ENGINEERING TRANSACTIONS • *Engng. Trans.* • **46**, 1, 115–129, 1998
Polish Academy of Sciences • Institute of Fundamental Technological Research
10.24423/engtrans.661.1998

OPTIMIZATION OF VARIABLE THICKNESS PLATES BY GENETIC ALGORITHMS

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The implementation of genetic algorithms to the optimal design of variable thickness plates is presented. Thin, elastic, piecewise constant thickness plates subjected to bending are investigated. The material distribution that minimizes the structural strain energy under constant volume constraint is searched. In numerical examples, square plates loaded by uniform normal pressure are optimized for different boundary conditions. The best designs are compared with the worst solutions, corresponding to the maximization of the strain energy. Significant changes in strain energy can be achieved by modifying thickness distribution for the same material volume. The performances of the approach are discussed.

1. Introduction

The development of sophisticated optimization methods and the multiplying performances of present computers have opened up the possibility of finding efficient design methods for many structural components. The structures which efficiently face loads, vibrations, shocks, etc., with the minimum possible cost or weight, are more and more required by modern engineering. A significant enhancement of mechanical performances can be achieved by modification of the shape. The minimization of the strain energy leads to rigid structures with deflections and stresses considerably reduced. Such optimal forms have higher stiffness and higher resistance against deformation. The paper is concerned with the optimization of a particular structural element – the plate.

Among several computational techniques, developed to deal with engineering optimization tasks, the promising applications of Genetic Algorithms (GAs) are explored in recent years. The GAs [1] are a class of computational models inspired by evolution, which may be used directly to solve unconstrained maximization problems. They are particularly effective for non-differentiable and discontinuous problems, or in the case of many locally optimal points. It cannot be mathematically shown that GAs always converge to a global optimum but they are very effective in finding "near optimal" solutions. Such propositions can be acceptable for engineering design and can introduce more realism into practical optimization [2]. The papers [3, 4, 5] are among the first applications of GAs to structural optimization. Numerous publications follow this trend, and show usefulness of this class of search procedures for various engineering problems.

Several authors have carried out optimization of plates subjected to bending. A review of optimal plate design can be found in [6]. In [7], the maximum rigidity plates have been investigated by minimizing the functional of the maximum plate deflection, and by analyzing a corresponding integral optimality criteria. A function-space gradient projection technique was proposed by [8] and minimum volume plates were searched under displacements constraints. In [9, 10] the structural shape optimization procedures are coupled with the finite strip method to analyze the optimal forms of plates and curved shells under different loading, boundary and design variable linking conditions. Discretized continuum-type optimality criteria method is applied by [11] to plates and shells. In [12] optimal, discrete variables plates and shells are searched by applying continuous and dual, discrete optimization approach.

The objective of the present work is to apply the GAs to the problem of optimal design of plates subjected to bending, and to investigate numerical performances of this optimization procedure. The thin, elastic, piecewise constant thickness plate model is considered. The design variables are plate segment thicknesses, which have to be selected among the given, available values. The thickness distribution that minimizes the strain energy for constant volume of the plate is searched. We want to allocate the material where it is needed for maximum performance, and remove it from where it is unnecessary. A brief presentation of genetic algorithms is followed by the implementation of GAs to plate design problem. Chromosome representation of design variables, characteristics of genetic operators, and fitness formulation are presented. In the numerical examples, square plates loaded by a normal uniform pressure are optimized for different boundary conditions. The best designs are compared with the worst material distribution, corresponding to the maximization of the strain energy. The cases of two and four available thicknesses are considered. The changes in the strain energy are examined. The efficiency of the proposed approach and possible enhancement of the method are discussed.

2. Foundations of genetic algorithms

Genetic algorithms (GAs) are adaptive search strategies based on biological observation. They imitate the mechanism of natural evolution, hereditary and survival of the fittest. The basic principles were first laid down by [1] and next extended and developed in many books, e.g. [13, 14, 15]. The genetic algorithms are naturally adapted to solve unconstrained discontinuous maximization problems. It cannot be shown mathematically that they always converge to a definite optimum but numerous results from the literature demonstrate their robustness and efficiency in various fields of application. GAs procedures are able to find near-optimal solutions of difficult problems, where standard optimization procedures cannot be successfully applied.

