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Research Paper

Development of a Hybrid Meta-Model for Material Selection Using Design of Experiments and EDAS Method

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Selection of materials for a specific application is one of the extremely demanding problems in a synchronised manufacturing environment as it directly determines perceptible quality and cost of the product. Material selection is a complex process, intending to choose the best material while satisfying a pre-decided set of requirements. Material selection decision is made during preliminary product design stage. An improperly chosen material leads not only to an early component failure but also to a redundant cost involvement. There are numerous materials and various criteria influencing the material selection process for a particular application. Although a good amount of multi-criteria decision-making (MCDM) methods are available to deal with this type of selection applications, this paper aims to propose a hybrid method of design of experiments (DOE) and evaluation based on distance from average solution (EDAS) to solve material selection problems in current industrial applications. DOE and EDAS are used jointly to determine the critical material selection criteria and their interactions by fitting a polynomial to the experimental data in a multiple linear regression analysis. A gear material selection problem is demonstrated to establish the application competence of the DOE-EDAS method. Application results were validated with the results of the previous researchers and they indicate that the proposed DOE-EDAS hybrid model is straightforward, robust and practical in solving complex MCDM problems.

Key words: multi-criteria decision-making; design of experiments; EDAS; hybrid meta-model; materials selection.

1. Introduction

Material selection has immense importance and crucial role in the manufacturing industry for product design and development. Moreover, the rapid progress of manufacturing technology in the last few decades has compelled the design engineers to select the best material among several alternatives. Proper selection of materials for a particular engineering application has several benefits, such as improved reliability and quality, reduced cost, enhanced product life, etc., whereas, improper material selection directs to poor performance and untimed product failure. Moreover, it also leads to enhanced production and operating cost from manufacturer and user perspective. The decision makers have to take a large number of factors, like mechanical, electrical, physical and economic considerations into account, which will affect the quality and application of a product in a particular domain. Furthermore, there is a large variety of manufacturing processes and machine operations available coupled with complex interrelationships among selection criteria which make the material selection process very complicated and time-consuming. The foremost prerequisite may be the material strength for designing and manufacturing a specific mechanical element, but based on the working conditions and functional need, several other attributes may have to be considered concurrently. The selection of the most suitable material involves the study of a large number thermal, electrical, mechanical, and physical properties with cost consideration, production process, market value, availability of resources and product performance [1]. For mechanical design, the mechanical properties of the materials are given the top priorities. However, due to the availability of over 40 000 metal alloys and almost the same number of non-metals, ceramics, polymers and composites, each having its characteristics, applications, advantages and limitations, it becomes difficult for designers to be able to make optimum decisions on selecting the best material for a specific application to meet in the best way the necessary criteria. So, before selecting the best material for a given application, a designer has to consider several alternatives with various conflicting criteria, which eventually leads to a multi-criteria decision-making problem (MCDM). Most of the MCDM methods have the potential to rank the alternatives from best to worst by considering several weighting criteria.

The remaining sections of the paper are organised as follows: Sec. 2 presents the literature review on materials selection, Sec. 3 describes the proposed hybrid DOE-EDAS model in detail; Sec. 4 demonstrates the applicability of the hybrid DOE-EDAS method in solving two material selection problems. Finally, Sec. 5 concludes the paper.

2. Literature review on materials selection

To evaluate the suitability of different materials for various engineering applications, several studies were carried out by the precedent researchers. MCDM techniques have proven its great potentiality in the field of material selection. In this section, the most relevant and recent past research works related to material selection problems are presented. Jahan *et al.* [2] proposed linear assignment method in order to help design engineers to choose the optimal material based on ordinal data for a given component and validated it with three real-life examples. Chatterjee *et al.* [3] explored the applicability and potentiality of the complex proportional assessment (COPRAS) and evaluation of mixed data (EVAMIX) methods for selecting the most appropriate material for a cryogenic storage tank used for transportation of liquid nitrogen and a product that operates in high-temperature oxygen-rich environment. Athawale *et al.* [4] elucidated the applicability of the utility additive (UTA) method for material selection problem. Huang *et al.* [5] explored the applicability of a technique for order preference by the similarity to ideal solution (TOPSIS) method for the environmentally conscious material selection problem. Chauhan and Vaish [6] proposed the potentiality of Vlse Kriterijumska Optimizacija I Kompromisno Resenje (VIKOR) and TOPSIS method for hard and soft magnetic material selection. The relative weights for different attributes were calculated using Shannon's entropy method, whereas hierarchical clustering was used to classify magnetic materials, and Pearson correlation coefficients were calculated between the attributes under study. Girubha and Vinod [7] proposed the applicability of the fuzzy-based VIKOR method for material selection of an automotive component. Chatterjee and Chakraborty [8] elucidated the applicability of four preference ranking-based MCDM methods including the extended preference ranking organisation method for enrichment evaluations (EXPROM2), gray system theory-based COPRAS (COPRAS-G), organisation, rangement Et synthese de donnes relationnelles (ORESTE), and operational competitiveness rating analysis (OCRA) for solving a gear material selection problems. Maity *et al.* [9] proposed a method for cutting tool material selection using the COPRAS-G method. Karande and Chakraborty [10] applied a multi-objective optimisation on the basis of the ratio analysis (MOORA) method for material selection problems, depicting some real-world problems. Liu *et al.* [11] proposed the induced ordered weighted averaging VIKOR (IOWA-VIKOR) operator for solving material selection problems. Çalişkan *et al.* [12] proposed a decision model including extended EXPROM2, TOPSIS, and VIKOR for the selection of the best material for the tool holder used in hard milling. Prasad and Chakraborty [13] developed a QFD-based software module to automate the material selection problem along with four real-world examples. Ilangku-

