r_1}{r_c}, \qquad \frac{dr_2}{dr_c} = \frac{r_2}{r_c}. \tag{3.14}
$$

Moreover, from these relations it follows that in the core argument of *arccos* is independent of the parameter t (it only depends on r_*).

Basing on the results of the paper [11], due to incompressibility of the materials we can obtain:

$$
V(t,r_*) = \lambda_{c(s)}(r_*)V_0\frac{\cos\phi_N(t,r_*)}{\cos[\phi(t,r_*)-\phi_N(t,r_*)]}, \qquad t\in[0,1],\quad r_*\in[r_c,R_c], \tag{3.15}
$$

where the values λ_c and λ_s are defined in (3.5) (see also (1)). The above equation is applicable to both proportional and non-proportional flows. The axial and radial components are

$$
V_r = -V\sin\phi,\quad V_z = -V\cos\phi, \tag{3.16}
$$

What is important to note is that condition (3.10) is equivalent to the following relation:

$$\phi(t, r_*) - \phi_N(t, r_*) \neq 0, \qquad t \in (0,1), \quad r_* \in (r_c, R_c). \tag{3.17}$$

The last fact shows us that there is no necessity to check the condition (3.10) separately, because it has to be done in the process of determination of the velocity field.

Now, the strain-rate tensor can be calculated

$$\dot{\varepsilon}_r = \frac{\partial v_r}{\partial r}, \quad \dot{\varepsilon}_z = \frac{\partial v_z}{\partial z}, \quad \dot{\varepsilon}_\theta = \frac{v_r}{r}, \quad \dot{\varepsilon}_{rz} = \frac{1}{2}\left(\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r}\right), \tag{3.18}$$

and finally, by the Saint-Venant – Levi – Mizes hypotheses (see for example [12]) the stress tensor $\sigma'_{ij} = \sigma_{ij} - \sigma\delta_{ij}$ with accuracy to an unknown hydrostatic pressure σ is of the form:

$$\sigma'_{ij} = \frac{\sqrt{2}k_{c(s)}}{\sqrt{\dot{\varepsilon}_{ij}\dot{\varepsilon}_{ij}}}\dot{\varepsilon}_{ij}. \tag{3.19}$$

Here k_c and k_s are maximal tangential stresses of the components of the composed material in the core and the sleeve.

4. Numerical results and discussion

Basing on the concept presented in the paper [10] the velocity field is described by using the parameters of the plastic zone [6, 8, 9]. The parameters R_0, R_c, r_0, r_c are calculated with sufficient accuracy (with relative error less than 1%), but the remaining ones (a, a', b, s, b) can be estimatelly found from the analysis of a kind of grid distortion and the changes of macrostructure [10].

Some remarks to numerical procedure are presented below. To find velocities in an arbitrary point (r_b, z_b) in the core (sleeve) we first localize this point in a small domain $[t_1, t_M] \times [r_1^*, r_M^*]$ and then the domain is parametrized as it is shown in Fig. 2. Further, velocities in each node (t_i, r_j^*) $(1 \leqslant i, j \leqslant M)$ which corresponds to the respective point (z_i, r_j) in the mentioned domain are calculated from the equations (3.15) – (3.16). During the calculations the core (the sleeve) is divided into 400 parts to localize the point under consideration and then the obtained domain is parametrized by 41 × 41 nodes $(M = 41)$.

Further, velocities and components of the strain rate tensor are calculated by the least square method. Namely, the velocities are approximated by linear relations:

$$\begin{aligned} v_r(r,z) &= A_r + B_r(r - r_b) + C_r(z - z_b), \\ v_z(r,z) &= A_z + B_z(r - r_b) + C_z(z - z_b), \end{aligned} \qquad \sqrt{(r - r_b)^2 + (z - z_b)^2} \leqslant \epsilon,$$

and then values of the parameters have to be found as the best approximation for the points (z_i, r_j) belonging to a small circle (with radius equal to ϵ) with the center in the point (z_b, r_b) shown in Fig. 2. Then the velocities and the strain rate components in the point (r_b, z_b) under consideration are defined by the equations:

$$v_r = A_r,\ v_z = A_z,\ \dot{\varepsilon}_{rr} = B_r,\ \dot{\varepsilon}_{zz} = C_z,\ \dot{\varepsilon}_{rz} = (B_z + C_r)/2.$$

What is important to note is that the strain-rate tensor is calculated, in fact, without any special numerical differentiation.

