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Evaluation of Critical Static Loads of Three-Layered Annular Plate with Damaged Composite Facings

Dorota PAWLUS

Faculty of Mechanical Engineering and Computer Science
University of Bielsko-Biała
Willowa 2, 43-309 Bielsko-Biała, Poland
e-mail: doro@ath.bielsko.pl

This paper presents the approach to evaluate the critical, static loads of three-layered annular plate with composite facings whose laminas are damaged. The plate is composed of thin, laminated fibre-reinforced composite facings and a thicker, foam core. The fibre and matrix cracks of facing laminas influence the plate critical state. The solution of such buckling problem was carried out using both analytical and numerical methods. Axisymmetrical and asymmetrical plate buckling modes were analysed. Numerous results show the stability behaviour of composite plate with failures.

Key words: composite annular plate, static stability, failure, finite element method.

1. Introduction

Composite layered structures are subjected to different forms of failure. These forms can be either local or global, and they can be treated as microscopic damage, e.g., fibre or matrix crack or macroscopic damage such as global buckling [5]. Combining these failures could be especially dangerous for the strength of construction element. The evaluation of the critical parameters for the three-layered, annular plate with damaged facings made of fibrous composite is presented. The wide range of the application of sandwich annular plates in, for example, mechanical and nuclear engineering or aerospace industry and a rapid development of composite structure technology create a practically important issue. Problem of the dynamic stability of sandwich or laminated annular plates is undertaken in numerous works, for example, in [1, 3].

2. Problem formulation

The cross-section structure of the plate is symmetric and is composed of thin composite facings and a thicker foam core (see Fig. 1). The accepted exemplary configuration of a laminated composite facing is expressed by the code $[0°/-45°/45°/90°]$. This configuration is characteristic for a composite called quasi-isotropic one. The composite facing consists of $n = 4$ laminas, each of thickness equal to $h_i = 0.000125$ m. Plate geometry is expressed by inner radius $r_i = 0.2$ m and outer one $r_o = 0.5$ m. The core thickness is equal to $h_2 = 0.005$ m. The material parameters of a glass/epoxy composite as a facing material are as follows: $E_1 = 53.781$ GPa, $E_2 = 17.927$ GPa, $G_{12} = 8.964$ GPa, $\nu_{12} = 0.25$ [4], and the parameters of a polyurethane foam as core material treated as isotropic one are $G_2 = 5$ MPa, $\nu_2 = 3$.

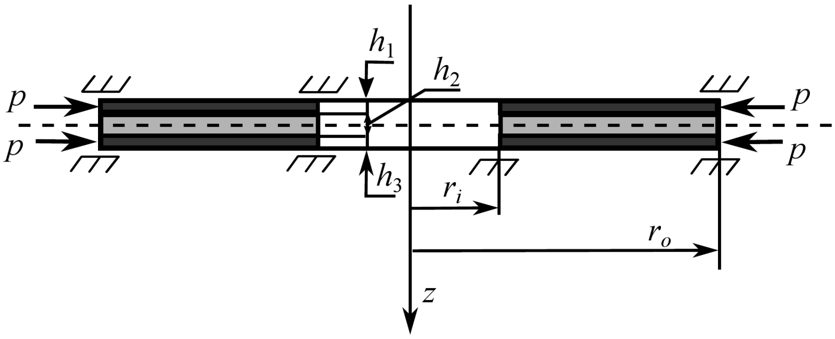


Fig. 1. Scheme of three-layered annular plate with composite facings.

Accepted model of the composite degradation is based on the correction parameter method presented in [5]. The matrix or fibre cracks change the mechanical properties of laminate. The matrix crack causes the rigidity elimination in a direction transverse to the fibres. This is expressed by the correction parameter η whose value is in the range of [0.1, 0.4]. Mathematically, this is described by the modification of the stiffness matrix. Its form for non-damaged lamina is

$$\begin{bmatrix} C_{11} & C_{12} & 0 & 0 & 0 & 0 \\ C_{12} & C_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix}, \tag{2.1}$$

where

$$C_{11} = \frac{E_1}{(1-\nu_{12}\nu_{21})}, \qquad C_{22} = \frac{E_2}{(1-\nu_{12}\nu_{21})}, \qquad C_{12} = \frac{E_1\nu_{21}}{(1-\nu_{12}\nu_{21})},$$

$$C_{21} = \frac{E_2\nu_{12}}{(1-\nu_{12}\nu_{21})}, \qquad C_{66} = G_{12}.$$

For the lamina with a matrix crack, the elements C_{11}, C_{12}, C_{22} take the following new values: $\mathbb{C}_{11} = \eta \cdot C_{11}$, $\mathbb{C}_{12} = C_{22} = 0$, but when the fibre crack occurs: $\mathbb{C}_{11} = C_{22}$ [5].