MARAN *et al.* [14] proposed the fuzzy analytic hierarchy process (FAHP) method along with PROMETHEE for selecting the appropriate material for manufacturing of automobile bumpers. CAVALLINI *et al.* [15] proposed a quality function deployment (QFD)-based VIKOR algorithm to deal with the material selection problems. MAITY and CHAKRABORTY [16] applied the fuzzy TOPSIS method for grinding wheel abrasive material selection. CHATTERJEE and CHAKRABORTY [17] applied COPRAS and additive ration assessment (ARAS) – based methods for solving gear material selection problem in a given manufacturing environment. KARANDE *et al.* [18] proposed a methodology combining utility concept and desirability function approach for solving several material selection problems. ANOJKUMAR *et al.* [19] described the applicability of four MCDM methods: FAHP-TOPSIS, FAHP-VIKOR, FAHP-ELECTRE, and FAHP-PROMTHEE, for solving pipe material selection problems in sugar industries. In addition, the effectiveness and flexibility of the VIKOR method were depicted by solving the material selection problem. DARJI and RAO [20] investigated the applicability of four MCDM methods: extended TODIM (an acronym in Portuguese for interactive and multicriteria decision making), ARAS, OCRA and EVAMIX, for pipe material selection in sugar industries. YAZDANI and PAYAM [21] applied the Ashby approach as a multi- objective decision making (MODM) technique as well as the TOPSIS and VIKOR method as a multiple attribute decision making (MADM) technique to select the most appropriate material for micro-electro-mechanical systems (MEMS) devices. ANOJKUMAR *et al.* [22] applied FAHP integrated with the TOPIS and VIKOR techniques for material selection in sugar industries. XUE *et al.* [23] proposed a method based on the interval-valued intuitionistic fuzzy sets (IVIFSs) and multi-attributive border approximation area comparison method (MABAC) to deal with material selection problems with incomplete weight information. CHANDRASEKAR and RAJA [24] applied a fuzzy TOPSIS methodology to select the material for automobile torsion bar selection. ZHAO *et al.* [25] proposed a methodology, combining grey relational analysis (GRA) and AHP to rank the alternative materials for sustainable design. SINGH *et al.* [26] proposed a methodology, combined with VIKOR and AHP, for selecting best brake friction material. NASAB and ANVARI [27] presented the applicability of TOPSIS, COPRAS and GRA to cope with the material selection problems.

From the above survey of referential literature, it can be observed that in most of the material selection papers, the researchers have mainly focused on the application of various MCDM techniques such as AHP, TOPSIS, QFD, VIKOR, ELECTRE and subsequent determination of the performance scores to evaluate and rank the candidate materials for different applications. However, these models have principally overruled the possibility of any interaction between the material properties, which may also exist. Additionally, in the case of many criteria and alternatives, this possibility may turn for the decision makers into difficulty in obtaining a clear view of the problem and evaluating the results due to the involvement of different preferential parameters such as preference functions, veto threshold, pair-wise comparison, which may be very difficult to define in real-time scenarios [28].

In this paper, a modest effort has thus been used to diminish this research gap while exploring the suitability of the hybrid DOE and EDAS method-based approach for identifying the best material under different engineering applications. A full factorial experimental design plan is first formulated with five replications and two levels for each material selection properties. Subsequently, a regression meta-model interrelating those material selection properties and EDAS score is developed showing the main effects, and the possible two-way and three-way interactions between those properties. The performance of this meta-model is observed to be quite promising in determining the performance scores of the materials.

3. Methods

3.1. DOE methodology

DOE is a systematic statistical method to determine the relationship between factors affecting a process and the output of that process. It is widely used for designing and analysing multi-variable experiments [29]. It enables designers to determine simultaneously the individual and interactive effects of several factors that could affect the output results in any process. A strategically planned and executed experiment may provide a great deal of information about the effect on a response variable due to one or more factors. Its main role in design is to identify the significant factors (independent variables) influencing the response (dependant variable) and the degree of this influence. The orthogonal experimental design along with the deployment of the orthogonal array and factor design is the primary step of DOE. A full factorial experiment requires measurements at each of all the possible level combinations of the input variables. Sometimes, the number of input variables and their levels become too large, making the application of full factorial experiments practically impossible. On these occasions, a suitable subset of the factor level combinations needs to be selected, resulting in fractional factorial experiment design. The orthogonal experiment design explores properties of the fractional factorial experiment to determine the best factor level combinations [30–32].

In this paper, two-level full factorial experiment plans are used to measure how the five considered material selection properties (input variables) affect the EDAS score (response/output variable). The corresponding mathematical meta-model presenting the relationship between n number of input variables and the measurable EDAS score can be given as follows:

$$Y = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i3} + ... + \beta_n x_{in} + \varepsilon, \tag{3.1}$$

where Y is the response variable (EDAS score), β_0 is the Y-intercept coefficient, β_1, ..., β_n are the effect coefficients, x_1, ..., x_n are the input variables, and ε is the error term.