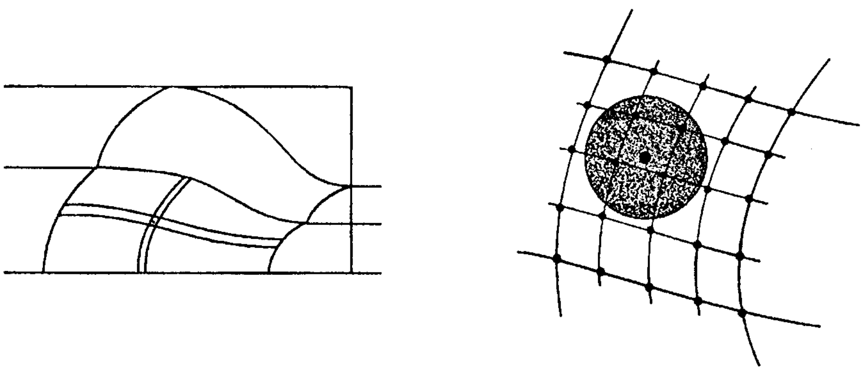


FIG. 2. Localization of an arbitrary point under consideration.

The comparison of the results obtained by the hypothesis of straight lines flow and the present approach is presented below.

In Fig. 3 plastic zone and the corresponding flow lines are presented for the extrusion of two different materials: the hard core (hard lead alloy – OT3) and

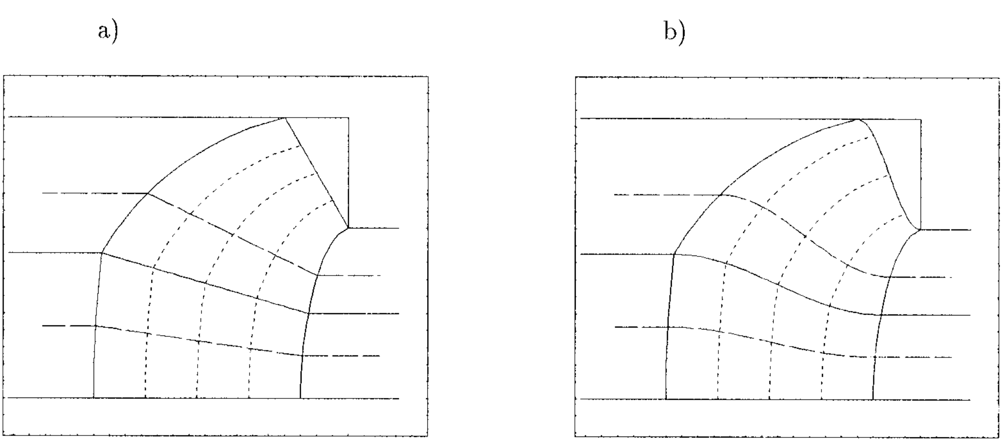


FIG. 3. Plastic zone and flow lines for both different models for Pb/OT3 composite (the hard core) $\lambda = 3$ and $R_0/R_c = 2$.

the soft sleeve (soft lead Pb). Exact dates for the experiment can be found in papers [6, 7, 8]. The definition of the boundaries of the plastic zone determined by the relations (3.2) – (3.4) was presented in the previous paper [10].

In the next figure grid distortions for both models for the similar composite are presented. As it follows from Fig. 4 grid distortions are of little difference.

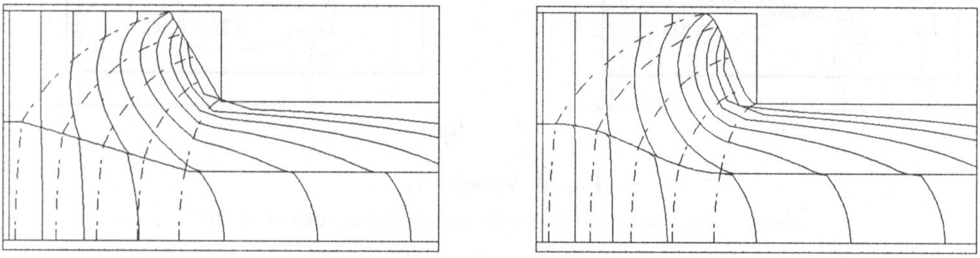


FIG. 4. Numerical grid distortion for both models.

