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Research Paper

Influence of Rotation on Transversely Isotropic Piezoelectric Rod Coated with a Thin Film

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In this paper, the influence of rotation on axisymmetric waves of a piezoelectric rod coated with a thin film is studied using the constitutive equation of linear theory of elasticity and piezoelectricity. Potential functions are introduced to uncouple the equations of motion in radial and axial directions. The surface area of the rod is coated by a perfectly conducting material. The frequency equations are obtained for longitudinal and flexural modes of vibration and are studied numerically for PZT-4 ceramics. The computed non-dimensional frequency, phase velocity, relative frequency shift, electromechanical coupling and electric displacement are presented in the form of dispersion curves. This type of study is important in the construction of rotating sensors and gyroscope.

Key words: wave propagation in piezoelectric cylinder/fiber; forced vibration; Bessel function; actuators/sensors; thin film.

1. Introduction

The piezoelectric materials are the important structural components in devices such as pressure transducers and accelerometers. Initially, piezoelectric materials were used as resonators for ultrasound sources in sonar devices. The piezoelectric materials such as barium titanate ($BiTiO_3$) are fabricated by the advancement of piezoelectricity in the engineering field. Coated piezoelectric polymers are used in a variety of compositions and geometrical shapes for a large variety of applications from transducers in acoustics, ultrasonics and hydrophone applications to resonators in bandpass filters, power supplies, delay lines, medical scans and some industrial non-destructive testing instruments. The rotational effect will attenuate the medium and energy transfer. Also, the thin film coating and rotation can greatly influence the performance of the wave medium.

The wave propagation in elastic solid was discussed extensively by Graff [1] and Achenbach [2]. A detailed account of the historical development of the problem was given by Meeker and Meitzler [3]. Wave propagation in transversely isotropic cylinders was developed by Farhang Honarvar *et al.* [4]. They verified the consistency of this model with the physics of the problem and a systematic solution to the corresponding equations was developed. Wang and Hao [5] developed modeling of guided wave propagation with a spectral element with application in structural engineering. Studies by Tiersten [6] should be mentioned among the notable early contributions to the topic of the mechanics of piezoelectric solids. The author developed modeling for small vibrations of piezoelectric bodies by the linear theory of piezoelectricity through Maxwell's equations. In piezoelectricity, the quasistatic electric field is coupled to the dynamic mechanical motion. Electroelastic governing equations of piezoelectric materials were presented by Parton and Kudryavtsev [7]. Paul and Venkatesan [8] studied the wave propagation in infinite piezoelectric solid cylinders of arbitrary cross-section using the Fourier expansion collocation method, formulated by Nagaya [9]. They used series solutions in which some boundary conditions were exactly satisfied, term-by-term. The remaining boundary conditions were satisfied in a mean-square sense. Ebenezer and Ramesh [10] analyzed axially polarized piezoelectric cylinders with arbitrary boundary conditions on the flat surfaces using the Bessel series. In their paper, the series expressions for displacements, potential, and stress converge rapidly and the result was validated with a finite element program. Later Botta and Cerri [11] extended this approach and compared their results with those in which the effect of variable electric potential was not considered. Kim and Lee [12] studied piezoelectric cylindrical transducers with radial polarization and compared their results with those obtained experimentally and numerically by the finite element method. Selvamani [13] developed modeling of elastic waves in a fluid-loaded and immersed piezoelectric circular fiber. He concluded that the effect of inner and outer fluid, as well as the anisotropy of the material with a thickness on the various considered wave characteristics, is more significant and dominant in the flexural modes of vibration. Rao and Rao [14] studied the nonlocal critical velocities of fluid conveying clamped-pinned single-walled carbon nanotubes subjected to an axial magnetic field. They concluded that higher the values of the non-local parameter the greater are the effects of reducing the stability of the system.

Abd-Alla and Bayones [15] discussed the effect of rotation, magnetic field, thermal relaxation time and pressure on the wave propagation in a generalized viscoelastic medium under the influence of time-harmonic source. Wauer [16] studied the propagation of waves in conducting piezoelectric solid for the case when the entire medium rotates with a uniform angular velocity. He concluded that the rotational speed influences the wave characteristics significantly. Roychoudhuri and Mukhopadhyay [17] studied the effect of rotation and relaxation times on plane waves in generalized thermo-visco-elasticity. Sergiu *et al.* [18] studied the energy dissipation and critical speed of granular flow in a rotating cylinder, and they found that the coefficient of friction has the greatest significance on the centrifuging speed. The one-dimensional analysis for magneto-thermo-mechanical response in a functionally graded annular variable-thickness rotating disk was presented by Bayat *et al.* [19]. Selvamani and Ponnusamy [20] investigated the effect of rotation in an axisymmetric vibration of a transversely isotropic solid bar immersed in an inviscid fluid. Wave propagation in a generalized piezothermoelastic rotating bar of circular cross-section was discussed by Selvamani and Ponnusamy [21] using Bessel functions. A long-wave model for the surface elastic wave in a coated half-space was investigated by Dai *et al.* [22]. Wang [23] discussed the axi-symmetric wave propagation in a cylinder coated with a piezoelectric layer. Research on its application for time-delay devices was investigated by Sun and Cheng [24]. A theoretical model of the coated structure was investigated by Minagawa [25] to predict attenuation characteristics for finding suitable modes for a guided wave inspection Viscous film flows coating the interior of a tube were discussed by Camassa and Ogrosky [26] for thin-film and long-wave models. They concluded the important contribution of cylindrical geometry to the behavior of the film flow. Barshinger [27] investigated the guided waves in pipes with viscoelastic coatings. He found that the presence of attenuative, viscoelastic coatings causes significant problems for developing a guided-wave, nondestructive inspection of coated pipes. Piezoelectric $BaTiO_3$ thin film nano-generator on plastic substrates was constructed by Park *et al.* [28]. Their results show that a nanogenerator can be used to power flexible displays by means of mechanical agitations for future touchable display technologies.