The main impact of each input variable is assumed to be independent of the remaining variables. The interaction effects are also available to determine the presence of interactions between the considered input variables. The following matrix formulation is of great help in representing the above-mentioned linear regression model more practically, and enables calculations of the intercept, main effect and interaction effect coefficients, and error term

$$\mathbf{Y} = \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix}, \quad \mathbf{X} = \begin{bmatrix} 1 & x_{11} & x_{12} & ... & x_{1n} \\ 1 & x_{21} & x_{22} & ... & x_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & x_{i1} & x_{i2} & ... & x_{in} \end{bmatrix}, \quad \boldsymbol{\beta} = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \vdots \\ \beta_n \end{bmatrix}, \quad \boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{bmatrix}. \tag{3.2}$$

Employing the least square method, the regression coefficient $\boldsymbol{\beta}$ is expressed as $\boldsymbol{\beta} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y}$, where $\mathbf{X}'$ is the transposed matrix of $\mathbf{X}$ and $(\mathbf{X}'\mathbf{X})^{-1}$ is the inverse of $\mathbf{X}'\mathbf{X}$. The error $\boldsymbol{\varepsilon}$ between the experiment and estimated model is given as $\boldsymbol{\varepsilon} = \mathbf{Y} - \widehat{\mathbf{Y}}$, where the estimated response is $\widehat{\mathbf{Y}} = \boldsymbol{\beta}\mathbf{X}$.

3.2. Evaluation based on distance from average solution (EDAS) method

This section presents a newly developed method called EDAS to deal with MCDM problems [33]. This newly developed method uses average solution for appraising the alternatives. Positive distance average (PDA) and negative distance average (NDA) are considered as two measures for the appraisal of alternatives. These measures can demonstrate the difference between each alternative and the average solution. Moreover, measures are calculated according to the type of criteria (beneficial or non-beneficial). The best solution in the EDAS method is calculated based on the distance from the average solution (AV). Higher values of PDA and/or lower values of NDA indicate that the alternative is better than the average solution. Moreover, the necessity for calculating the ideal and nadir solution is eliminated in the proposed methodology, as required in other MCDM techniques, such as VIKOR and TOPSIS.

The main steps of the EDAS method are presented as follows [33, 34]:

Step 1. Select the most important criteria that describe alternatives.

Step 2. Construct the decision-making matrix **X**, shown as follows:

$$\mathbf{X} = [x_{ij}]_{n\times m} = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1m} \\ x_{21} & x_{22} & \cdots & x_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{nm} \end{bmatrix}, \tag{3.3}$$

where x_{ij} ($x_{ij} \geq 0$) denotes the performance value of i-th alternative on j-th criterion ($i \in \{1, 2, ..., n\}$ and $j \in \{1, 2, ..., m\}$).

Step 3. Determine the average solution according to all criteria, shown as follows:

$$\mathbf{AV} = [\mathrm{AV}_j]_{1\times m}, \tag{3.4}$$

where

$$\mathrm{AV}_j = \frac{\sum\limits_{i=1}^{n} X_{ij}}{n}. \tag{3.5}$$

Step 4: Calculate the positive distance from average (PDA) and the negative distance from average (NDA) matrixes according to the type of criteria (benefit and cost), shown as follows:

$$\mathbf{PDA} = [\mathrm{PDA}_{ij}]_{n\times m}, \tag{3.6}$$

$$\mathbf{NDA} = [\mathrm{NDA}_{ij}]_{n\times m}, \tag{3.7}$$

If j-th criterion is beneficial

$$\mathrm{PDA}_{ij} = \frac{\max\left(0, (X_{ij} - \mathrm{AV}_j)\right)}{\mathrm{AV}_j}, \tag{3.8}$$

$$\mathrm{NDA}_{ij} = \frac{\max\left(0, (\mathrm{AV}_j - X_{ij})\right)}{\mathrm{AV}_j}. \tag{3.9}$$

And if j-th criterion is non-beneficial

$$\mathrm{PDA}_{ij} = \frac{\max\left(0, (\mathrm{AV}_j - X_{ij})\right)}{\mathrm{AV}_j}, \tag{3.10}$$

$$\mathrm{NDA}_{ij} = \frac{\max\left(0, (X_{ij} - \mathrm{AV}_j)\right)}{\mathrm{AV}_j}, \tag{3.11}$$

where PDA_{ij} and NDA_{ij} denote the positive and negative distance of i-th alternative from average solution in terms of j-th criterion, respectively. The graphical representation of PDA and NDA values in a sample condition with four alternatives and two beneficial criteria is shown in Fig. 1 [33].

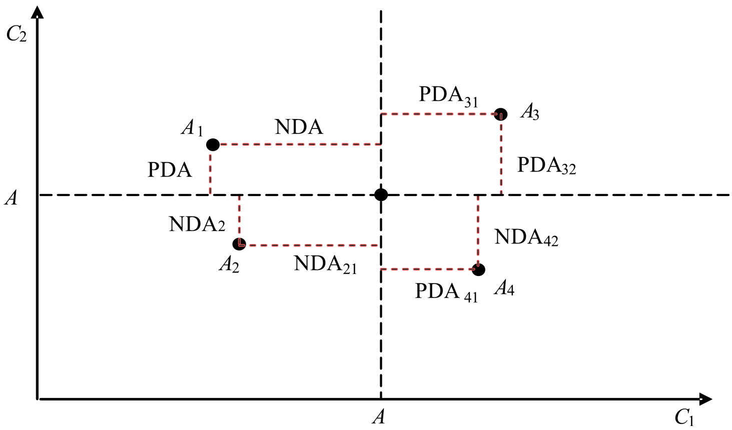

FIG. 1. Graphical representation of PDA and NDA values in a simple situation [33].