GAs manipulate the coded information. Each potential solution is encoded in a string data structure and represented in the form analogous to the chromosomes of the biological individual in an evolutionary chain. In the presented approach, a one-chromosome individual model is applied. Design variables can be expressed as finite length substrings using, for example, binary representation. All substrings are concatenated head-to-tail to form one "chromosome", representing a potential design. A decoding procedure is used to obtain physical values of design variables, and to evaluate individuals. A measure of performance, called fitness, is directly related to the value of the objective function of a chromosome.

All each iteration Genetic Algorithm explores a fixed number of points called population. The process of evolution is simulated using a set of biologically inspired operators (like selection, reproduction and mutation) defined over the population itself. Each iterative step creates a new generation of individuals. The potential designs in a population compete with each other to pass to next generations. According to evolutionary theories, only the most suited elements are likely to survive. The chromosomes with high fitness values will be chosen to form next generation. New "children" chromosomes are generated by recombining the genes of "parents" strings, analogous to sexual reproduction in nature.

The structure of a simple GA program is presented in Fig. 1. At the first stage, an initial population of potential solutions $P(0)$, encoded into chromosome forms, is generated at random. The population size is, however, to be kept moderate. After evaluating the fitness of each individual of the population, a selection mechanism chooses "parents" for reproduction. They will generate offspring and transmit their biological heredity to new generations. The fitness is a measure of the reproductive efficiency. The selection policy must assume survival of more fitted individuals. The best strings get more copies, the average stay even, the worst die off. The crossover and the mutation operators are applied to create new population of individuals. Crossover is a procedure of exchange of information between two parent strings. In the simplest model, a crossing site is chosen randomly. Two new strings (children) are produced from the parent genetic material by interchanging substrings behind the crossing site. The mutation operator alters randomly the selected genes to prevent the premature convergence of the population, and to vary genetic features. Crossover and mutation probabilities are defined to determine whether operators should be implemented or not. The most promising strings are searched for improved solutions and new populations

n=0

initialize population of individuals P(n)

evaluate fitness of all individuals of P(n)

while (not *termination criterion*) do

n=n+1

select individuals for reproduction from P(n-1)

recombine selected individuals (crossover, mutation)

select the survivors to generate new population P(n)

evaluate fitness of all individuals of P(n)

end

FIG. 1. The structure of a genetic algorithm program.

$P(n)$ are created, keeping only the fittest individuals from the past generation. New designs are usually better and they replace members of old populations. The cycle selection – reproduction – evaluation is repeated until a satisfactory solution of the problem is found. The process can be stopped, for example, when there is no improvement in the fitness average of a population, when a given percentage in population uniformity has been obtained, or when a given number of generations has been evaluated.

3. Formulation of the optimization problem

The paper is concerned with the optimization of plates subjected to bending. The distribution of thicknesses within the plate has to be found. The total material volume V of the structure should remain constant. Thin, elastic plates are considered. The surface of the modeled structure is divided into N regions of area a_i and piecewise constant thickness t_i $(i = 1, ..., N)$, which are design variables. The problem is formulated as minimization of the structural elastic strain energy SE of the plate, due to the action of transverse loads:

find t_i $(i = 1, ...N)$,
to minimize

$$SE(t_i) \to \min,$$

subject to:

$$V = \sum_{i=1}^{N} t_i a_i = V_{\text{const}} = \text{const},$$

$$t_i \in \{t_i^1; t_i^2; ...t_i^{k_i}\}.$$

In the applied plate model, each thickness t_i has to be selected from a corresponding set of k_i values, where k_i corresponds to the number of available thicknesses for the i-th variable. For practical reasons, additional features of the expected design can be easily introduced by linking the design variables. This can be needed, for example, to take into account symmetry conditions and technological or manufacturing constraints.