In this paper, the effect of rotation on axisymmetric waves of piezoelectric rod coated with a thin film is studied using the constitutive equation of linear theory of elasticity and piezoelectricity. Potential functions are introduced to uncouple the equations of motion. The surface area of the rod is coated by a perfectly conducting material. The frequency equations are obtained for longitudinal and flexural modes of vibration and are studied numerically for PZT-4 ceramics. The non-dimensional frequency, phase velocity, relative frequency shift, electromechanical coupling and electric displacement are investigated and presented as dispersion curves.

2. Modeling of the problem

A homogeneous transversely isotropic piezoelectric circular rod of infinite length coated by a thin film is considered for this problem. The medium is assumed to be rotating with uniform angular velocity $\mathbf{\Omega}$. The displacement equation of motion has the additional terms with a time-dependent centripetal acceleration $\mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{u})$ and $2(\mathbf{\Omega} \times \mathbf{u}_{,t})$, where $\mathbf{u} = (u, 0, w)$ is the displacement vector and $\mathbf{\Omega} = (0, \Omega, 0)$ is the angular velocity. The governing equations of axisymmetric motion in the absence of body forces with the coordinates (r, z) are given as

$$\begin{aligned} &\frac{\partial}{\partial r}\sigma_{rr} + \frac{\partial}{\partial z}\sigma_{rz} + \frac{\sigma_{rr}}{r} + \rho(\mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{u}) + 2(\mathbf{\Omega} \times \mathbf{u}_{,t}))_r = \rho\frac{\partial^2 u_r}{\partial t^2}, \\ &\frac{\partial}{\partial r}\sigma_{rz} + \frac{\partial}{\partial z}\sigma_{zz} + \frac{\sigma_{rz}}{r} + \rho(\mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{u}) + 2(\mathbf{\Omega} \times \mathbf{u}_{,t}))_z = \rho\frac{\partial^2 u_z}{\partial t^2}. \end{aligned} \tag{2.1}$$

The Gauss electric conduction equation without free charge is taken as

$$\frac{1}{r}\frac{\partial}{\partial r}(rD_r) + \frac{\partial D_z}{\partial r} = 0. \tag{2.2}$$

The coupled form of stress equations in axisymmetric directions are given as

$$\begin{aligned} \sigma_{rr} &= c_{11}e_{rr} + c_{12}e_{\theta\theta} + c_{13}e_{zz} - e_{31}E_z, \\ \sigma_{zz} &= c_{13}e_{rr} + c_{13}e_{\theta\theta} + c_{33}e_{zz} - e_{33}E_z, \\ \sigma_{rz} &= 2c_{44}e_{rz} - e_{15}E_r, \end{aligned} \tag{2.3}$$

$$\begin{aligned} D_r &= e_{15}e_{rz} + \varepsilon_{11}E_r, \\ D_z &= e_{31}(e_{rr} + e_{\theta\theta}) + e_{33}e_{zz} + \varepsilon_{33}E_z, \end{aligned} \tag{2.4}$$

where σ_{rr}, $\sigma_{\theta\theta}$, σ_{zz}, $\sigma_{r\theta}$, $\sigma_{\theta z}$, σ_{rz} are the stress components, e_{rr}, $e_{\theta\theta}$, e_{zz}, $e_{r\theta}$, $e_{\theta z}$, e_{rz} are the strain components, c_{11}, c_{12}, c_{13}, c_{33}, c_{44}, $c_{66} = (c_{11} - c_{12})/2$ are the elastic constants, e_{31}, e_{15}, e_{33} are the piezoelectric constants, ε_{11}, ε_{33} are the dielectric constants, and ρ is the mass density.

The strain e_{ij} and the displacements are related as

$$\begin{aligned} &e_{rr} = u_{r,r}, \qquad e_{\theta\theta} = r^{-1}(u_r + u_{\theta,\theta}), \qquad e_{zz} = u_{z,z}, \\ &e_{r\theta} = u_{\theta,r} + r^{-1}(u_{r,\theta} - u_\theta), \qquad e_{z\theta} = \left(u_{\theta,z} + r^{-1}u_{z,\theta}\right), \\ &e_{rz} = u_{z,r} + u_{r,z}, \end{aligned} \tag{2.5}$$

where u_r, u_θ, u_z are the displacements along radial, circumferential and axial directions, respectively. The comma in the subscripts denotes the partial differentiation with respect to the variables. Substituting Eqs. (2.3), (2.4) and (2.5) in Eqs. (2.1) and (2.2), results in the following three-dimensional equations of motion and electric conduction equation:

$$
\begin{aligned}
&c_{11}\left(u_{rr,r}+r^{-1}u_{r,r}-r^{-2}u_r\right)+c_{44}u_{r,zz}+\left(c_{44}+c_{13}\right)u_{z,rz}\\
&\qquad\qquad+\left(e_{31}+e_{15}\right)V_{,rz}+\rho\left(\Omega^2u+2\Omega w_{,t}\right)=\rho u_{r,tt},\\
&c_{44}\left(u_{z,rr}+r^{-1}u_{z,r}\right)+r^{-1}\left(c_{44}+c_{13}\right)\left(u_{r,z}\right)+\left(c_{44}+c_{13}\right)u_{r,rz}\\
&\qquad+c_{33}u_{z,zz}+e_{33}V_{,zz}+e_{15}\left(V_{,rr}+r^{-1}V_{,r}\right)+\rho\left(\Omega^2u+2\Omega w_{,t}\right)=\rho u_{z,tt},\\
&e_{15}\left(u_{z,rr}+r^{-1}u_{z,r}\right)+\left(e_{31}+e_{15}\right)\left(u_{r,zr}+r^{-1}u_{r,z}\right)\\
&\qquad\qquad+e_{33}u_{z,zz}-\varepsilon_{33}V_{,zz}-\varepsilon_{11}\left(V_{,rr}+r^{-1}V_{,r}\right)=0.
\end{aligned}
\tag{2.6}
$$