Step 5. Determine the weighted sum of PDA and NDA for all alternatives, shown as follows:

$$\mathrm{SP}_i = \sum_{j=1}^{m} w_j \mathrm{PDA}_{ij}, \tag{3.12}$$

$$\mathrm{SN}_i = \sum_{j=1}^{m} w_j \mathrm{NDA}_{ij}, \tag{3.13}$$

where w_j is the weight of j-th criterion.

Step 6. Normalise the values of SP and SN for all alternatives, shown as follows:

$$\mathrm{NSP}_i = \frac{\mathrm{SP}_i}{\max_i(\mathrm{SP}_i)}, \tag{3.14}$$

$$\mathrm{NSN}_i = 1 - \frac{\mathrm{SN}_i}{\max_i(\mathrm{SN}_i)}. \tag{3.15}$$

Step 7. Calculate the appraisal score (AS) for all the alternatives, shown as follows:

$$\mathrm{AS}_i = \frac{1}{2}\left(\mathrm{NSP}_i + \mathrm{NSN}_i\right), \tag{3.16}$$

where $0 \leq \mathrm{AS}_i \leq 1$.

Step 8. Rank the alternatives according to the descending values of AS. The alternative with the highest AS value is the best choice among the candidate alternatives.

3.3. Hybrid DOE-EDAS model

In this section, a new hybrid MCDM model, i.e., the DOE-EDAS model is proposed. DOE and EDAS methods are used jointly to identify critical criteria and their interactions by fitting a polynomial to the experimental data in a multiple linear regression analysis. The hybrid DOE-EDAS method comprises of four steps, as depicted in Fig. 2.

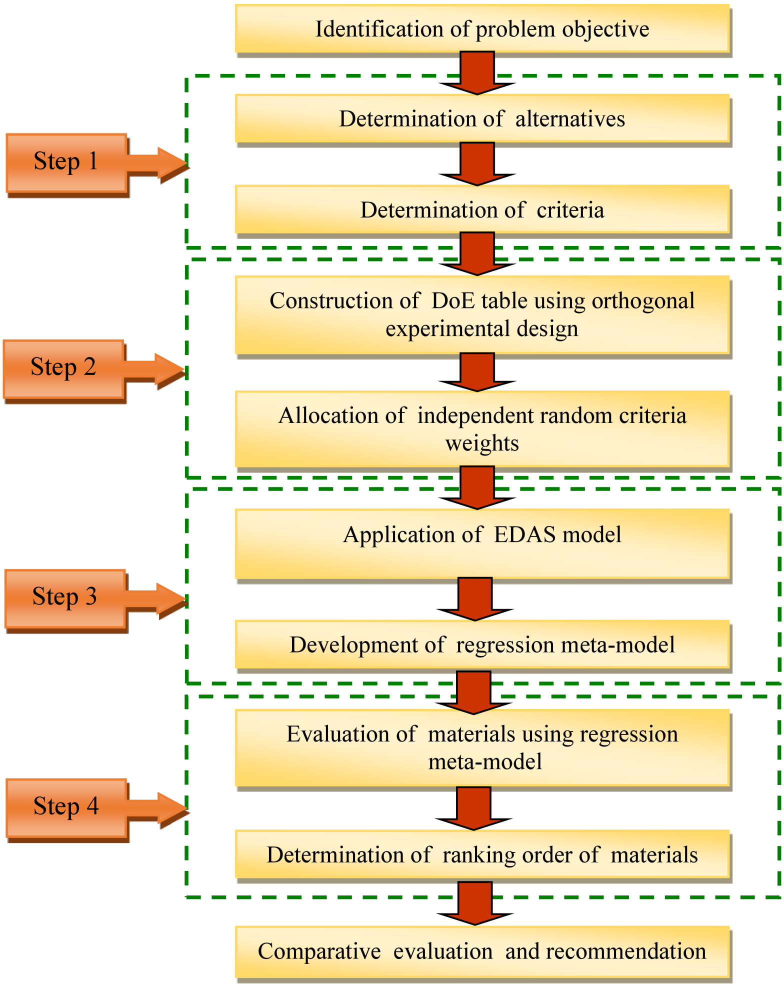

FIG. 2. Four steps of the hybrid DOE-EDAS model.

4. Illustrative examples

Two illustrative examples are presented to demonstrate the computational accuracy, application approach and convenience of the hybrid DOE-EDAS model and validation of its result in solving material selection problems.

4.1. Example 1: gear material selection problem

This example of gear material selection for high speed and high stress applications is taken from Milani *et al.* [35], where nine alternative materials, i.e., cast iron, ductile iron, SG iron, cast alloy steel, through hardened alloy steel, surface hardened alloy steel, carburized steel, nitride steel and through hardened carbon steel were considered. The performance of those nine material alternatives was measured with respect to five selection criteria, i.e., core hardness (CH) (in Bhn), surface hardness (SH) (in Bhn), surface fatigue limit (SFL) (in N/mm^2), bending fatigue limit (BFL) (in N/mm), and ultimate tensile strength (UTS) (in N/mm^2). Among those five criteria, SH, SFL, BFL, and UTS are beneficial where higher values are preferred, and on the other hand, the lower value of CH is usually desired as it is a non-beneficial criterion. Milani *et al.* [35] dealt with a gear material selection method employing the TOPSIS method and observed the ranking of the material alternatives as 9–8–6–5–4–3–1–2–7. Hence, based on their findings, carburized steel was the best choice followed by nitride steel. Cast iron was the least preferred material for gear manufacturing. Using the EXPROM2 method, Chatterjee and Chakraborty [8] also obtained the same rankings for the most preferred and the least preferred alternative materials. Table 1 shows the decision matrix for this gear material selection problem.