This formulation corresponds to the discrete non-linear constrained optimization problem. The complete enumeration of all combinations needs the analysis of $k_1 * k_2 * ... * k_N$ possible variants, and it seems to be rather prohibitive even for moderate size problems. The search methods based on GAs are able to find a "good" solution. The optimal design of square plates under different support conditions at the edges, subjected to uniform normal pressure, is carried out in numerical examples. The material has to be removed from where it is useless, and allocated where it is needed to obtain the best design. The presented results can help in understanding the behavior, and in designing this type of structural components.

4. Implementation of GAs for plate optimization

4.1. Chromosome representation

Two-stage mapping process is carried out to express each design variable as a finite length string constructed over the binary alphabet $\{0, 1\}$. First, the discrete variable t_i is mapped to an integer number k $(1 \leq k \leq k_i)$, corresponding to the position of the value in the given catalogue of thicknesses. Next, k is represented in the form of binary digit string. The chromosome representation consists in concatenating head-to-tail all the N design variables coded as binary strings. The inverse decoding process enables us to obtain real values of thicknesses. The chromosome length depends on the number of design variables, and on the number of genes necessary to encode discrete values for each variable.

4.2. Evaluation of the fitness

A standard GAs procedure resolves unconstrained maximization problems, searching for the maximal fitness value. The classical constrained optimization problem can be formulated as

$$F(\mathbf{x}) \rightarrow \min, \qquad g_i(\mathbf{x}) \leq 0, \quad i = 1, ..., M,$$

where F is the objective function to be minimized, $\mathbf{x}$ is the vector of design variables, and $g_i(\mathbf{x})$ are constraints. The fitness function f can be represented as a combination of the objective function and the penalty term. The following formulation, based on the exterior penalty approach, has been applied

$$f = C - \left(F^* + \sum_{i=1}^{M} \alpha_i g_i^* \right),$$

where C and α_i $(i = 1, ..., M)$ are constant penalty parameters. The values F^* and g_i^* are given by

$$F^* = \frac{F}{F_{\max}},$$
$$g_i^* = \max\left(\frac{g_i}{|g_{i\,\max}|}, 0 \right),$$

where $F_{\max}$ and $g_{i\,\max}$ are constant, maximum reference values.

4.3. Selection, crossover and mutation operators

Many different genetic operators have been developed to mimic the mechanism of natural evolution and survival of the fittest. In the presented approach, chromosomes are selected as parents according to their fitness values by using the ranking selection scheme [15]. For the population of N_{pop} individuals, which are first sorted according to descending order of their fitness values, the n-th chromosome is selected for reproduction. The integer index n is determined by

$$n = \text{int}\left(\frac{\psi - \sqrt{\psi^2 - 4(\psi - 1)r}}{2\psi - 2} N_{\text{pop}} \right),$$

where r is a random number between 0 and 1, and ψ is the bias, which is given the value 1.5.

New "children" strings are reproduced pair by pair from two parent strings. A variant of the two-point crossover, using the uniform crossover technique, has been proposed in the present study. First, two crossover sites are chosen randomly and a binary crossover mask is generated at random for each gene between cut points. Next, the genes of parents are changed according to the corresponding mask value between two cut sites, as illustrated in Fig. 2.

The standard mutation operator, which alters a randomly chosen gene, has been applied to introduce new genetic features and diversity into the population.

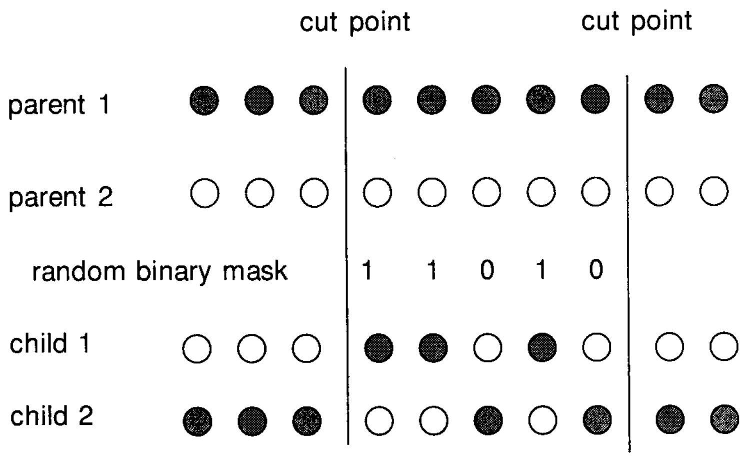

FIG. 2. Crossover operator.