3. Solutions of the field equation

Equation (2.6) represents the coupled partial differential equations of the two displacement components. To obtain the propagation of free harmonic electroelastic waves in the piezoelectric circular rod, we assume the solutions of the displacement components to be expressed in terms of derivatives of potentials from Paul and Venkatesan [8]

$$
\begin{aligned}
u_r\left(r,z,t\right)&=\left(\phi_{,r}\right)\mathrm{e}^{i(kz+\omega t)},\\
u_z\left(r,z,t\right)&=\left(\frac{i}{a}\right)W\mathrm{e}^{i(kz+\omega t)},\\
V\left(r,z,t\right)&=iV\mathrm{e}^{i(kz+\omega t)},\\
E_r\left(r,z,t\right)&=-E_{,r}\mathrm{e}^{i(kz+\omega t)},\\
E_z\left(r,z,t\right)&=E_{,z}\mathrm{e}^{i(kz+\omega t)},
\end{aligned}
\tag{3.1}
$$

where $i=\sqrt{-1}$, k is the wave number, ω is the angular frequency, $\phi(r)$, $W(r)$ are the displacement potentials, $V\left(r,\theta\right)$ is the electric potentials, and a is the geometrical parameter of the rod.

By introducing the dimensionless quantities such as $x=r/a$, $\zeta=ka$, $\varpi^2=\rho\omega^2a^2/c_{44}$, $\bar{c}_{11}=c_{11}/c_{44}$, $\bar{c}_{13}=c_{13}/c_{44}$, $\bar{c}_{33}=c_{33}/c_{44}$, $\bar{c}_{66}=c_{66}/c_{44}\bar{\varepsilon}_{11}=\varepsilon_{11}c_{44}/e_{33}^2$, $\bar{e}_{31}=e_{31}/e_{33}$, $\bar{e}_{15}=e_{15}/e_{33}$, $\mathit{\Gamma}=\frac{\rho\mathit{\Omega}^2R^2}{2+\bar{\lambda}}$ and substituting Eq. (3.1) in Eq. (2.6), we can get the following non-dimensional form of equations:

$$\left(\bar{c}_{11}\nabla^2+\left(\varpi^2+\mathit{\Gamma}-\zeta^2\right)\right)\phi-\zeta\left(1+\bar{c}_{13}\right)W-\zeta\left(\bar{e}_{31}+\bar{e}_{15}\right)V=0,$$

$$\zeta\left(1+\bar{c}_{13}\right)\nabla^2\phi+\left(\nabla^2+\left(\varpi^2+\mathit{\Gamma}-\zeta^2\bar{c}_{33}\right)\right)W+\left(\bar{e}_{15}\nabla^2-\zeta^2\right)V=0, \tag{3.2}$$

$$\zeta\left(\bar{e}_{31}+\bar{e}_{15}\right)\nabla^2\phi+\left(\bar{e}_{15}\nabla^2-\zeta^2\right)W+\left(\zeta^2\bar{\varepsilon}_{33}-\bar{\varepsilon}_{11}\nabla^2\right)V=0,$$

where

$$\nabla^2=\frac{\partial^2}{\partial x^2}+x^{-1}\frac{\partial}{\partial x}+x^{-2}\frac{\partial^2}{\partial\theta^2}. \tag{3.3}$$

Equation (3.2) can be rewritten in the following determinant form:

$$\begin{vmatrix} a^* & -\zeta\left(1+\bar{c}_{13}\right) & -\zeta\left(\bar{e}_{31}+\bar{e}_{15}\right) \\ \zeta\left(1+\bar{c}_{13}\right)\nabla^2 & b^* & \left(\bar{e}_{15}\nabla^2-\zeta^2\right) \\ \zeta\left(\bar{e}_{31}+\bar{e}_{15}\right)\nabla^2 & \left(\bar{e}_{15}\nabla^2-\zeta^2\right) & \left(\zeta^2\bar{\varepsilon}_{33}-\bar{\varepsilon}_{11}\nabla^2\right) \end{vmatrix}(\phi,W,V)=0, \tag{3.4}$$

where

$$a^*=\left(\bar{c}_{11}\nabla^2+\left(\varpi^2+\Gamma-\zeta^2\right)\right),$$

$$b^*=\left(\nabla^2+\left(\varpi^2+\Gamma-\zeta^2\bar{c}_{33}\right)\right).$$

Evaluating the determinant given in Eq. (3.4), we obtain the following differential equation:

$$\left(P\nabla^6+Q\nabla^4+R\nabla^2+S\right)(\phi,W,V)=0, \tag{3.5}$$

where

$$P=c_{11}\left(\bar{e}_{15}^2+\varepsilon_{11}\right),$$

$$Q=\left[\left(1+\bar{c}_{11}\right)\bar{\varepsilon}_{11}+\bar{e}_{15}^2\right]\varpi^2+\left\{\begin{array}{l} 2\left(\bar{e}_{31}+\bar{e}_{15}\right)\bar{c}_{13}\bar{e}_{15}-\left(1+\bar{\varepsilon}_{11}\bar{c}_{33}\right)\bar{c}_{11} \\ +\bar{c}_{13}^2\bar{\varepsilon}_{11}+2\bar{c}_{13}\bar{\varepsilon}_{11}-2\bar{e}_{15}\bar{c}_{11}+2\bar{e}_{13}^2 \end{array}\right\}\varsigma^2,$$