Table 1. Decision matrix for the gear material selection problem [35].

Material	CH	SH	SFL	BFL	UTS
Cast iron (A_1)	200	200	330	100	380
Ductile iron (A_2)	220	220	460	360	880
SG iron (A_3)	240	240	550	340	845
Cast alloy steel (A_4)	270	270	630	435	590
Through hardened alloy steel (A_5)	270	270	670	540	1190
Surface hardened alloy steel (A_6)	240	585	1160	680	1580
Carburized steel (A_7)	315	700	1500	920	2300
Nitride steel (A_8)	315	750	1250	760	1250

Now to illustrate and validate the proposed procedure of gear material selection through the DOE-EDAS application, various steps of the methodology, as given in Sec. 3, are carried out as described below.

Step 1. *Determination of the criteria levels.*

Based on the data in Table 1, criteria levels were determined, among which CH with minimum level of 185 and maximum level of 315, SH with minimum level of 185 and maximum of 750, SFL with minimum level of 330 and maximum level of 1500, BFL with minimum level of 100 and maximum level of 920, and UTS with minimum level of 380 and maximum level of 2300 are considered as the two factor levels for the subsequent development of the mathematical meta-model for this material selection process.

Step 2. *Development of the experiment design plan.*

For the development of the mathematical meta-model, five material selection properties, i.e., CH, SH, SFL, BFL, and UTS are treated as the input variables, and the calculated EDAS score is regarded as the output variable. To represent the two-level combinations for these five input variables, a 2^5 full factorial design plan requiring a total of 160 experiments is to be employed which requires 160 combinations, where only the minimum and maximum values of each input variable are used in the experiment plan for data collection in the form of EDAS scores. Each combination is run for five times in the EDAS model and, at each run, there is an independent random criteria weight set to ensure the independence of each combination. It signifies that for each combination of factor levels, five EDAS scores are derived as five replications, each replication considering a separate criteria weight set [30]. These criteria weight sets are based on the adopted 10-point scale. The five criteria weight sets, as required for the replications of the EDAS scores, are exhibited in Table 2.

Table 2. Five weight sets for the replication of EDAS score.

Criteria	Weight set 1	Weight set 2	Weight set 3	Weight set 4	Weight set 5
CH	7	4	2	9	5
SH	3	2	5	2	9
SFL	9	7	3	6	2
BFL	2	9	7	4	6
UTS	5	3	9	7	3

The full factorial experimental design plan based on two levels for each of the five factors and the calculated EDAS scores for the five replications are provided in Table 3. The assignment of different sets of criteria weights results in different EDAS scores.

Table 3. Results of the 2^5 full factorial design.

Design of experiment points	CH	SH	SFL	BFL	UTS	Replication 1	2	3	4	5
						EDAS scores				
1	315	750	330	920	380	0.4751	0.7282	0.6212	0.6548	0.6607
2	185	750	1500	920	380	0.4169	0.3547	0.1630	0.2961	0.3463
3	185	185	1500	100	2300	0.8942	0.8753	0.8900	0.8740	0.8374
4	315	750	1500	100	2300	0.3910	0.2102	0.2368	0.5397	0.2892
5	185	185	1500	100	380	0.3113	0.0971	0.4590	0.1804	0.4162
6	315	750	330	100	380	0.0537	0.4683	0.4309	0.0000	0.2988
7	315	750	330	920	380	0.1061	0.2219	0.1893	0.2705	0.1402
8	185	185	1500	100	2300	0.4414	0.0971	0.3723	0.0372	0.0656
9	315	750	330	920	2300	0.2359	0.4680	0.0282	0.3966	0.3641
10	315	750	1500	100	2300	0.3911	0.3436	0.1860	0.3073	0.1880
11	315	750	330	100	380	0.0536	0.1133	0.2454	0.5398	0.4159
12	315	185	330	920	380	0.0527	0.3840	0.3245	0.3347	0.2935
13	185	185	1500	100	380	0.3113	0.4345	0.4586	0.0892	0.5270
14	185	185	330	920	380	0.1562	0.5313	0.2967	−0.0001	0.1225
15	315	185	1500	920	380	0.2602	0.1878	0.1354	0.0003	0.0483
16	185	185	330	100	380	0.1035	0.2303	0.0515	0.2437	0.3683
17	315	185	1500	100	380	0.2078	0.3843	0.1382	−0.0001	0.5007
18	185	750	1500	920	380	0.4170	0.2105	0.3488	0.2434	0.2892
19	315	750	1500	920	2300	0.4437	0.3209	0.1378	0.3482	0.2282
20	185	185	330	920	380	0.1559	0.3842	0.2133	0.0891	0.2938
21	185	750	330	920	380	0.2095	0.2934	0.3470	0.2703	0.4123
22	315	185	1500	100	380	0.2080	0.3011	0.3245	0.1804	0.1135
23	185	185	330	920	2300	0.2862	0.2219	0.4822	0.1261	0.0482
24	315	750	1500	100	2300	0.3911	0.2595	0.3471	0.2324	0.1228
25	185	185	330	920	2300	0.2860	0.0341	0.0281	0.3868	0.2939
26	185	185	330	100	2300	0.2335	0.4180	0.3723	0.4508	0.2892
27	185	750	330	920	2300	0.3394	0.3350	0.1870	0.3484	0.5267
28	185	750	330	100	2300	0.2871	0.3211	0.1869	0.1261	0.0000
29	315	185	330	100	2300	0.1301	0.2379	0.1865	0.1162	0.2722
30	315	750	330	100	2300	0.1834	0.0634	0.4001	0.3871	0.2722
31	185	750	1500	920	2300	0.5471	0.4180	0.5099	0.0370	0.4166
32	185	750	1500	920	380	0.4171	0.4682	0.1353	0.2806	0.5266