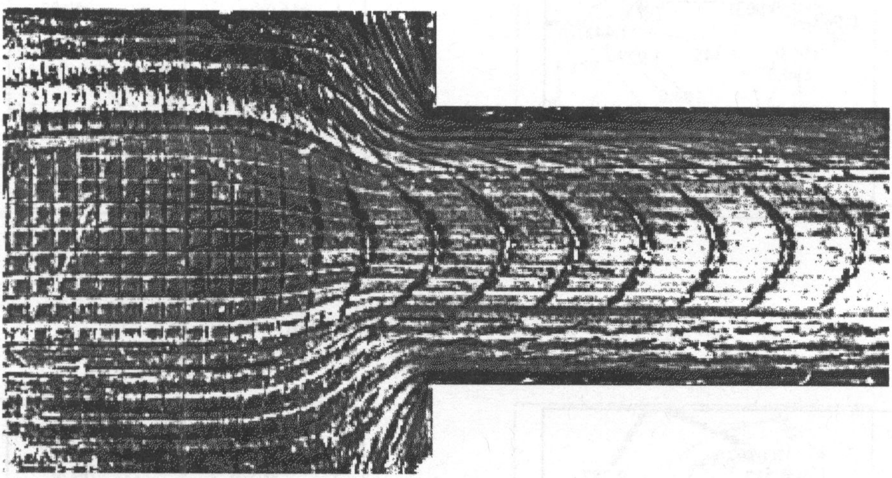
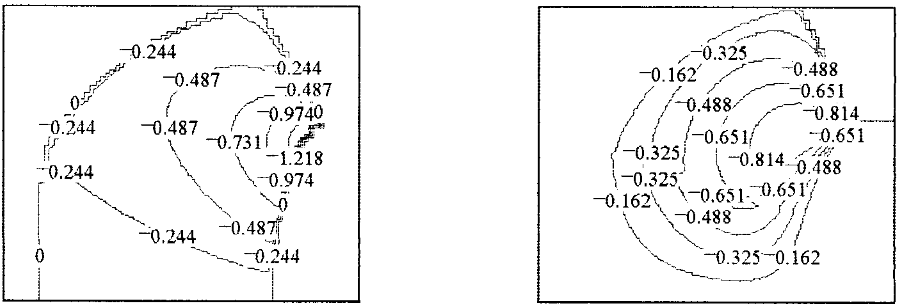


FIG. 5. Experimental grid distortion for Pb/OT3 composite (the hard core).

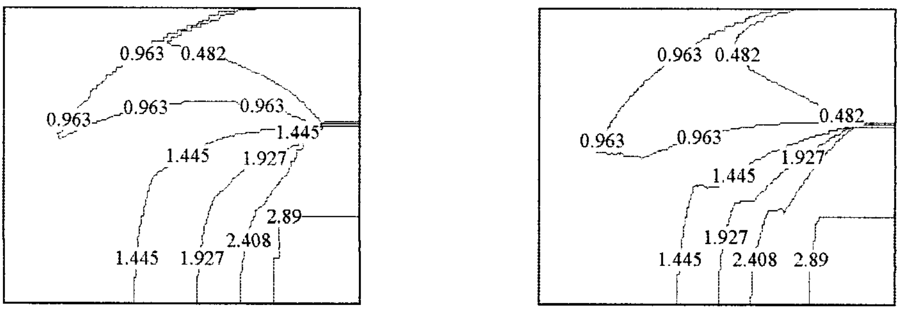
In Fig. 6, 7 distributions of the velocities V_r, V_z are shown. We do not present here results for the strain-rate tensor, because its components are drastically different for straight lines model and for the smooth flow lines model near the boundaries of the plastic zone, which is the natural consequence of the fact that in the case of the straight lines model these boundaries are discontinuity boundaries, in fact.

In Fig. 8 – 11 numerical results for normed components of the stress deviator are presented.



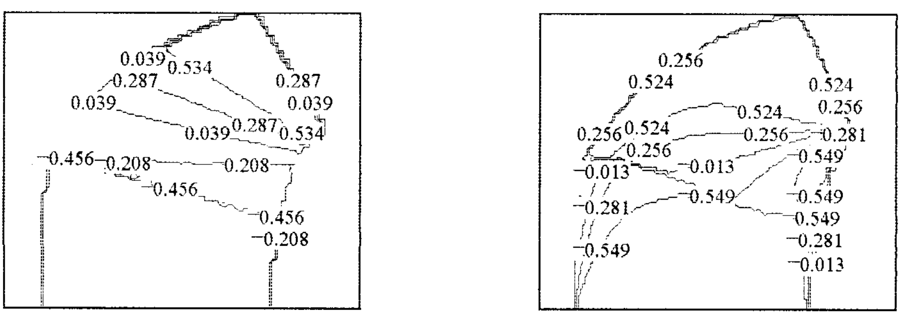
a) b)

FIG. 6. Velocity V_r.