$$\begin{aligned}R=\bar{\varepsilon}_{11}\varpi^4-\left[\left(1+\bar{c}_{13}\right)\bar{\varepsilon}_{11}+\left(1+\bar{c}_{11}\right)+\left(\bar{e}_{31}+\bar{e}_{15}\right)+2\bar{e}_{15}\right]\varsigma^2\varpi^2 \\ +\left\{\bar{c}_{11}\left(1+\bar{c}_{33}\bar{\varepsilon}_{33}\right)-\left[\left(\bar{e}_{31}+\bar{e}_{15}\right)^2+\bar{\varepsilon}_{11}\right]\right. \\ \left.-2\bar{e}_{31}\left(1+\bar{c}_{13}\right)-\bar{c}_{13}\bar{\varepsilon}_{33}\left(\bar{c}_{33}+\bar{c}_{13}\right)+2\bar{e}_{15}\right\}\varsigma^4,\end{aligned}$$

$$S=-\left\{\left(1+\bar{c}_{33}\right)\varsigma^6-\left[2\left(1+\bar{c}_{33}\right)\bar{\varepsilon}_{33}+1\right]\varsigma^4\varpi^2+\bar{\varepsilon}_{33}\varsigma^2\varpi^4\right\}.$$

Solving Eq. (3.5), we get the solutions for a piezoelectric circular rod as

$$\phi = \sum_{i=1}^{3} A_i J_n\left(\alpha_i a x\right)\cos n\theta, \qquad W = \sum_{i=1}^{3} a_i A_i J_n\left(\alpha_i a x\right)\cos n\theta,$$

(3.6)

$$V = \sum_{i=1}^{3} b_i A_i J_n\left(\alpha_i a x\right)\cos n\theta,$$

where $\left(\alpha_i a\right)^2 > 0$ $(i = 1, 2, 3)$ are the roots of the algebraic equation

$$A\left(\alpha_i a\right)^6 - B\left(\alpha_i a\right)^4 + C\left(\alpha_i a\right)^2 + D = 0. \tag{3.7}$$

The Bessel function J_n is used when the roots $\left(\alpha_i a\right)^2 (i = 1, 2, 3)$ are real or complex and the modified Bessel function I_n is used when the roots $\left(\alpha_i a\right)^2$ $(i = 1, 2, 3)$ are imaginary. If $\left(\alpha_4 a\right)^2 < 0$, the Bessel function J_n is replaced by the modified Bessel function I_n.

The constants a_i, b_i defined in Eq. (3.6) can be calculated from the following equations:

$$\left(1 + \bar{c}_{13}\right)\varsigma a_i + \left(\bar{e}_{31} + \bar{e}_{15}\right)\varsigma b_i = -\left(\bar{c}_{11}\left(\alpha_i a\right)^2 - \varpi^2 - \mathit{\Gamma} + \varsigma^2\right), \tag{3.8}$$

$$\left(\left(\alpha_i a\right)^2 - \varpi^2 - \mathit{\Gamma} + \varsigma^2 \bar{c}_{33}\right) a_i + \left(\bar{e}_{15}\left(\alpha_i a\right)^2 + \varsigma^2\right) b_i = -\left(\bar{c}_{13} + 1\right)\varsigma\left(\alpha_i a\right)^2. \tag{3.9}$$

4. Boundary conditions and frequency equations

In this problem, the free axisymmetric vibration of the transversely isotropic piezoelectric rotating rod of circular cross-section coated with the thin film is considered. For the coated surface, the mechanical boundary conditions can be written as

$$\sigma_{rj} = -\delta_{jb}\, 2\mu' h' \left[\left(\frac{3\lambda' + 2\mu'}{\lambda' + 2\mu'}\right)\left(u_r\right)_{a,ab} + \left(u_r\right)_{b,aa}\right] + 2h'\rho'\left(\ddot{u}_r\right)_j$$

and the shorted electrical boundary condition is

$$V = 0, \tag{4.1}$$

where λ', μ', ρ' and h' are Lame's constants, density, and the thickness of the coated material, respectively, δ_{jb} is the Kronecker delta function in which a, b takes the value of θ and z, and j takes r, θ, and z. In order to get the axisymmetric waves a, b can take only z. Then, the transformed boundary conditions along an axisymmetric direction are as follows:

$$\sigma_{rr} = 2h'\rho'\ddot{u}_r,$$

$$\sigma_{rz} = -2h'\mu' G^2 W_{,zz} + 2h'\rho'\ddot{W}, \tag{4.2}$$

$$V = 0 \qquad \text{at} \qquad r = a,$$

where $G^2 = \frac{1+C'_{12}}{C'_{11}}$.

Substituting the solutions given in Eqs. (3.6), (3.9) in the boundary condition Eq. (4.2), we obtain a system of linear algebraic equations as follows:

$$[B]\{X\} = \{0\}, \tag{4.3}$$

where $[B]$ is a 3×3 matrix of unknown wave amplitudes, and $\{X\}$ is an 3×1 column vector of the unknown amplitude coefficients B_1, B_2, B_3. The solution of Eq. (4.2) is nontrivial when the determinant of the coefficient of the wave amplitudes $\{X\}$ vanishes, that is,

$$|B| = 0. \tag{4.4}$$

The components of $|B|$ are obtained as

$$B_{1i} = 2\overline{c}_{66}\left\{n(n-1) - \overline{c}_{11}(\alpha_i a)^2 - \varsigma(\overline{c}_{13}a_i + \overline{e}_{31}b_i)\right\}J_n(\alpha_i a) + 2\overline{c}_{66}(\alpha_i a)J_{n+1}(\alpha_i a), \quad i = 1,2,$$