Step 3. *Determination of the regression meta-model.*

The polynomial regression meta-model considering six input variables is provided in Eq. (4.1). In this equation, in addition to main effects of the six factors, interactions (two and three ways) between the factors are also included

$$Y = \beta_0 + \sum_{i=1}^{6} \beta_i x_i + \sum_{i} \sum_{j_{i<j}} \beta_{ij} x_i x_j + \sum_{i} \sum_{j} \sum_{k_{i<j<k}} \beta_{ijk} x_i x_j x_k + \beta_{12345} x_1 x_2 x_3 x_4 x_5 + \varepsilon, \tag{4.1}$$

where Y is the EDAS score, β_0 is the overall mean response or intercept coefficient, β_i is the main or first-order effect of factor i, β_{ij} is the two-factor interaction between factors i and j with $i \neq j$, β_{ijk} is the three-factor interaction between factors i, j and k with $i \neq j \neq k$, and β_{12345} is the five-factor interaction between all the factors.

Now, based on the data in Table 3 and using Eq. (4.1), the coefficients β are determined using a MINITAB (R15) software package, as given in Table 4. The calculated EDAS scores are also analysed by the analysis of variance (ANOVA) procedure. The ANOVA results, as exhibited in Table 5, provide a summary of the main effects and interactions between various factors. In Table 4, the p-values determine which of the effects in the regression model are statistically significant. If the p-value is less than or equal to 0.05, it can be concluded that the effect is significant; otherwise, it is not significant. The 'term' column in Table 5 represents the main effects, and all the two-way and three-way interactions. The 'effect' column displays the relative strength of the effects of the terms. The β coefficients and their standard errors (SE) are shown in the third and fourth column respectively. The last two columns provide the corresponding t- and p-values. In Table 4, the rows of all the significant factors ($p \leq 0.05$) are shown in boldface. The p-values in Table 4 thus lead to the following conclusions:

a) all the main factors, i.e., CH, SH, SFL, BFL, and UTS ($p = 0$) are statistically significant;
b) SH, SFL, BFL, and UTS are the positive contributors, whereas CH contributes negatively to estimating the EDAS score;
c) among the two-way interactions, CH $\times$ SH ($p = 0.026$) is statistically significant, while others are not, and
d) the three-way interactions are not at all statistically significant.

Thus, the developed polynomial regression meta-model for determining the performance score of the alternative materials in terms of the EDAS score can be expressed as follows:

$$Y = 0.29765 - 0.05031 \cdot \mathrm{CH} + 0.04024 \cdot \mathrm{SH} + 0.06100 \cdot \mathrm{SFL} + 0.08039 \cdot \mathrm{BFL} + 0.06554 \cdot \mathrm{UTS} + 0.02226 \cdot \mathrm{CH} \cdot \mathrm{SH}. \tag{4.2}$$

Table 4. Estimated effects and coefficients for the gear material selection problem.

Term	Effect	Coefficient	SE of coefficient	t-value	p-value
Constant		0.29765	0.009865	30.17	**0.000**
CH	−0.10062	−0.05031	0.009865	−5.10	**0.000**
SH	0.08048	0.04024	0.009865	4.08	**0.000**
SFL	0.12199	0.06100	0.009865	6.18	**0.000**
BFL	0.16078	0.08039	0.009865	8.15	**0.000**
UTS	0.13108	0.06554	0.009865	6.64	**0.000**
CH · SH	0.04451	0.02226	0.009865	2.26	**0.026**
CH · SFL	0.01692	0.00846	0.009865	0.86	0.393
CH · BFL	−0.01303	−0.00651	0.009865	−0.66	0.510
CH · UTS	−0.01624	−0.00812	0.009865	−0.82	0.412
SH · SFL	−0.01031	−0.00515	0.009865	−0.52	0.602
SH · BFL	0.00731	0.00366	0.009865	0.37	0.711
SH · UTS	−0.01684	−0.00842	0.009865	−0.85	0.395
SFL · BFL	−0.02966	−0.01483	0.009865	−1.50	0.135
SFL · UTS	−0.01075	−0.00538	0.009865	−0.54	0.587
BFL · UTS	−0.01417	−0.00709	0.009865	−0.72	0.474
CH · SH · SFL	0.03889	0.01944	0.009865	1.97	0.051
CH · SH · BFL	−0.02367	−0.01183	0.009865	−1.20	0.233
CH · SH · UTS	0.02006	0.01003	0.009865	1.02	0.311
CH · SFL · BFL	0.03087	0.01543	0.009865	1.56	0.120
CH · SFL · UTS	−0.03769	−0.01884	0.009865	−1.91	0.058
CH · BFL · UTS	0.01666	0.00833	0.009865	0.84	0.400
SH · SFL · BFL	−0.00030	−0.00015	0.009865	−0.02	0.988
SH · SFL · UTS	−0.02594	−0.01297	0.009865	−1.31	0.191
SH · BFL · UTS	0.00335	0.00167	0.009865	0.17	0.866
SFL · BFL · UTS	−0.00556	−0.00278	0.009865	−0.28	0.778
CH · SH · SFL · BFL	−0.01925	−0.00962	0.009865	−0.98	0.331
CH · SH · SFL · UTS	−0.00384	−0.00192	0.009865	−0.19	0.846
CH · SH · BFL · UTS	−0.01971	−0.00986	0.009865	−1.00	0.320
CH · SFL · BFL · UTS	0.01081	0.00541	0.009865	0.55	0.585
SH · SFL · BFL · UTS	0.01290	0.00645	0.009865	0.65	0.514
CH · SH · SFL · BFL · UTS	−0.01216	−0.00608	0.009865	−0.62	0.539