5. OPTIMIZATION EXAMPLES

5.1. Plate model

Finite element approach was applied to model the plate structures. The thin plate element ACM [16], based on the Kirchhoff theory, has been chosen for the FEM analysis. For this study, the structural strain energy $SE = \frac{1}{2}\mathbf{q}^T\mathbf{K}\,\mathbf{q}$ of the plate was taken as the objective function, where $\mathbf{K}$ is the stiffness matrix, and $\mathbf{q}$ is the displacement vector.

A square plate of 1 m by 1 m, divided into 100 constant thickness, rectangular elements, is optimized in numerical examples. The modulus of elasticity $E = 2 \cdot 10^{11}\,\mathrm{N/m^2}$ and Poisson's ratio $\nu = 0.3$ have been chosen. The plate is subjected to a uniform pressure load of intensity $p = 10^6\,\mathrm{N/m^2}$. Only one quarter of the plate is modeled with 5×5 element mesh. The constant piecewise thickness areas are limited to the dimension of finite elements. All-round simply supported plates, all-round clamped plates, and plates with mixed boundary conditions (with two opposite edges clamped and two others opposite edges simply supported) are considered in numerical examples. Optimal design of plates having two different thicknesses {0.03m, 0.04 m}, or four available thickness values {0.025 m, 0.03 m, 0.035 m, 0.04 m} is presented. For simply supported and clamped plates, the thicknesses have been linked to 15 independent design variables, necessary to maintain the symmetry of solution. In the case of mixed boundaries, 25 design variables were needed to represent thicknesses of one quarter of the plate.

5.2. Fitness formulation and GAs parameters

The objective function to be minimized for a plate is formulated as

$$F^* = \frac{SE}{SE_{\max}},$$

where SE is the strain energy of the plate, and the reference value $SE_{\max}$ corresponds to the strain energy calculated for the plate of minimal available thickness.

The constant volume constraint has been expressed in the form

$$g_1^* = \left| \frac{V - V_{\text{const}}}{V_{\max} - V_{\text{const}}} \right|,$$

where V is the plate volume, V_{const} is the constant volume value, and $V_{\max}$ is the maximal possible volume. The fitness f (to be maximized by GA) is defined by

$$f = C - F^* - \alpha_1 g_1^*,$$

where constants $C = 12$ and $\alpha_1 = 10$ have been applied in numerical examples.

For the population size of 30 individuals, 400 and 600 generations have been analyzed respectively, in the first, and in the second example. The crossover probability 0.75, and the mutation probability 0.003 have been taken. In order to evaluate the statistical performances of the method, 100 runs of the GAs optimization program have been carried out.

5.3. Optimal design of two-thicknesses plates

The constant volume condition $V_{\text{const}} = 0.034\,\text{m}^3$ has been taken in this example. It imposes the constant ratio 2/3 between the elements of different thickness. The best designs for two-thicknesses plates are presented in Fig. 3 for different boundary conditions. Computational results indicate the characteristic areas of the greatest concentration of the material. The distribution of thicknesses depends on the boundary conditions, and on the discrete values of available thicknesses. Table 1 summarizes the results for SE minimization. The SE for the constant thickness plates of identical volume is included as well. The average SE value, the standard deviation, the average number of the generation, when the best solution has been found, and the percentage reduction in the average strain energy with respect to the constant thickness plate of the same volume, are presented. The best designs from Table 1 correspond to the material distributions presented in Fig. 3.