$$B_{13} = 2\overline{c}_{66}n\{(n-1)J_n(\alpha_3 a) - (\alpha_3 a)J_{n+1}(\alpha_3 a)\},$$

$$B_{14} = 2(\alpha_4 a)\left[\left(\rho'h'\big/a\rho(ca)^2 - \overline{c_{66}}\right)\right]J_n(\alpha_4 a),$$

$$B_{2i} = 2n\{(n-1)J_n(\alpha_i a) + (\alpha_i a)J_{n+1}(\alpha_i a)\}, \quad i = 1,2,$$

$$B_{23} = \left\{\left[(\alpha_3 a)^2 - 2n(n-1)\right]J_n(\alpha_3 a) - 2(\alpha_3 a)J_{n+1}(\alpha_3 a)\right\},$$

$$B_{24} = 2(\alpha_4 a)\left[\left(\rho'h'\big/a\rho(ca)^2 - \overline{c_{66}}\right)\right]J_n(\alpha_4 a),$$

$$B_{3i} = ((\varsigma + a_i) + \overline{e}_{15}b_i)\{nJ_n(\alpha_i a) - (\alpha_i a)J_{n+1}(\alpha_i a)\}, \quad i = 1,2,$$

$$B_{33} = n\varsigma J_n(\alpha_3 a),$$

$$B_{34} = 0.$$

4.1. Relative frequency shift

Relative frequency shift plays an important role in the construction of rotating gyroscope, acoustic sensors and actuators. The frequency shift of the wave due to the rotation is defined as $\Delta\varpi = \varpi(\Omega) - \varpi(0)$. Ω is the angular rotation, and the relative frequency shift (RFS) is defined by

$$\text{RFS} = \left|\frac{\Delta\varpi}{\varpi}\right| = \left|\frac{\varpi(\Omega) - \varpi(0)}{\varpi(0)}\right|, \tag{4.5}$$

where $\varpi(0)$ is the frequency of the waves in the absence of rotation.

4.2. Electromechanical coupling

The electromechanical coupling (π^2) for a cylindrical rod is important for alteration of structural responses through applied electric fields in the design of sensors and surface acoustic damping wave filters. The electromechanical coupling is defined as

$$\pi^2 = \left|\frac{\Psi_e - \Psi_f}{\Psi_e}\right|, \tag{4.6}$$

where Ψ_e and Ψ_f are the phase velocities of the wave under electrically shorted and charge-free boundary conditions at the surface of the rod.

5. Numerical results and investigations

The frequency equation given in Eq. (4.4) is transcendental in nature with unknown frequency and wave number. The solutions of the frequency equation are obtained numerically by fixing the wave number. The material chosen for the numerical calculation is PZT-4 ceramics coated with a gold material. The material properties of PZT-4 and gold are taken from Berlincourt *et al.* [29]. The material constants are as follows:

$$c_{11} = 13.9 \cdot 10^{10}\ \text{N} \cdot \text{m}^{-2}, \qquad c_{12} = 7.78 \cdot 10^{10}\ \text{N} \cdot \text{m}^{-2},$$

$$c_{13} = 7.43 \cdot 10^{10}\ \text{N} \cdot \text{m}^{-2}, \qquad c_{33} = 11.5 \cdot 10^{10}\ \text{N} \cdot \text{m}^{-2},$$

$$c_{44} = 2.56 \cdot 10^{10}\ \text{N} \cdot \text{m}^{-2}, \qquad c_{66} = 3.06 \cdot 10^{10}\ \text{N} \cdot \text{m}^{-2},$$

$$e_{31} = -5.2\ \text{C} \cdot \text{m}^{-2}, \qquad e_{33} = 15.1\ \text{C} \cdot \text{m}^{-2},$$

$$e_{15} = 12.7\ \text{C} \cdot \text{m}^{-2}, \qquad \varepsilon_{11} = 6.46 \cdot 10^{-9}\ \text{C}^2 \cdot \text{N}^{-1} \cdot \text{m}^{-2},$$

$$\varepsilon_{33} = 5.62 \cdot 10^{-9}\ \text{C}^2 \cdot \text{N}^{-1} \cdot \text{m}^{-2}, \qquad \rho = 7500\ \text{kg} \cdot \text{m}^{-2}.$$

For the gold material $\rho' = 19.283 \quad \mathrm{g \cdot cm^{-3}}$, $\lambda' = 1.63 \cdot 10^{10}\ \mathrm{N \cdot m^{-2}}$, $\mu' = 0.42 \cdot 10^{10}\ \mathrm{N \cdot m^{-2}}$.

In the following dispersion curves, we discuss the results of non-dimensional frequency, wave number and electric displacement for longitudinal and flexural modes. The notation used in the figures, namely Lm, Fsm, and FAsm respectively denote the longitudinal mode, symmetric flexural mode and antisymmetric flexural mode. The first and second modes are represented by 1 and 2.

The influence of rotation is presented in the variation of non-dimensional frequency ϖ versus the dimensionless wave number $|\varsigma|$ for flexural modes of the piezoelectric circular rod with and without thin film coating in Figs. 1 and 2.

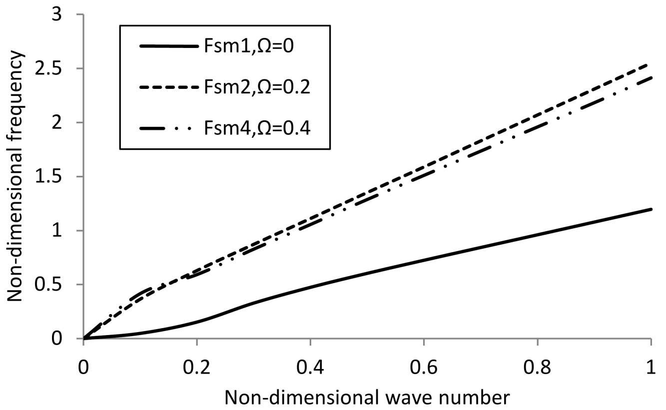

Fig. 1. Non-dimensional wave number $|\varsigma|$ versus non-dimensional frequency ϖ of flexural symmetric modes of the piezoelectric cylindrical rod with the coating.