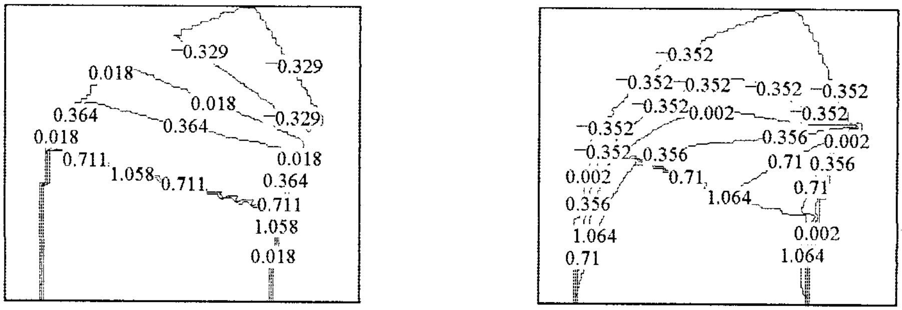
a) b)

FIG. 7. Velocity V_z.



a) b)

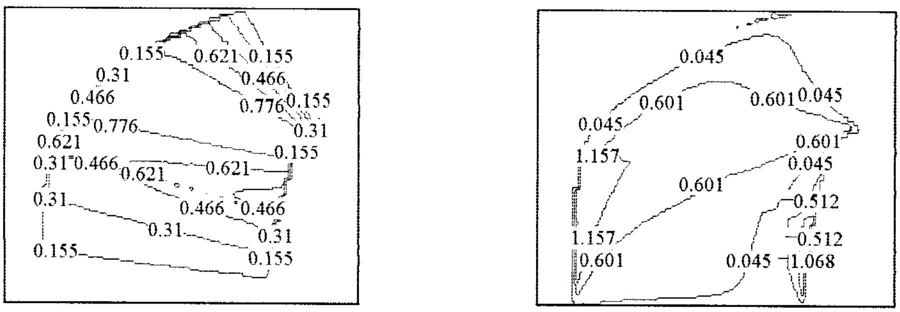
FIG. 8. Normed component of stress deviator $(\sigma_{rr} - \sigma)/k_s$.



a)

b)

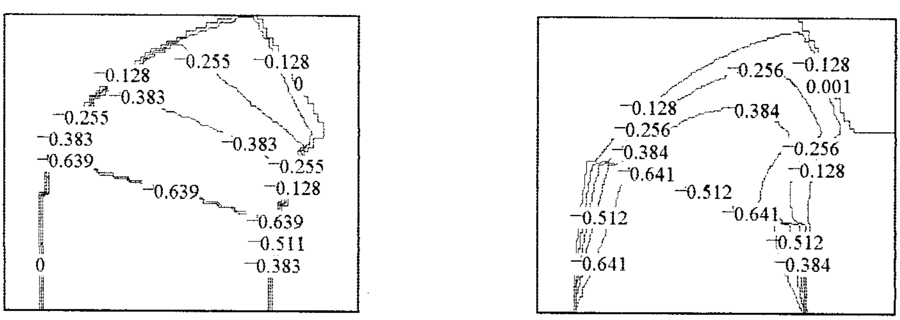
FIG. 9. Normed component of stress deviator $(\sigma_{zz} - \sigma)/k_s$.



a)

b)

FIG. 10. Normed component of stress deviator σ_{rz}/k_s.



a)

b)

FIG. 11. Normed component of stress deviator $(\sigma_{\theta\theta} - \sigma)/k_s$.

5. Conclusions

The presented numerical results of the modelling composite plastic flow demonstrate velocity field, grid distortion and stress distribution in the deformation zone. Very good agreement with experimental dates confirms the concept of the model and the chosen solution. The results indicate that it is possible to prove the design of composite products obtained in the course of the metal forming processes by taking into account the features of the actual non-uniform deformation of composite material. Assumptions for modelling are rather simple, but results are very close to the actual ones, so, such a solution may be useful from the practical point of view.

1. The proposed model of a kinematically admissible velocity field of composite bimaterial can be constructed for an arbitrarily determined boundaries of the plastic zone and always lead to smooth flow lines.

2. The presented method is not time consuming and the results of solutions give sufficiently exact information on the character of the flow of the composite material. Moreover, the numerical procedure is as simple as it has been in the case of the straight line model.

3. The conditions of existence of the proposed kinematically admissible field (3.10) or (3.17) are checked in the numerical procedure itself so it is not necessary to do it separately.

4. Numerical results concerning velocities and the grid distortion show good agreement with the experiments [6, 7, 8]. They may indicate the way in which modelling and engineering of this type of the composite material can be improved.

5. Basing on the exposed relations it is possible to predict the mode of deformation in the extrusion and the permissible degree of deformation of the components and the composite. Advantages of this model let us develop and apply it to obtain a better optimization of the extrusion process e.g. including the upper bound method. It makes it possible to include friction conditions into the consideration.

6. The proposed kinematically admissible velocity field for bimaterial composite can be generalized in the case when the composite consists of more than two materials. In such a case all materials except one should be considered as the next consecutive layer.

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