In the ANOVA results in Table 5, as the p-values for the main effects and only one two-way interaction are smaller than 0.05, they become the significant factors. In this table, R is the correlation between the predicted values and the observed values of EDAS score, and R^2 is the square of this coefficient which

Table 5. Analysis of variance results for the EDAS scores.

Source	Degrees of freedom	Adj SS	Adj MS	F-value	p-value
Main effects	5	2.9806	0.5961	38.29	0.000
2-way interactions	10	0.1736	0.0173	1.12	0.356
3-way interactions	10	0.2336	0.0233	1.50	0.146
4-way interactions	5	0.0422	0.00845	0.54	0.743
5-way interactions	1	0.0059	0.0059	0.38	0.539
Error	128	1.9929			
Total	159	5.4290			
$S = 0.1247$, $R^2 = 63.29\%$, R^2 (adj) $= 54.40\%$ $S = 48.4711$, $R^2 = 76.93\%$, R^2 (adj) $= 71.34\%$					

indicates the percentage of variation explained by the developed regression line out of the total variation. This value may tend to increase when additional predictors are included in the model.

Thus, a higher R^2 value may be artificially obtained by increasing the number of terms in the model. To penalise this effect, R^2(adj) is considered, which is the percentage of response variable variation that is explained by its relationship with one or more predictor variables, adjusted for the number of predictors in the model. It indicates how well terms fit a regression model and adjusts for the number of terms in the model. If more and more useless variables are added to the model, then the R^2(adj) value will decrease. If more useful variables are added to the model, the R^2(adj) value will increase. The adjusted R^2 is always less than or equal to the R^2 value. From Table 5, it can be concluded that 54.40% of the variation in the dependant variable Y (EDAS score) is explained by the variation of the independent variables in this regression meta-model, and it is appropriate to satisfy the model.

Step 4. *Validation of the developed meta-model.*

To assure the validity of the developed meta-model, the decision matrix of the original gear material selection from Table 1 is considered here. Table 6 provides the normalised EDAS scores based on the meta-model and the corresponding rank orderings for nine gear materials along with the ranking preorder as obtained by Milani *et al.* [35] using the TOPSIS method and the EXPROM2 method as proposed by Chatterjee and Chakraborty [8]. In all these methods, carburised steel (A_7) and Nitride steel (A_8) obtained the top two ranks, and cast iron (A_1) obtained the last rank respectively. This table also exhibits a substantial agreement between the intermediate rankings of the candidate materials. Very high Spearman's rank correlation coefficient (0.9833 between the

Table 6. Comparison of the ranking results for the gear material selection problem.

Sl. No.	EDAS-based meta models score	Rank	TOPSIS [35]	EXPROM2 [8]
1.	0.3108	9	9	9
2.	0.3348	8	8	8
3.	0.3348	7	6	6
4.	0.3359	5	5	5
5.	0.3528	4	4	4
6.	0.3911	3	3	3
7.	0.4239	1	1	1
8.	0.3933	2	2	2
9.	0.3356	6	7	7

meta-model, TOPSIS and EXPROM2 methods) validates the application of this mathematical meta-model for determining the performance scores of materials to aid the manufacturing industries.

4.2. Example 2: material selection for an automobile bumper

A material selection problem for an automobile bumper is now considered here [36] to further illustrate the application of the proposed DOE-EDAS method. A bumper is a construction integrated with the front and rear ends of an automobile to absorb impact in a minor collision which ideally minimises repair costs. The selection of bumper material is extremely important as the improper bumper material selection leads to damage of critical components such as radiator cap, fan, engine manifold, etc. The mechanical properties playing critical roles in the bumper material selection are compressive yield strength (CYS), flexural modulus (FM), hardness (H), the Charpy impact strength (CIS), elongation (E) and cost (C) as identified by Ilangkumaran *et al.* [36]. Thus, the decision matrix for this bumper material selection problem consists of five materials: polyethylene (A_1), polypropylene (A_2), acrylonitrile butadiene styrene (A_3), polyamide (A_4) and polystyrene (A_5) and six criteria, as shown in Table 7. For the given problem, CYS, FM, H, and CIS are the benefitial attributes, and E and C of the material are the non-beneficial attributes. Ilangkumaran *et al.* [36] used the fuzzy AHP method to determine the normalised criteria weights as $W_{CYS} = 0.154$, $w_{FM} = 0.200$, $w_H = 0.155$, $w_{CIS} = 0.282$, $w_E = 0.115$, $w_C = 0.132$ and applied the PROMETHEE-GAIA method to derive the ranking of the considered alternatives as $A_4 > A_3 > A_2 > A_5 > A_5$ indicating polyamide (A_4) as the best material and polystyrene (A_5) as the worst material for the considered problem.