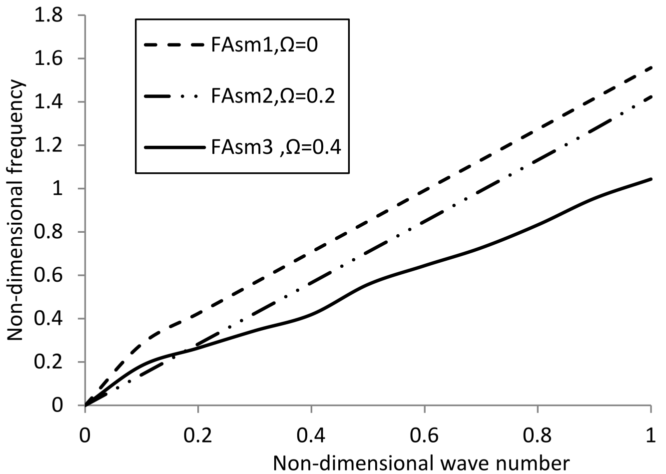

Fig. 2. Non-dimensional wave number $|\varsigma|$ versus non-dimensional frequency ϖ of flexural antisymmetric modes of the piezoelectric cylindrical rod with the coating.

Figure 1 shows the linear propagation of non-dimensional frequencies with respect to its wave number in different rotational speeds of the rod with the coating. But in Fig. 2, there is a small energy transfer between the modes in the lower range of wave number which might happen due to the coating and rotational effect of the rod. The rotation increases the magnitude of the non-dimensional frequency in Figs. 1 and 2. Figures 3 and 4 exhibit the dispersion of phase velocity modes with non-dimensional wave number for different rotational speeds in the absence of the coating. From Figs. 3 and 4, it is observed that, as the wave number increases the phase velocity is decreasing to the lower limit and travels in the wave propagation for $\Omega = 0.5$ rad/s in antisymmetric flexural mode. The cross-over point between the flexural modes of phase velocity shows that there is

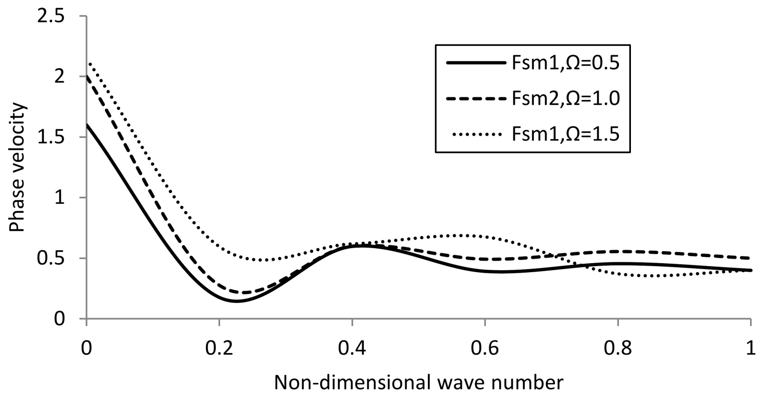

Fig. 3. Non-dimensional wave number $|\varsigma|$ versus phase velocity of flexural symmetric modes of the piezoelectric cylindrical rod without the coating.

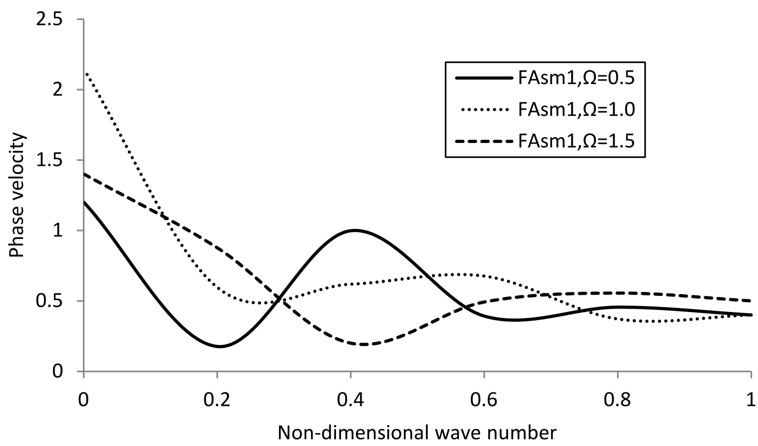

Fig. 4. Non-dimensional wave number $|\varsigma|$ versus phase velocity of flexural antisymmetric modes of the piezoelectric cylindrical rod without the coating.

variation in the energy level between the modes of vibrations due to the impact of rotation. The rotation decreases the magnitude of the phase velocity in the rod.