The mechanics of fibrous composite is based on the classical lamination theory using the following expressions:

$$\begin{aligned} A_{ij} &= \sum_{k=1}^{N} \left(\overline{Q}_{ij}\right)_k (z_k - z_{k-1}), \\ B_{ij} &= \frac{1}{2}\sum_{k=1}^{N} \left(\overline{Q}_{ij}\right)_k \left(z_k^2 - z_{k-1}^2\right), \\ D_{ij} &= \frac{1}{3}\sum_{k=1}^{N} \left(\overline{Q}_{ij}\right)_k \left(z_k^3 - z_{k-1}^3\right), \end{aligned} \tag{2.2}$$

where A_{ij}, B_{ij}, D_{ij} are extensional, coupling, bending stiffnesses, respectively, Q_{ij} is the transformed reduced stiffness of lamina, and N is the number of layers.

The elastic, engineering constants E, G, ν of quasi-isotropic composite were calculated according to the expressions presented in [2]:

$$E = 2\frac{A_{66}}{t}\left(1 + \frac{A_{12}}{A_{11}}\right), \qquad G = \frac{A_{66}}{t}, \qquad \nu = \frac{A_{12}}{A_{11}}, \tag{2.3}$$

where E – Young's modulus, G – Kirchhoff's modulus, ν – Poisson's ratio.

The values of these three parameters for the analysed quasi-isotropic glass/epoxy composite are as follows: $E = 31.1$ GPa, $G = 12.5$ GPa, $\nu = 0.24$.

3. Plate models

The problem was solved analytically and numerically using the finite difference method (FDM) and numerically using the finite element method (FEM). While solving the buckling problem of the sandwich plates with quasi-isotropic composite facings, the eigenvalue task was formulated. As the result of calculation, the minimal value of stress being the critical static load p_{cr} was obtained.

The description of the solution to the problem is presented in detail in [6, 7]. The main equation is expressed as follows:

$$\det\left(\mathbf{M}_{\mathrm{APDG}} - p^{*}\mathbf{M}_{\mathrm{AC}}\right) = 0, \tag{3.1}$$

where $\mathbf{M}_{\mathrm{APDG}}$, $\mathbf{M}_{\mathrm{AC}}$ are the matrices of elements composed of geometric and material parameters of plate, the quantity b (the length of the interval in the FDM), mode number is m, and coefficients δ, γ are determining the differences of radial and circumferential displacements of the points in middle surfaces of facings, respectively.

The calculations using the FEM were carried out at the Academic Computer Centre CYFRONET-CRACOW using the ABAQUS system (KBN/SGI_ORIGIN_2000/PŁódzka/030/1999). Two kinds of plate models were built: a circular, symmetrical, annular form called the basic model and simplistic one built from axisymmetrical elements (see Fig. 2). The facings are built of the shell elements with composite option. The core mesh is built from the solid elements. The grids of the facing elements are tied with the grid of the core elements using the surface contact interaction.

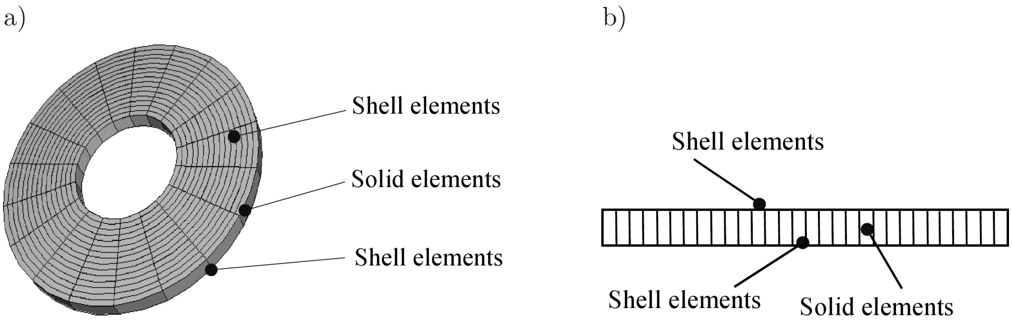


Fig. 2. FEM plate models: a) basic, b) simplistic.