In Fig. 4 and Table 2 the worst solutions of the problem are presented for comparison. They have been obtained by the maximization of the strain energy for the same constant volume. The zones where the material is necessary

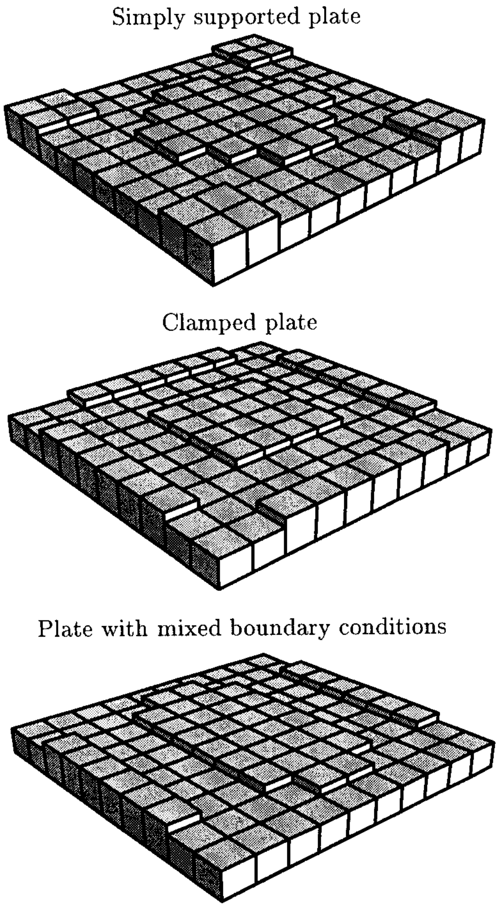

FIG. 3. The best material distribution – SE minimization (two-thicknesses plates).

Table 1. Minimization of the strain energy for two-thicknesses plate.

100 runs of GA program $V = 0.034\,\mathrm{m}^3$	simply supported plates	clamped plates	mixed boundary cond. plates
Average SE value [Nm]	266.734	55.654	101.235
Standard deviation [Nm]	5.939	1.545	2.039
Average SE decrease (with respect to *)	−10.73%	−19.17%	−16.73%
Avg. best generation No.	60	108	228
The best design SE [Nm]	257.204	53.044	97.946
SE decrease for the best design (with respect to *)	−13.92%	−22.96%	−19.44%
* SE for the constant thickness plate	298.781	68.851	121.579

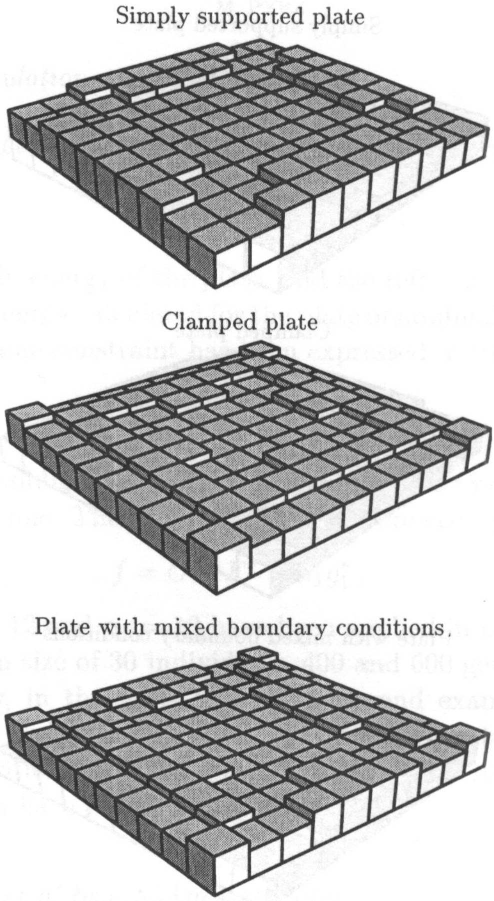

FIG. 4. The worst material distribution – SE maximization (two-thicknesses plates).

Table 2. Maximization of the strain energy for two-thicknesses plate.