Figures 5 and 6 reveal that the variation of relative frequency shifts with the wave number $|\varsigma|$ for the longitudinal and flexural modes of the piezoelectric rod with and without the coating layer. The relative frequency shift is quite high at lower wave number (higher wavelength) and becomes steady with increasing wave number (lower wavelength). The relative frequency shift profiles are dispersive in trend for the rod with the coating and also, the symmetric modes are getting high amplitude in the rod without the coating. The crossover points between the

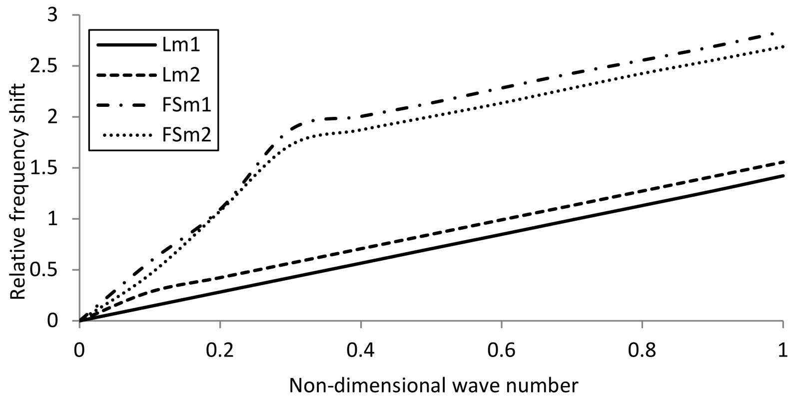

FIG. 5. Non-dimensional wave number $|\varsigma|$ versus relative frequency shift of the piezoelectric cylindrical rod with the coating.

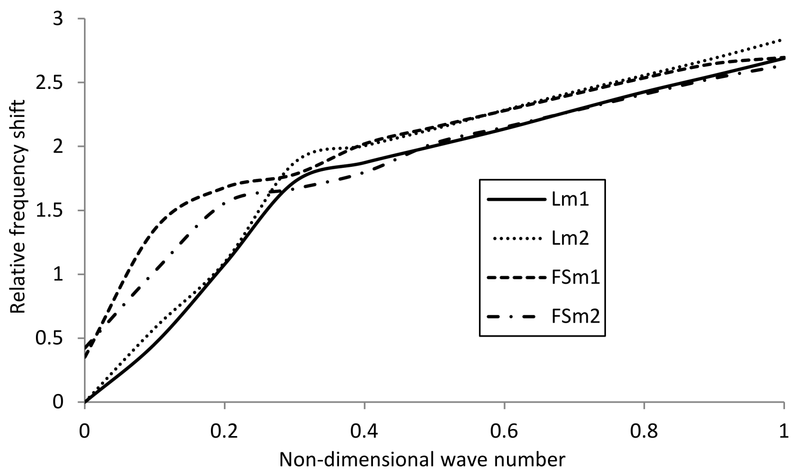

FIG. 6. Non-dimensional wave number $|\varsigma|$ versus relative frequency shift of piezoelectric cylindrical rod without coating.

vibration modes represents the transfer of energy due to the added mass effect of coating and rotational speed.

A comparison is made between the electro-mechanical coupling versus the wavenumber $|\varsigma|$ of a piezoelectric rotating rod for the longitudinal and flexural modes with and without the coating and is shown in Figs. 7 and 8. From Figs. 7 and 8, it is observed that both the Lm and Fsm mode increase as wave number increases. But in Fig. 8 the wave propagation merges for $|\varsigma| < 0.4$ and increases monotonically for the remaining range of wave number. Also, it is observed that in symmetric modes the coupling effect gets higher in the presence of the coating.

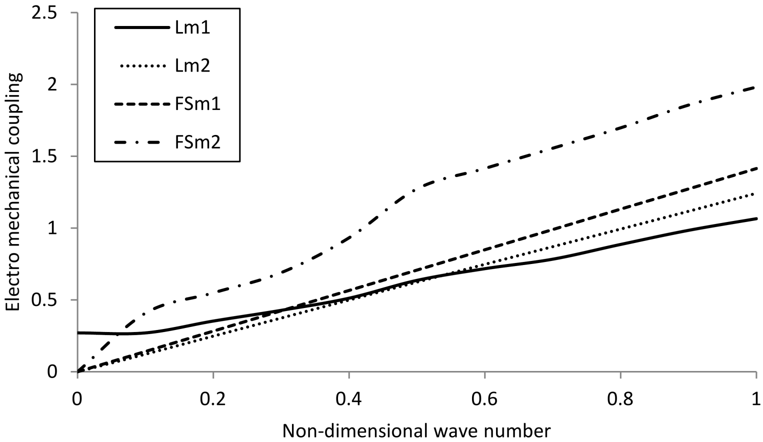

Fig. 7. Non-dimensional wave number $|\varsigma|$ versus electro mechanical coupling of the piezoelectric cylindrical rod with the coating.

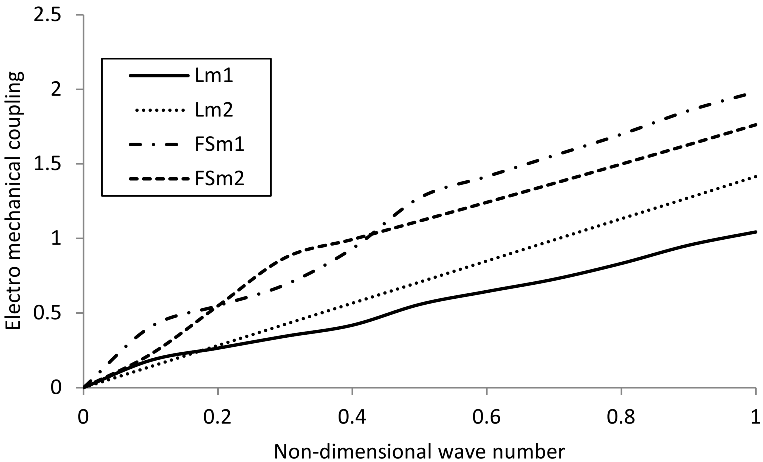

Fig. 8. Non-dimensional wave number $|\varsigma|$ versus electro mechanical coupling of the piezoelectric cylindrical rod without the coating.

This shows the strong bondage between the interfacial boundaries of the rod and the coating layer.

Figures 9–12 represent the propagation of electric displacement with respect to the thickness of the coated layer with different rotational speeds. Whenever the thickness of the rod increases the electric displacement is decreasing and again increasing and travels in the wave propagation. Also, it is noticed in all the figures that the trend of the curve is oscillating when the rotational speed increases. These trends of the curves admit the elastic properties of the solid due to rotational effect and coating of the material.