4. Results

Table 1 shows the minimal values of critical static loads p_{cr} of plates loaded on the inner or outer edge. The case of matrix crack is examined for different combinations of laminas. Laminas are expressed as lamina 1, 2, 3 and 4 according to a composite code [0°/−45°/45°/90°] (for example: lamina 1 is for fibres arranged with angle 0°). The correction parameter η is equal to $\eta = 0.1$. The presented results are for the basic and simplistic FEM composite plate models. The part of Table 1 concerning the plates without the failures shows the range of critical static loads p_{cr} calculated for the FEM plate model whose facings are

Table 1. Values of critical loads with corresponding buckling modes of plate models.

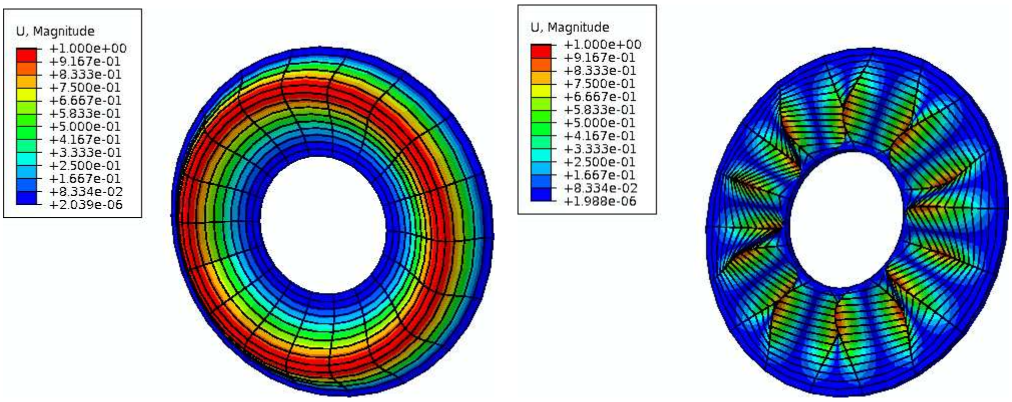
Critical static load p_{cr} [MPa]/buckling mode m									
Plate model		simplistic	basic	simplistic	basic	simplistic	basic	simplistic	basic
damaged lamina									
		1		2		3		4	
Edge loading	inner	32.93/0	34.28/0	36.26/0	35.90/0	35.75/0	35.90/0	33.61/0	34.28/0
	outer	22.00/0	–	23.42/0	–	23.36/0	–	22.91/0	–
		–	12.90/≈2	–	13.03/≈2	–	13.03/≈2	–	12.90/≈2
combination of damaged laminas									
		1 + 2		1 + 4		1 + 3 + 4		all layers	
Edge loading	inner	31.85/0	32.61/0	30.37/0	32.15/0	28.98/0	30.62/0	26.98/0	27.54/0
	outer	20.50/0	–	19.99/0	–	17.55/0	–	13.46/0	–
		–	11.42/≈2	–	12.70/5	–	10.05/≈2	–	8.04/5

		non-damaged laminas		
Plate model		simplistic	basic	FDM
Edge loading	inner	36.64/0 (39.96/0)	36.84/0 (40.76/0)	(38.89/0)
	outer	24.07/0 (26.21/0)	24.02/0 (26.22/0)	(29.15/0)
		–	14.63/6 (15.45/9)	(17.39/6)

modelled as both composite and quasi-isotropic. This enables to compare the values with the values calculated for the FDM quasi-isotropic plate model (the values are presented in brackets).

One can observe the good compatibility of the values calculated using the two kinds of the FEM models: simplistic and basic. The values of loads p_{cr} obtained for composite and quasi-isotropic plates are comparable. This shows that some approximation analyses for special laminated composites with quasi-isotropic configuration could be carried out using their simplistic quasi-isotropic notation. The buckling form of plates with non-damaged laminas is global axisymmetrical or with several circumferential waves (see Fig. 3a). The results, which are presented in Table 1, show the decrease in value of critical load p_{cr} for plate models with damaged laminas. The notation (≈2) means that the buckling mode is not axisymmetrical. The loss of stability of plates with damaged laminas could be in irregular form or global, circumferentially regular form (see Fig. 3b). The theoretical example, in which the matrix of all the laminas is damaged, discloses the plate case with the minimal value of the static, critical load p_{cr}.