100 runs of GA program $V = 0.034\,\mathrm{m}^3$	simply supported plates	clamped plates	mixed boundary cond. plates
Average SE value [Nm]	343.713	84.255	147.064
Standard deviation [Nm]	12.138	1.937	3.124
Average SE increase (with respect to *)	+15.04%	+22.37%	+20.96%
Avg. "worst" generation No.	71	78	208
The worst design SE [Nm]	367.106	86.498	152.504
SE increase for the worst design (with respect to *)	+22.87%	+25.63%	+25.44%
* SE for the constant thickness plate	298.781	68.851	121.579

and needless are complementary. Significant changes in the value of SE can be achieved by the modifications of thickness distribution.

A steady convergence of the best fitness, with "relatively good" results at the beginning, and small enhancements at final generations, has been observed.

5.4. *Optimal design of four-thicknesses plates*

For this example, the constant volume $V_{\text{const}} = 0.036\,\text{m}^3$ has been taken. The best material distributions for the strain energy minimization problem are shown in Fig. 5. In Table 3 the statistical parameters of 100 runs of the GA optimization program are given. "Near optimal" distributions of material are very similar to the continuous optimization solutions, presented for example in [7, 8, 9].

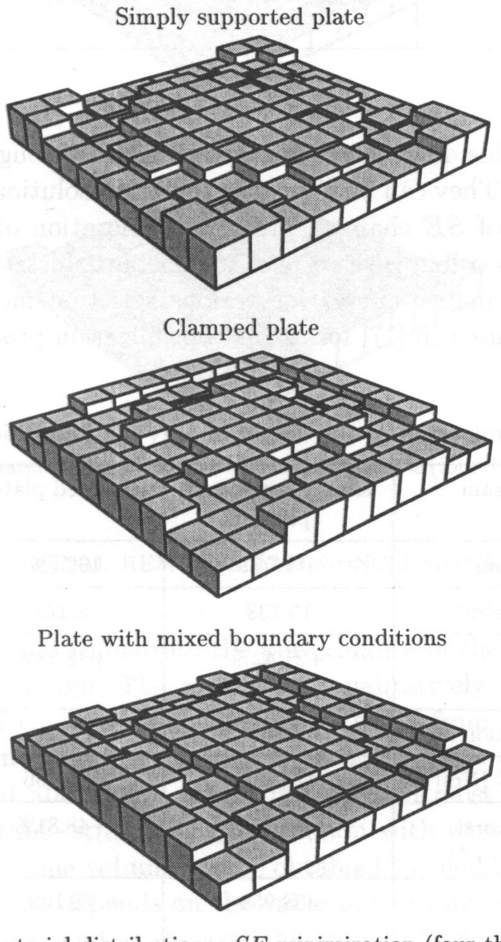

FIG. 5. The best material distribution – SE minimization (four-thicknesses plates).

Table 3. Minimization of the strain energy for four-thicknesses plates.

100 runs of GA program $V = 0.034\,\mathrm{m}^3$	simply supported plates	clamped plates	mixed boundary cond. plates
Average SE value [Nm]	292.404	60.148	108.102
Standard deviation [Nm]	8.946	2.564	2.311
Average SE decrease (with respect to *)	−13.73%	−22.99%	−21.62%
Avg. best generation No.	300	275	448
The best design SE [Nm]	270.012	55.064	104.231
SE decrease for the best design (with respect to *)	−20.34%	−29.50%	−24.42%
* SE for the constant thickness plate	338.951	78.108	137.924

In Fig. 6 and Table 4 the worst designs, obtained through the SE maximization, are presented. They can be viewed as the worst solution of the minimization problem. In terms of SE changes, greater deterioration of the design than its improvement was possible with respect to constant thickness plate of the same volume. The performance curves for various selection methods and crossover operators are presented in [17] for similar optimization problem.

Table 4. Maximization of the strain energy for four-thicknesses plates.