Table 7. Decision matrix for the automobile bumper material selection problem [36].

Material	CYS	FM	H	CIS	E	C
Polyethylene (A_1)	20	700	92	1.00	500	78
Polypropylene (A_2)	40	1500	92	1.00	100	84
Acrylonitrile butadiene styrene (A_3)	65	2500	105	2.18	30	114
Polyamide (A_4)	130	3100	93	3.00	50	153
Polystyrene (A_5)	70	2500	90	0.60	7	1300

When the DOE-EDAS method is used, a six-factor polynomial regression equation (Eq. (4.3)) is developed for this problem as expressed below:

$$\begin{aligned}(4.3)\quad Y = 0.3641 - 0.1008 \cdot \mathrm{E} - 0.0963 \cdot \mathrm{C} + 0.0548 \cdot \mathrm{CYS} \\ + 0.0398 \cdot \mathrm{FM} + 0.0250 \cdot \mathrm{H} + 0.0186 \cdot \mathrm{CIS}.\end{aligned}$$

The ANOVA results are presented in Table 8. The ranking results of the DOE-EDAS method are compared with the ones obtained by Ilangkumaran *et al.* [36] given in Table 9. These results show that the rankings are exactly the same as those using different methods with a Spearman's rank correlation coefficient of 1, which leads to the conclusion that the DOE-EDAS model can be used to determine the ranking preorder of materials in a systematic and effective manner. Hence, mapping diverse material selection examples to the

Table 8. Analysis of variance results for the EDAS scores.

Source	Degrees of freedom	Adj SS	Adj MS	F-value	p-value
Main effects	6	3.99800	0.6663	48.54	0.000
2-way interactions	15	0.19900	0.0132	0.97	0.494
3-way interactions	10	0.14466	0.0144	1.05	0.403
Total	159	1.75711			
$S = 0.1171$, $R^2 = 71.19\%$, R^2 (adj) $= 64.21\%$					

Table 9. Ranking results for the automobile bumper material selection problem.

Material	DOE-EDAS meta model score	Rank	PROMETHEE rank [36]
A_1	0.2878	5	5
A_2	0.3789	3	3
A_3	0.4136	2	2
A_4	0.4353	1	1
A_5	0.3241	4	4

meta-model is not at all troublesome. A comparative analysis of quantitative characteristics between the proposed hybrid DOE-EDAS model and some well-established MCDM methods is presented in Table 10.

Table 10. Ranking results for the problem of automobile bumper material selection.

Method	Methodological aspect	Computation time	Simplicity	Transparency	Flexibility	Output
TOPSIS	Rank-problem statement	Low	Critical	Reasonable	Moderate	Total pre-order
EXPROM2	Rank-problem statement	Moderate	Very simple	Good	High	Total pre-order
OCRA	Rank-problem statement	Low	Very simple	Low	High	Total pre-order
EVAMIX	Rank-problem statement	High	Critical	Reasonable	Moderate	Total pre-order
COPRAS	Rank-problem statement	Low	Very simple	Good	High	Total pre-order
DOE-EDAS hybrid model	Criteria interaction and rank-problem statement	Low	Very simple	Good	High	Total pre-order

However, an acceptable and convenient model can be achieved if other material selection properties are also included in the developed EDAS-based meta-model. Thus, the main focus should not lie on the selection of the most appropriate method to be adopted, but on proper structuring of the decision problem considering relevant criteria and candidate alternatives while identifying the most appropriate choice.

5. Conclusions

This paper presents a new DOE-EDAS-based meta-model for materials selection problems from a set of candidate alternatives in the manufacturing environment. This combined application is based on a straightforward experimental design analysis which involves the least amount of mathematical calculations. The developed meta-models tender the interrelationships that exist between the measured EDAS scores and the considered material properties. DOE and EDAS methods are used jointly to identify critical criteria and their interactions by fitting a polynomial to the experimental data in a multiple linear regression analysis. Two real-time material selection examples are considered to demonstrate the application competence and suitability of the proposed model. In the first case study of gear material selection, it is observed that for selection of the gear materials, the following are contributing factors: optimal core hardness, surface hardness, surface fatigue limit, bending fatigue limit and ultimate tensile strength, and there is only one two-way interaction which is statistically significant. However, no three-way interaction is statistically significant. Similarly, in the second example, all the considered automobile bumper material properties, like compressive yield strength, flexural modulus, hardness, the Charpy impact strength, elongation, and cost are statistically significant, and there is no statistically significant two-way or three-way interaction between them. The results of the DOE-EDAS method almost substantiate with those derived by the past researchers, which signifies that the DOE-EDAS method is an uncomplicated and efficient approach as compared to other well-established material selection methods such as AHP, VIKOR, PROMETHEE, TOPSIS, ELECTRE, etc, as most of these techniques either require very lengthy computations involving pair-wise comparisons or they need some preferential parameters to be defined, which may be very complicated for the decision makers in practical situations.

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