Additionally, the loads p_{cr} for plate with the laminas damaged in the form of fibre crack are shown in Fig. 4. Figure 4 presents the distribution of values of loads p_{cr} for plates with non-damaged facings (case marked as 0), failure laminas and all damaged laminas in one of the analysed forms: fibre or matrix crack (case marked as 5). The level of values of load p_{cr} for the quasi-isotropic composite a)



b)

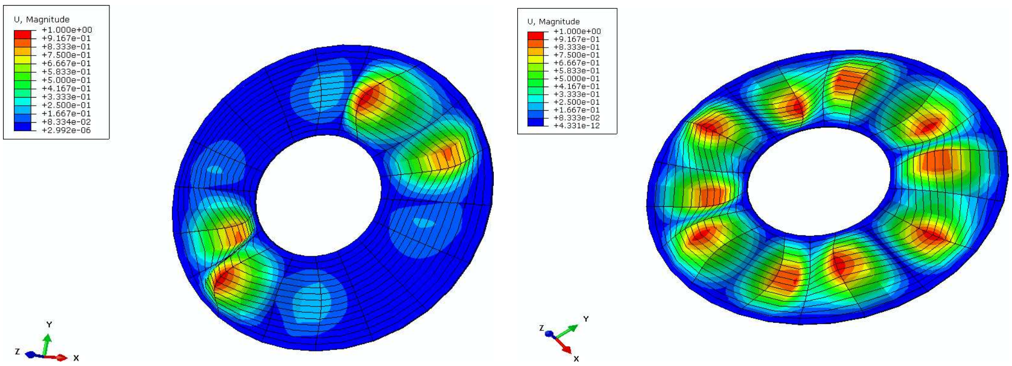


Fig. 3. Plate buckling forms for: a) non-damaged laminas, b) damaged laminas.

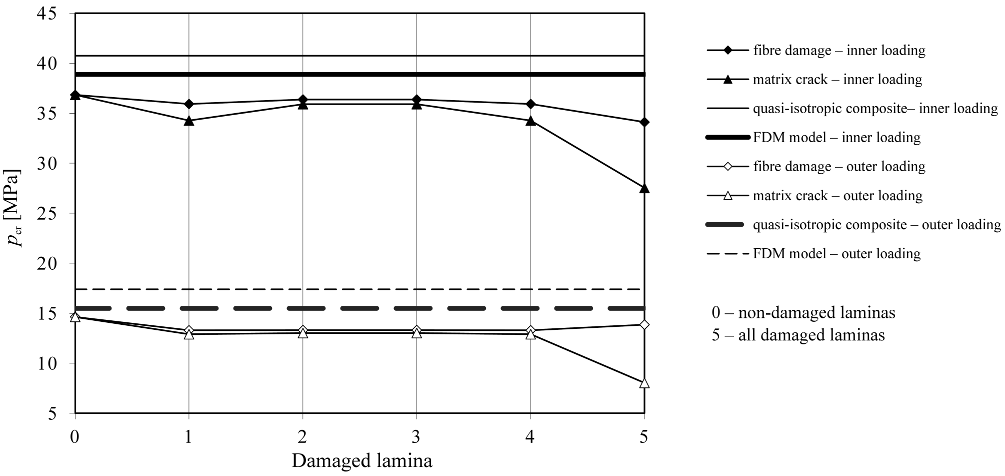


Fig. 4. Distribution of the critical static stress for plates with damaged laminas of facings.

FEM model and the FDM model are shown, too. The presented values were calculated for the FEM basic model.

5. Conclusions

Presented solutions are an attempt to evaluate the stability response of the composite plate with damaged facings. In the case of the damages of the single lamina, small decreases in values of critical static loads are observed. The form of plate buckling can be different strongly depended on the configuration of the non-damaged laminas of facings. Configuration of non-damaged laminas arranged as [0°/90°] seems to have a higher structure resistance to the stability loss of plate loaded on the inner or outer edge. Particular choice of the structure of laminate, which consists of only four laminas, enabled the evaluation of buckling behaviour of the plate. The presented approach to the analysed problem can be useful in similar engineering issues. Further researches, especially some experimental investigations, could be very important.

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