100 runs of GA program $V = 0.034\,\mathrm{m}^3$	simply supported plates	clamped plates	mixed boundary cond. plates
Average SE value [Nm]	411.551	106.096	191.711
Standard deviation [Nm]	19.738	6.379	6.527
Average SE increase (with respect to *)	+21.42%	+35.83%	+39.00%
Avg. "worst" generation No.	286	312	467
The worst design SE [Nm]	477.091	116.236	203.140
SE increase for the worst design (with respect to *)	+40.755%	+48.81%	+47.28%
* SE for the constant thickness plate	338.951	78.108	137.924

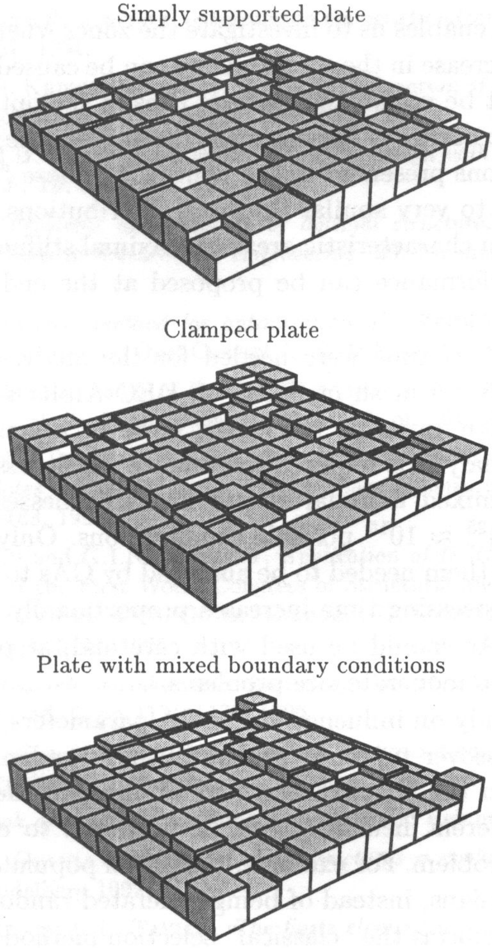

FIG. 6. The worst material distribution – SE maximization (four-thicknesses plates).

6. Final remarks and conclusions

The presented solutions illustrate the performance of the genetic algorithms in the optimal plate design. The results are qualitatively in good agreement with the solutions of continuous optimization known from the literature. The optimization procedure removes material from where little contribution to the stiffness is made, and allocates it where it is needed most for maximum performance. A considerable strain energy reduction with respect to the constant thickness plate of the same volume can be obtained by modification of the form. The percentage decrease depends on the value of constant material volume, on the boundary conditions, and on the available values of discrete thicknesses. The strain maximization enables us to investigate the zones where the material is useless. A significant increase in the strain energy can be caused by a wrong material distribution. It must be noted, that several runs of the optimization procedure, due to the stochastic nature of the approach, have given the results slightly worse than the best solutions presented in the figures. All these "near-optimal" plates correspond however to very similar thickness distributions, keeping the concentration of material in characteristic areas of maximal stiffness. A set of potential designs of good performance can be proposed at the end of the optimization procedure for the designer.

About 400 sec CPU time were needed for the analysis of 400 generations of 30 plates (using 5×5 mesh in FEM) on DEC Alpha workstation. GAs can handle discrete design variables efficiently, however a considerable number of complete problem analysis is required to evaluate the fitness of all chromosomes. In the example of mixed boundaries and four thicknesses plate, the research space size reached $4^{25} \approx 10^{15}$ possible combinations. Only 1.8×10^4 (which is about $2 \times 10^{-9}\%$) of them needed to be analyzed by GAs to obtain the presented results. Since the processing time increases proportionally to the complexity of the FEM model, GAs should be used with care and, at present, they may be applied reasonably to moderate size problems.

An additional study on influence of the GA parameters, like population size, selection policy, crossover types, or probabilities of random operators, can give more information on the effectiveness, and to enhance the possibilities of this search method. Different heuristics can be proposed to enhance the GA approach for a given problem. For example, the initial population can be composed of known feasible designs, instead of being generated randomly. The use of "elitist" selection outperforms the "classical" selection methods [17]. Some "repair" procedures may be applied to correct "unfeasible" chromosomes. The research on evolutionary algorithms [15, 18], adapting evolutive operators to "natural" problem representation, seems to be a promising direction for further investigation, and can lead to an efficient approach to practical engineering optimization problems.

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Received February 6, 1997.