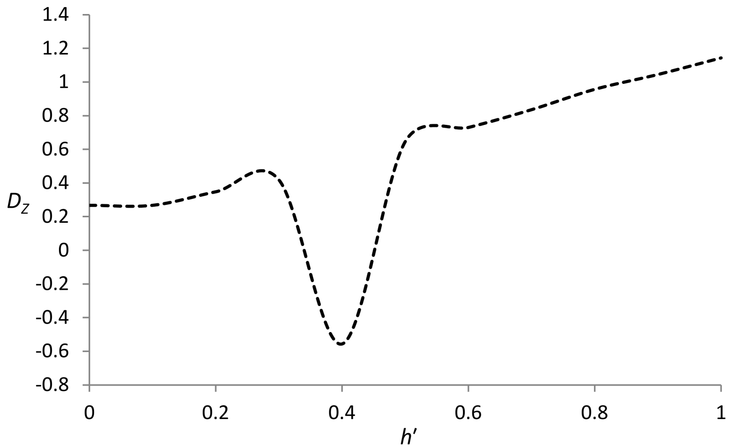

FIG. 9. Variation of electric displacement versus thickness of the coating material h' for $\Omega = 0$.

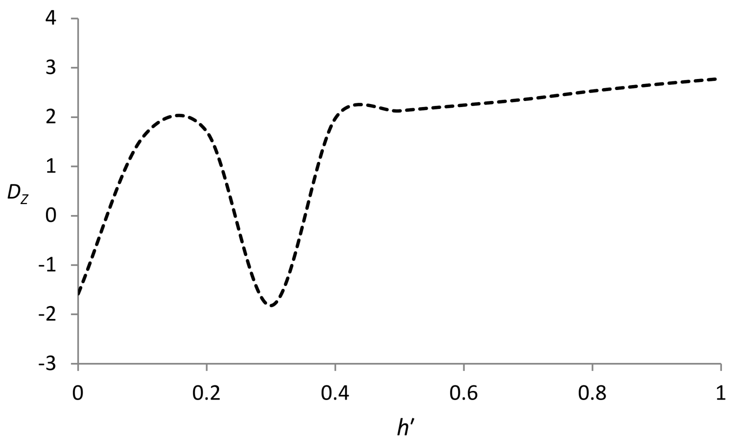

FIG. 10. Variation of electric displacement versus thickness of the coating material h' for $\Omega = 0.2$.

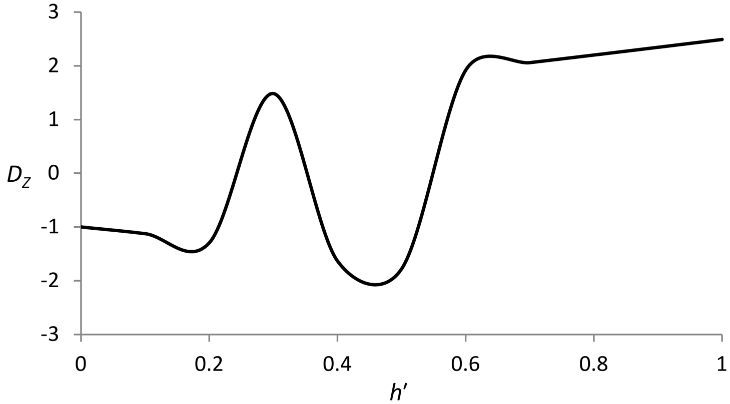

Fig. 11. Variation of electric displacement versus thickness of the coating material h' for $\Omega = 0.3$.

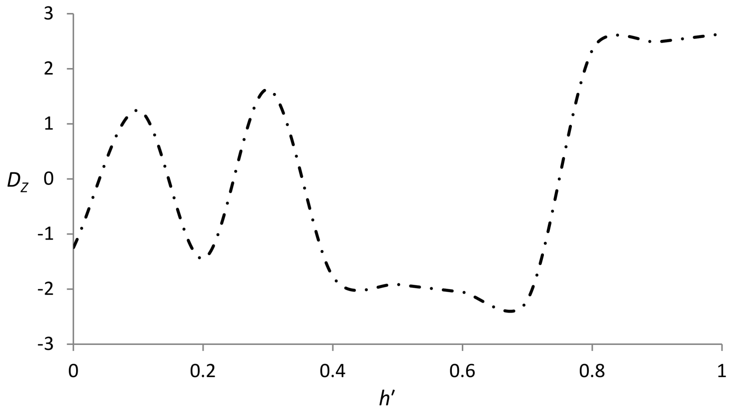

Fig. 12. Variation of electric displacement versus thickness of the coating material h' for $\Omega = 0.5$.

6. Conclusions

The influence of rotation in an axisymmetric wave propagation in a piezoelectric circular rod coated with the thin film is discussed using the three-dimensional theory of piezoelectricity. Three displacement potential functions are introduced to uncouple the equations of motion, electric conduction. The frequency equations are obtained for longitudinal and flexural modes of vibration and are studied numerically for the PZT-4 material rod with the gold coating. The computed non-dimensional frequency, phase velocity, relative frequency shift, electro-mechanical coupling and electric displacements are presented in the form of dispersion curves. From the graphical pattern, it is observed that the rotation and the coating of the piezoelectric rod are greatly influenced by the variations of the mechanical parameters in symmetric and anti-symmetric flexural modes. Also, numerical results show that the inclusion of thin film coating has a certain effect on the dispersion relation of wave propagation in the rotating piezoelectric rod. Thus in the design of rotating sensors which are used in navigation field, the coupling between the thin film coating and the sensors should be taken into account for constructive modeling

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